Prediction of Hydraulic Conductivity Function Parameters of Slurries Using Hybrid Metaheuristics Approach

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Abstract. The self-weight consolidation behavior of slurries like dredged clay and mine tailings is governed by the finite-strain consolidation theory. This theory considers the hydraulic conductivity and compressibility of slurries as functions of the void ratio. The determination of these material functions is essential for understanding the settlement behavior of the concerned materials which is necessary for the safe and efficient disposal of these materials. The experimental procedures for the determination of these functional relations need elaborate setups and are time-consuming. Therefore, it is convenient to adopt methods based on numerical techniques for this purpose. In this paper, an inverse analysis method is proposed for the estimation of hydraulic conductivity function parameters from the settlement versus time behavior and initial conditions of slurry materials obtained from the settling column test. The finite difference solution of the governing equation for finite-strain consolidation was used for the forward analysis. The inverse analysis was carried out using Particle Swarm Optimization (PSO) algorithm combined with a gradient-based optimization algorithm *fmincon*. The method was tested using the synthetic settlement response of two slurries. The shortcomings of *fmincon* and PSO algorithms were discussed. The proposed method of back analysis estimated the hydraulic conductivity function parameters accurately when the compressibility function is known and is assumed to be time-invariant.

Keywords: Hydraulic conductivity, finite-strain consolidation, Particle swarm optimization,

1 Introduction

The land is an exhaustible natural resource in huge demand and is depleting at a very fast rate all around the globe. Reclamation of lands by backfill using dredged clay slurries, for infrastructural development, is a practice observed near water bodies and offshore regions [12]. It is important to understand the consolidation behavior of the slurry material to ensure the safe construction and utilization of such reclaimed land. Global industries are heavily dependent on the extracted mineral resources for their functioning. Mining of minerals results in the production of tonnes of waste slurry called mine tailings which are stored in huge reservoirs called tailings storage facilities.

Such reservoirs hold on to a tremendous amount of water and land at the same time [10]. The most accurate method for estimating the speed of consolidation and the ultimate height of the waste deposits in these facilities has therefore been the subject of considerable research. Deposited slurries undergo very slow settlement under self-weight which may take years to complete. The self-weight consolidation behavior of slurries is governed by the finite strain consolidation theory [6]. The theory considers hydraulic conductivity and compressibility as functions of the void ratio.

These constitutive functions have a significant influence on the settlement behavior of soft deposits [3],[13]. Determination of hydraulic conductivity function from laboratory experiments such as large strain consolidation tests, seepage-induced consolidation tests, etc is tedious. The settlement of interface in a settling column test, on the other hand, is the simplest and most direct data that can be obtained from experiments on the slurries. Due to the high dependency of consolidation settlement values on the constitutive relationships, is possible to back-predict these functional parameters from the settlement data using the concepts of large strain consolidation [3],[13]. The parameters thus predicted can help in understanding the consolidation behavior of the corresponding slurries in the storage facility or land reclamation sites.

Such back prediction process has two components, a forward analysis method using the relationships governing the large strain consolidation and an effective parameter-search method to reduce the errors between numerical predictions and the available experimental results. Optimization algorithms such as the Nelder-Mead simplex algorithm [13], Particle Swarm Optimisation (PSO) [4],[3], Genetic Algorithm (GA) [9] gradient based algorithms [1], etc are generally used for inverse analysis problems. The swarm intelligence-based methods are effective in exploration within a domain but tend to converge prematurely [2]. The gradient-based methods are fast but need proper initial values to converge to the correct optimal solution [4]. A hybrid optimization technique that can combine the speed gradient-based method and exploration capabilities of swarm intelligence-based methods can effectively overcome the disadvantages of these methods and reduce the computational expense.

This study focuses on back-predicting the hydraulic conductivity function parameters from the settlement behavior of slurries when the compressibility function for the slurry is known and is assumed to be time-invariant. A Hybridized Particle Swarm Optimisation (HPSO) algorithm, combining the conventional PSO algorithm with the gradient-based method *fmincon* (constrained non-linear optimization technique), was used for the inverse analysis, in MATLAB. The forward analysis was carried out using the finite difference method. The inverse analysis method was tested on two synthetic settlement data cases. The data was also used to discuss about the disadvantages of using PSO and *fmincon*. An appreciable match was observed between the optimized parameter values and the actual values when HPSO was used. The settlement curves simulated using the precited parameters were observed to match well with the simulated synthetic settlement behavior.

2 Materials and Input Data

Self-weight consolidation behaviors of two slurries were considered in this study. The settlement versus time data along with material properties such as specific gravity, initial void ratio, depth of the sample, and compressibility function parameters of these materials are given in table 1.

Table 1. Relevant input properties of slurries

Material	Depth (m)	Specific gravity	Initial Void Ratio	Compressibility Function $e = A(\sigma' + B)^c$			Hydraulic Conductivity Function* $k = Me^P$	
				A (1/kPa)	B (kPa)	C	M (m/s)	P
MFT 5	0.30	2.65	6.00	3.27	0.07	-0.19	$2.50\text{x}10^{-11}$	3.39
Rutgers	0.50	2.58	4.89	2.86	0.13	-0.21	$2.00\text{x}10^{-11}$	6.00

* Fitted function is taken from literature [14],[7]

The inverse analysis technique using the HPSO algorithm was tested on two synthetic data sets. Properties from the literature for Matured fine tailings of oil sands (MFT 5) and dredged clay sediment (Rutgers) were used for the same. In the literature, the constitutive relationships for MFT 5 [14], and Rutgers [7] were obtained by conducting seepage-induced consolidation tests. For these two materials, the synthetic temporal settlement behaviors were simulated using the finite difference solution of the LSC governing equation. A maximum of 5%white noise was added to this numerically simulated settlement data so that the data can represent the experimental data. for using the input properties given in Table 1.

3 Forward Analysis using Finite Difference Method

The large strain consolidation (LSC) theory [6] overcomes the limitations of Terzaghi's one-dimensional consolidation theory by considering hydraulic conductivity and compressibility as functions of the void ratio. The relative motion of the solid and fluid phases was incorporated in Darcy–Gersevanov's relationship for flow through porous media to incorporate the self-weight factor. The differential equation governing the finite strain consolidation of slurries is given as

$$\left(\frac{\rho_s}{\rho_f} - 1\right)\frac{d}{de}\left[\frac{k(e)}{1+e}\right]\frac{\partial e}{\partial z} + \frac{\partial}{\partial z}\left[\frac{k(e)}{\rho_f(1+e)}\frac{d\sigma'(e)}{de}\frac{\partial e}{\partial z}\right] + \frac{\partial e}{\partial t} = 0 \quad (1)$$

where ρ_s is the density of solids in the slurry, ρ_f is the density of the pore fluid, e is the void ratio, $k(e)$ is the hydraulic conductivity function and $\sigma'(e)$ is the compressibility function. The equation (1) was derived using the material coordinate system [11] to account for the moving solid-fluid interface. z is the volume of solids in unit cross-sectional area between the point considered and the datum plane. Cargill [5] developed the finite difference solution for the LSC governing equation as given below.

$$e_{i,j} = e_{i,j-1} - \frac{\tau}{\gamma_w}\left[\begin{matrix}\left\{(\gamma_s - \gamma_w)\,\beta(e_{i,j-1}) + \frac{\alpha(e_{i+1,j-1}) - \alpha(e_{i-1,j-1})}{2\,\delta}\right\}\frac{e_{i+1,j-1} - e_{i-1,j-1}}{2\,\delta} + \\ \alpha(e_{i,j-1})\left\{\frac{e_{i+1,j-1} - 2\,e_{i,j-1} + e_{i-1,j-1}}{\delta^2}\right\}\end{matrix}\right] \quad (2)$$

where τ is the time-step, δ is the space-step in the reduced/material coordinate system, γ_w is the unit weight of water and γ_s is the unit weight of solids. The terms α and β are short-forms expanded as given in Equations (3) and (4).

$$\alpha(e) = \frac{k(e)}{1+e}\frac{d\sigma'}{de} \tag{3}$$

$$\beta(e) = \frac{d}{de}\left[\frac{k(e)}{1+e}\right] \tag{4}$$

From the finite difference solution, the void ratio distribution within the slurry can be derived at any point in time. The settlement at the corresponding time can be computed using the void ratio profile using the equation

$$S(z,t) = \int_0^z [1 + e(z,0)]dz - \int_0^z [1 + e(z,t)]dz \tag{5}$$

where S is the settlement, z is the depth of location in the material coordinate system and t is the time of settlement. The settlement versus time curves for the process of self-weight consolidation of slurries were numerically simulated using equation (5) during the inverse analysis using a MATLAB code. The input data required for the forward analysis include the specific gravity of solids, the initial void ratio distribution (assumed to be uniform throughout the depth), the initial height of the sample, the time of consolidation, the spacing of time and space meshes, the compressibility function, and the permeability function. For this study, a single drainage condition with free drainage at the top of the sample was taken into consideration. While setting the space step and time step values, the stability and convergence conditions for the numerical method was taken into due consideration. This ensures that the error due to truncation does not get magnified over successive steps.

4 Inverse Analysis using Slurry Settlement Data

Qi and Simms [13] observed that constant parameters of the hydraulic conductivity in the power influenced the shape and spread of the settlement versus time plots of large strain consolidation of slurries. The final settlement value is generally observed to be influenced by the compressibility function. Therefore, it is logical to back-predict these functions from the observed settlement behavior of slurries. The compressibility and hydraulic conductivity relationships were expressed as modified power and power form representations respectively as given by

$$e = A(\sigma' + B)^c \tag{6}$$

$$k = Me^P \tag{7}$$

Where A, B, C, M, and P are constant parameters. The inverse analysis for hydraulic conductivity function was initiated using the input data such as observed settlement versus time data from laboratory or field tests, time step value, space step value, specific gravity of the slurry solids, the compressibility function parameters, depth of sample and the initial void ratio distribution data. The function parameters M and P were predicted from these input data using the optimization algorithm.

The steps of inverse analysis can be explained as follows. The optimization algorithm considers the M and P as the variables to be optimized. Within prescribed bounds/domain, the algorithm generated random values and used these as inputs for the forward analysis algorithm. The forward analysis algorithm generated settlement versus time curves using these input parameters. The optimization algorithm works by comparing the settlement values generated using forward analysis and the settlement data given as input, to minimize the objective function, given as

$$O = \sqrt{\left(\frac{1}{N}\right) \sum_{j=1}^{N} \left(S_o^i - S_p^i\right)^2} \tag{8}$$

where S_o is the settlement value observed (input data) and S_p is the settlement value obtained from the forward analysis, corresponding to the t_i^{th} time point, for a set of data having N points. The objective function calculates the root mean square error (RMSE) between the predicted and observed settlement curves. The termination condition for the algorithm was the number of iterations. The optimization code was run three times to check the robustness of the algorithm.

5 Hybrid Particle Swarm Optimisation (HPSO) Algorithm

Particle swarm optimization (PSO) is an evolutionary algorithm that emulates the behavior of a group of animals such as a flock of birds or a school of fishes. It is a stochastic optimization strategy based on population introduced by Kennedy and Eberheart [8]. The algorithm initializes a population of particles randomly, within a prescribed domain or search space. Each of the particles has individual properties of position, velocity, and fitness. The position is the value of variables to be optimized, fitness is the value of the objective function at that position. These particles move towards the global best *(g_{best})* value of position using the velocity term while keeping track of their personal best value (p_{best}) continuously. According to its own experience and the experience of its environment, each particle seeks to enhance its performance, by updating its position and velocity. The update equations for the position and velocity values for each variable to be optimized are of the following form:

$$\begin{gathered} v_j(i+1) = w * v_j(i) + c_1 * rand(N) * (p_{best_j}(i) - p_j(i)) + \\ c_2 * rand(N) * (g_{best}\ (i) - p_j(i)) \end{gathered} \tag{9}$$

$$p_j(i+1) = p_j(i) + v_j(i+1) \tag{10}$$

where i is the number of the current iteration and j is the particle number, N is the number of variables to be optimized, $v_j(i+1)$ is the updated velocity value for the $(i+1)^{th}$ iteration for the j^{th} particle, w is the inertia weight used to control the influence of preceding particle velocities on the optimization process. This weight is damped by using a damping coefficient w_{damp} at the end of each iteration. The value $p_j(i)$ is the value of the position of the particle and $p_{best_j}(i)$ is the personal best of the particle till the current iteration. $g_{best}\ (i)$ is the global best value of the variable till the current iteration. c_1 is the personal learning coefficient and c_2 is the global learning coefficient. These coefficients control the influence of the personal best and global best on the optimization process. $p_j(i+1)$ is the updated position value for the particle considered.

Fmincon is a gradient-based constrained non-linear optimization algorithm available in the optimization toolbox of MATLAB, which can work for unconstrained problems as well. It is a fast method to find the minima of an optimization problem when a proper initial point can be provided. It can be accessed through function call and run with input details such as the initial point and the upper and lower bounds of the variables to be optimized, objective function, etc. The default algorithm of optimization is the interior-point algorithm.

For exploring the domain, stochastic optimization algorithms like the PSO algorithm use weighted random searching. Stochastic optimization techniques may explore flat areas and break out of local minima by using random searches, but they are computationally costly and have slower convergence rates. Deterministic algorithms, based on gradients, converge more quickly by determining a desirable search path using derivative information, but they tend to become trapped in local minima. Deterministic methods also struggle when attempting to minimise functions where the global minimum is surrounded by flat areas with little gradient. These methods' capability to eventually converge is highly dependent on the starting point. Therefore, a novel method called Hybrid Particle Swarm Optimisation (HPSO), combining the exploration capability of PSO and the capability of *fmincon* to converge fast, when a proper initial guess was provided, was developed.

The method runs PSO and *fmincon* algorithms alternately such that the global minimum value from PSO after a fixed number of iterations was considered as the initial guess value for running the Fmincon function. The global minimum was further updated based on the output after a fixed number of iterations of Fmincon. This reduces the computational expense for PSO allowing the algorithm to reach the location of optimal values faster, where exploration can be done better. The pseudocode for the HPSO algorithm is given below

Begin
Load data for inverse analysis
Initialize *swarm_size, maxitr, n, m, domain, N, w,* w_{damp}, c_1 and c_2
For each particle of the swarm
  Initialize *position* randomly within the *domain*
  Initialize *velocity* as zero
  Evaluate fitness using equation (8)
  Set p_{best} as the current position
  Set g_{best} based on values of p_{best}
End of for
Set *iteration =0;*
Do
  For iteration <n
  For each particle
  Update velocity using equation (9)
  Update position using equation (10)
    Evaluate fitness for updated positions
  Update p_{best}
  Update g_{best}
   End of for
   Update *iteration*
   Set *w=w*wdamp*
  End of for
  If *iteration + m > maxitr*
  m= maxitr - iteration
  End if
  Run *Fmincon* using the initial value as g_{best} for *m* iterations
  Update g_{best}

Update *iteration*
While (*iteration* < *maxitr*)
End of begin

6 Results and Discussion

The hydraulic conductivity function parameters of two slurries were predicted using the HPSO algorithm. The material properties given in Table 1 were used as input along with the settlement data points for the inverse analysis. The settlement versus time data for MFT5 and Rutgers were simulated numerically using the input data from Table 1, using the forward analysis code. To the settlement values, a random white noise, up to a maximum of +/- 5%, was added so that the data is not too smooth and can therefore simulate the behaviour of experimental data. This synthetic data was used as input for the inverse analysis.

Table 2. P and M values predicted using FMINCON using different initial guesses

Case	Initial guess		Iterations	RMSE	Optimized values	
	P	M			P	M
Rutgers	6.66	2.49E-12	25	0.741	6.41	1.51E-13
	10.00	6.91E-18	38	0.741	6.39	1.64E-13
	6.25	1.19E-11	10	1.4E07	2.96	7.03E-06
MFT 5	3.71	1.60E-11	35	0.167	3.55	2.14E-11
	8.00	5.00E-07	40	1.21E07	3.46	2.89E-06
	3.33	2.14E-08	88	1.23E07	3.33	3.67E-06

Table 2 shows the *M* and *P* values predicted using various initial guess values for MFT5 and Rutgers, using *fmincon*. It was observed that the optimised values were heavily dependent on the initial guess supplied. The predictions were well beyond the tolerance level, even when the values close to the actual values were used as initial guesses.

The PSO parameter c_1 was set to 1.5, c_2 was set to 2, inertia weight w was set to 1, and damping coefficient w_{damp} was set as 0.99. The swarm size used was 40. The algorithm was run for three runs, for 60 iterations. Figures 1(a) and (b) show the robustness issue generally observed when PSO algorithm is used for optimisation. PSO was observed to experience premature convergence in the case of MFT5, within the first few iterations itself. For the case of Rutgers, two runs converged to a minimum however, one failed to reach optima within 60 iterations. The average RMSE for three runs was observed to be 0.332 and 0.185 for Rutgers and MFT5 respectively.

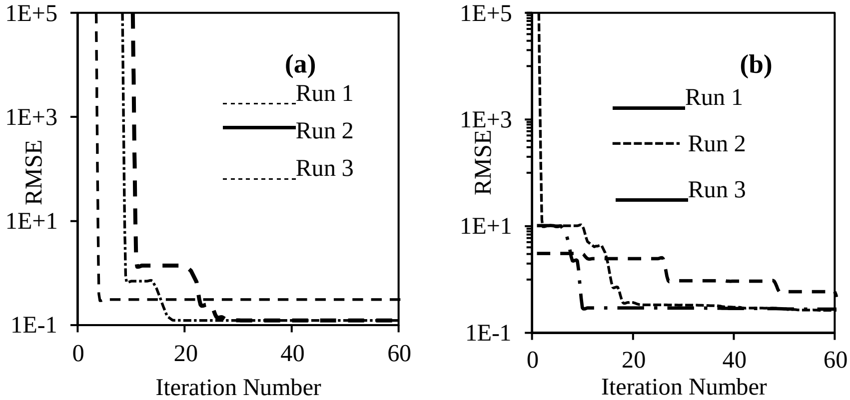

Figure 1. Iteration versus RMSE plots when PSO was used for optimization for the cases of (a) MFT5 (b) Rutgers

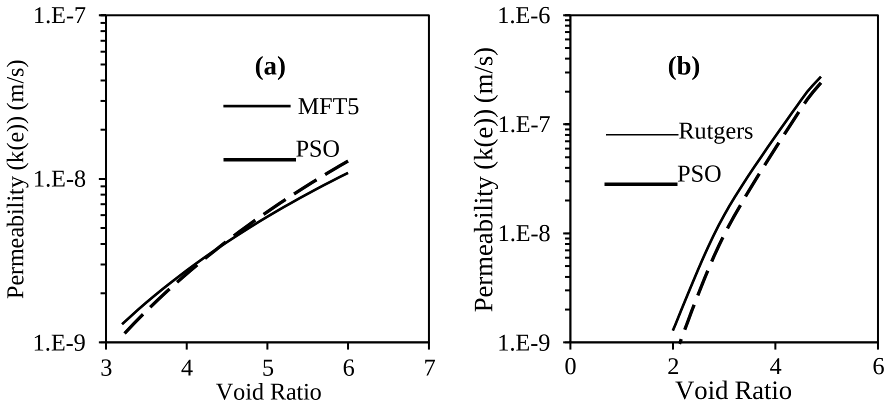

Figure 2 Comparison of the worst value of hydraulic conductivity function parameters predicted using PSO to the relation from corresponding literature of (a) MFT5 (b) Rutgers

Figure 2(a-b) compares the worst values of *M* and *P* predicted using PSO for the cases of MFT5 and Rutgers. The worst case of Rutgers had the predicted value of *P* as 6.585 and *M* as $6.937\text{x}10^{-12}$. The same for MFT5 had values of *P* and *M* as 3.918 and $1.147\text{x}10^{-11}$ respectively. Figures 3(a-b) show the comparison of the settlement curves simulated using the worst runs and the actual simulated synthetic data. It was observed that the curve simulated varied from the synthetic data points at a few places. The maximum error in settlement value predicted is 16.4% of actual/input value at around 71days for MFT5. The value of the same is 16.6% at 4 days time step for Rutgers.

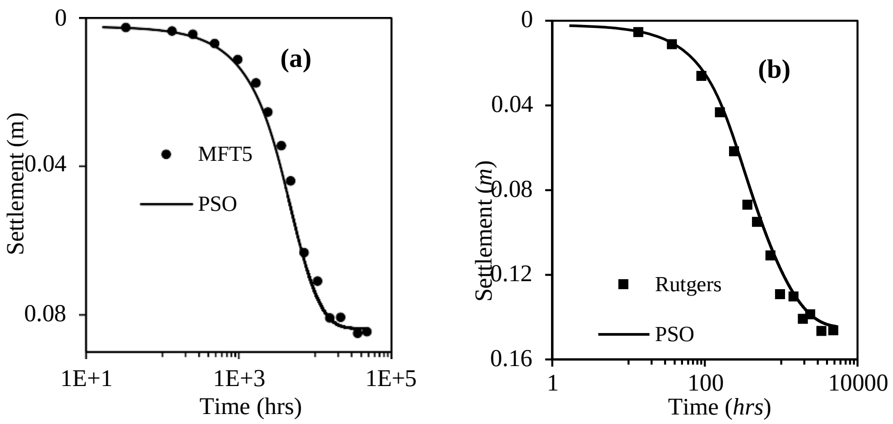

Figure 3. Comparison of settlement curves simulated using worst hydraulic conductivity function parameters predicted using PSO and the input settlement data for (a) MFT5 (b) Rutgers

The HPSO was developed to overcome the above-discussed shortcomings of *fmincon* and PSO. The PSO parameters were maintained at the values mentioned above. The number of iterations using PSO was set to 20 and the same for *fmincon* was set to 20 as well. A square domain was used, which was defined individually for each of the tested cases. The code was run thrice for each case to check the robustness, for a maximum of 100 iterations per run. Appropriate time and space step values for each case were given as input for the forward analysis to ensure stability and convergence criteria of the numerical technique.

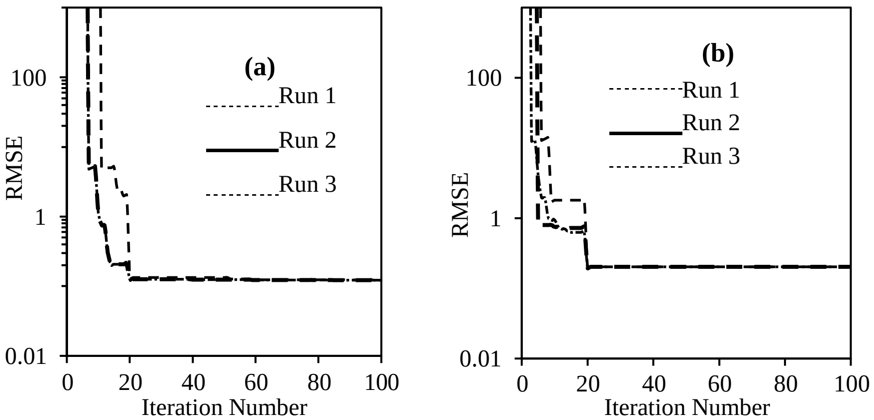

Figure 4. Iteration versus RMSE plots using HPSO for the cases of (a) MFT5 and (b) Rutgers

Therefore, the PSO algorithm initialized a random set of a population of 40 particles at the beginning of a run and proceeded to optimize for 20 iterations. The global best values at the end of 20 PSO iterations were used as the initial guess in the *fmincon* function. The function continued to optimise the objective function for the next 20 iterations, resulting in a possible change in the global optimum value. The overall number of iterations was 40 at that stage which is less by 60 iterations for a case having a maximum number of iterations set at 100. The PSO algorithm then continued its search based on the global optimum updated after the *fmincon*, for another 20 iterations after which the cycle continued till the overall iteration number reached the set maximum value.

Figure 4(a-b) shows the RMSE versus iteration number plot for all the cases. It was observed that the three runs consecutively resulted in the same or very close objective function values and optimal values for *M* and *P* for both the cases. In all three runs, the RMSE value had reached its minimum within 20 iterations suggesting that the initial guess values for *M* and *P* provided to *fmincon* after 20 PSO iterations were appropriate. In the first run of, MFT5 and Rutgers, the PSO was able to improve its prediction based on the output from *fmincon* after 40 iterations, as seen in Figures 4(a) and 4(b) respectively. All the three runs converged to a common minimum, at 20 iterations itself. The issue of robustness and problem of proper initial values for PSO and fmincon were overcome successfully by HPSO.

Table 3. Hydraulic conductivity function parameters predicted using HPSO

Material	Mean RMSE	Predicted Values	
		P	M (m/s)
MFT5	0.122	3.634	$1.62\text{x}10^{-11}$
Rutgers	0.259	6.237	$1.39\text{x}10^{-11}$

Figures 5(a) and 5(b) shows the comparison of the predicted *M* and *P* parameters with the function from the corresponding literature. Table 3 shows the optimal values of *M* and *P* predicted and the mean value of the objective function from three runs.

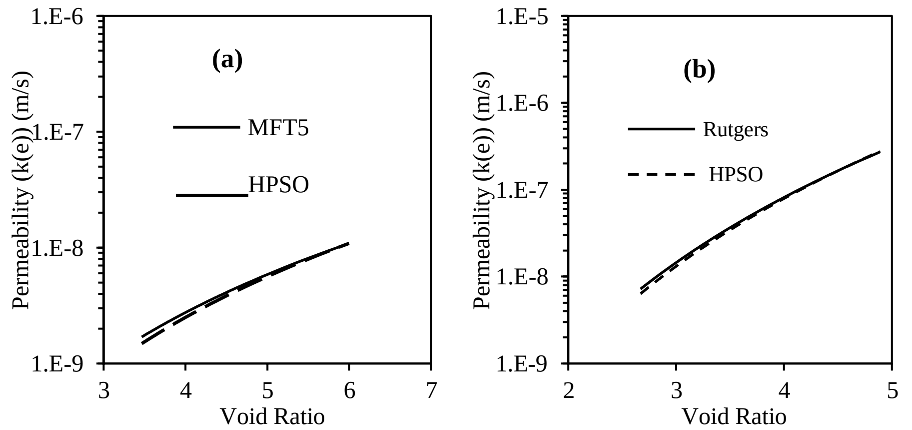

Figure 5. Comparison of hydraulic conductivity function predicted using HPSO to the relation from corresponding literature of (a) MFT5 (b) Rutgers

The predicted function was observed to match well with the actual functions. For the MFT 5 and Rutgers, the predicted functions slightly deviated from the actual at relatively lower void ratios. This could be because the input settlement data for these cases show a larger deviation towards the end of consolidation as shown in Figure 6(a) and Figure 6(b). The predicted function was over-predicting the hydraulic conductivity in the initial stages of consolidation that is at larger void ratios. The curves passed through the data points well for both cases. The maximum error in settlement value is 2.2% for MFT5 and 4.7% for Rutgers.

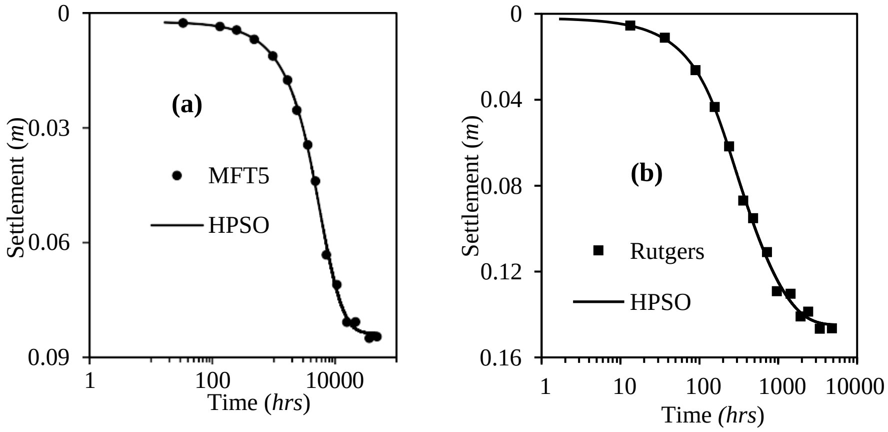

Figure 6. Comparison of settlement curves simulated using predicted hydraulic conductivity function from HPSO to the input settlement data for (a) MFT5 (b) Rutgers

7 Conclusions

Determination of hydraulic conductivity function is a regular process in the management of slurried waste materials. Many studies showed that the parameters of this constitutive function influenced the settlement behavior of the slurries significantly. Therefore it is very important to estimate the parameters accurately to ensure the safe and efficient use or disposal of slurries. These parameters can be estimated by inverse analysis using an optimization algorithm from the settlement data of slurries instead of resorting to time-consuming experimental procedures. Conventional algorithms tend to be computationally expensive, heavily dependent on the initial guess, or lack the robustness of prediction. The current study proposed a hybrid algorithm combining the best capabilities of the PSO algorithm and *fmincon* function in the optimisation toolbox of MATLAB. The study resulted in the following conclusions:

- The hybridization resulted in a better algorithm compared to the conventional PSO or *fmincon* function.
- The lack of robustness of conventional PSO was reduced and faster convergence was ensured by combining it with *fmincon.*
- PSO successfully provides an appropriate initial guess value for the *fmincon* function which helps *fmincon* to establish the best path towards optimal values.
- HPSO was observed to converge faster than PSO to the robust solution. Therefore, HPSO has reduced the run-time for inverse analysis.
- In one of the cases, the M value predicted by the PSO algorithm deviated from the real value by an order, although the same value was closer to the actual value when predicted using the HPSO algorithm.

- The inverse analysis using HPSO successfully predicted the hydraulic conductivity parameters from the settlement data when the compressibility function was known and was independent of time.
- The optimal parameter values from HPSO successfully simulated the observed settlement behavior of slurries

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