Pseudo-static Stability Analysis of Multilayered Slopes Using Sarma's Method of Non-Vertical Slices

Somenath Mukherjee[1], Subhadeep Metya[2*], and Gautam Bhattacharya[3]

[1] C.E. Testing Company Pvt. Ltd., Kolkata, West Bengal - 700092, India
[2] NIT Jamshedpur, Jharkhand - 831014, India
[3] IIEST, Shibpur. Howrah, West Bengal - 711103, India
smetya.ce@nitjsr.ac.in

Abstract. Stability analysis of multi-layered slopes with complex geometry and pore pressure conditions has always been a topic of immense interest to both the researchers and the practitioners in the field of geotechnical engineering. Over the decades, within the framework of limit equilibrium methods of slices, a number of rigorous methods have been proposed which are valid for general non-circular slip surfaces, satisfy all conditions of equilibrium, and take full account of the interslice forces. Among these the Sarma method is the only method in which the slices are not necessarily vertical, the critical inclinations of the slices are found as part of the solution, and uses the internal strength of material for the solution of the problem. Because of the number of iterations involved in finding the critical set of slice inclinations, Sarma method is not suitable for finding the critical slip surface. In order to compare the results obtained by using the other rigorous methods with those using Sarma's method, the latter can be used to reanalyze the critical slip surfaces determined for the other methods. In this study involving three complex slope stability problems, the critical slip surfaces determined using the Morgenstern and Price method and the Spencer method are re-evaluated using the Sarma method. The GEO 5 software is utilized for the purpose. The comparison of results have brought out that compared to the Sarma method, both the Morgenstern and Price and the Spencer method consistently yield conservative values of factor of safety for each of the three example problems.

Keywords: Slope stability analysis, Limit equilibrium method, Factor of safety, Critical slip surface, Sarma method, Pseudo-static approach of analysis.

1 Introduction

Slope stability analysis has over the decades remained a topic of overwhelming importance not only to the geotechnical practitioners but also to the researchers. It is widely known that failures of slopes are caused by ground movements such as falls, topples, slides, spreads, and flows. In engineered slopes, in particular, the slide type of ground movement is predominant and such a mechanism is more amenable to theoretical analysis as compared to the other types of ground movements (Huang, 1983).

Various approaches to solving slope stability problems include the limit equilibrium approach, the limit analysis approach, and the finite element approach. Among these, it is the limit equilibrium approach which has all along received the researchers' attention and earned almost exclusive preference of the practicing engineers.

The widely used rigorous methods of slices valid for slip surfaces of general noncircular or arbitrary shapes include the Morgenstern and Price method (1965), the Spencer method (1973), the Janbu's method (1973), and the Sarma method (1979). The Sarma method, however, is a class of its own. In this method, perhaps for the first time in the history of development of the limit equilibrium method of slices, the slices are not necessarily vertical and the critical inclinations of the interslice boundaries are found out as part of the solution. According to Sarma (1979), the reason for using inclined slices is that vertical slice interfaces are sometimes not suitable for an evaluation of internal stresses which is the real purpose of a sophisticated limit equilibrium analysis. Furthermore, this is the only method that uses the internal strength of material for the solution of the problem.

As commented by Sarma (1979), because of the large number of iterations involved in finding the critical set of inclinations of the interslice boundaries, the Sarma method is not really suitable for the analysis of sections where large numbers of probable slip surfaces have to be analyzed to determine the critical slip surface.

2 Critical Slip Surface using Optimization

The slope stability analysis within the framework of limit equilibrium approach is essentially a problem of optimization, namely, finding the critical slip surface having the minimum factor of safety. A number of softwares are currently available for efficiently carrying out this optimization. In this study, all slope stability computations are carried out using the geotechnical software GEO5. In this software, the optimization-based search for the critical slip surface proceeds through sequential changing of locations of discrete points on a slip surface and noting the change which results in the largest reduction in factor of safety. The two end points of the slip surface are moved along the slope surface while the remaining points are moved in the vertical and the horizontal directions. The initial step size is selected as one-tenth of the smallest distance between adjacent discrete points on the slip surface. In each of the subsequent cycles of search, the step size is reduced by one half. The locations of the discrete points from left to right are considered to be the optimum when there is negligible displacement of any of the points during the last cycle. For further details, the manual of the GEO5 software may be referred.

3 Illustrative Examples

In order to draw a comparison of results of slope stability analysis of multi-layered slopes based on some of the most widely used rigorous limit equilibrium methods of slices valid for general slip surfaces, three benchmark slope examples have been selected from the literature. These are described in the subsequent sections followed by their analyses and results as obtained by using the GEO5 software.

4 Example 1: Case Study of the Congress Street Cut

4.1 Description

This example concerns a failed slope reported in the literature pertaining to the case study of the 1952 historic Congress Street open cut in Chicago. As reported by Ireland (1954), in 1952 in Chicago, during the open cut construction of the Congress Street ‘superhighway’, in which deep benched cutting was undertaken in saturated glacial clay, a large rotational slope failure occurred on the south side of the cut for a length of about 60 m when the excavation reached a depth of nearly 14.3 m (Metya, 2017). An approximate cross-section of the cut at the time of failure, together with an approximate position of the observed failure surface was originally presented by Ireland (1954). Recently, Ji and Low (2012) have reported the dimensions of the cut at failure as in Fig. 1.

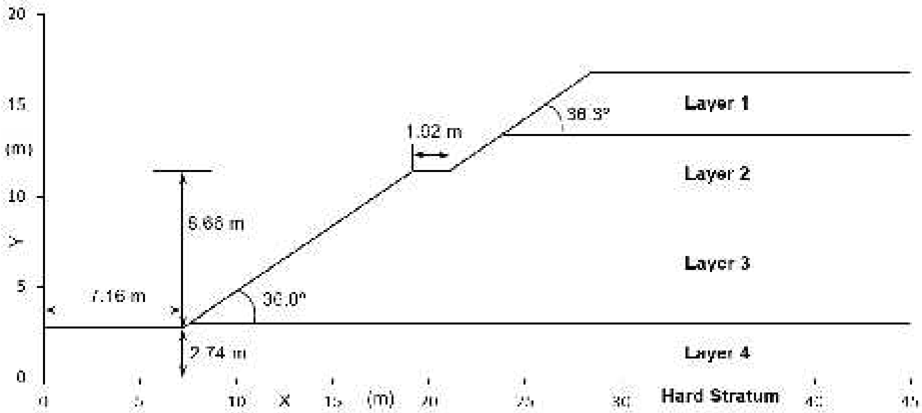

Fig. 1. Slope Section for Example 1: Cross-section of the Congress Street open cut [after Ji and Low (2012)].

According to Ji and Low (2012), the soil profile at the excavation site comprised a 3.35 m thick layer of sand and miscellaneous fill (layer 1) underlain by a 13.42 m thick layer of gritty blue clay. The failure occurred in this layer and is sub-divided into three sub-layers: a 4.27 m thick upper layer of stiff gritty blue clay (layer 2: Clay 1), a 6.1 m thick middle layer of medium gritty blue clay (layer 3: Clay 2), and a 3.05 m lower layer of medium gritty blue clay (layer 4: Clay 3). The layer of gritty blue clay is underlain by a layer of stiff to very stiff gritty blue clay (hard stratum). The water table was located at a depth of nearly 2.2 m from the slope top.

4.2 Present Study

As stated before, the purpose of the present study is to draw a comparison among the results of deterministic slope stability analyses carried out on the basis of three of the widely used limit equilibrium methods of slices valid for general slip surfaces. These three methods are: (1) the Morgenstern and Price method (Morgenstern and Price, 1965); (2) the Spencer method (Spencer, 1973); and (3) the Sarma method (Sarma, 1979). To this end, the following analyses have been carried out.

(i) Determination of the critical slip surface and the associated minimum factor of safety using the Morgenstern and Price method;
(ii) For the specific slip surface obtained in (i) above, determination of the factor of safety using the Sarma method;
(iii) Determination of the critical slip surface and the associated minimum factor of safety using the Spencer method
(iv) For the specific slip surface obtained in (iii) above, determination of the factor of safety using the Sarma method.

Soil Properties: Table 1 presents the soil properties considered in the present study. These are taken from Ji and Low (2012). The values of unit weight are, however, taken from Liang et al. (1999).

Table 1. Soil Properties for Example 1 [Ji and Low (2012); Liang et al. (1999)]

Layer	Material	Parameter	Unit	Value
1	3.35 m Sand and Miscellaneous Fill	c_1	kN/m^2	0.00
		ϕ_1	degree	30.00
		γ_1	kN/m^2	17.28
2	4.27 m Stiff Gritty Blue Clay (Clay 1)	c_2	kN/m^2	55.00
		ϕ_2	degree	0.00
		γ_2	kN/m^2	20.75
3	6.10 m Medium Gritty Blue Clay (Clay 2)	c_3	kN/m^2	43.00
		ϕ_3	degree	0.00
		γ_3	kN/m^2	20.11
4	3.05 m Medium Gritty Blue Clay (Clay3)	c_4	kN/m^2	56.00
		ϕ_4	degree	0.00
		γ_4	kN/m^2	20.11

Analysis I: Critical Slip Surface by Morgenstern & Price Method

Based on the Morgenstern and Price (1965) method, the critical slip surfaces and the associated minimum factors of safety (F_{min}) have been determined for the seismic coefficient K_h ranging from 0.0 to 0.2. The total number of slices considered in the analysis is 10. The values of F_{min} are presented in Table 2. A typical critical slip surface, for the seismic coefficients K_h = 0.10 is shown in Fig. 2.

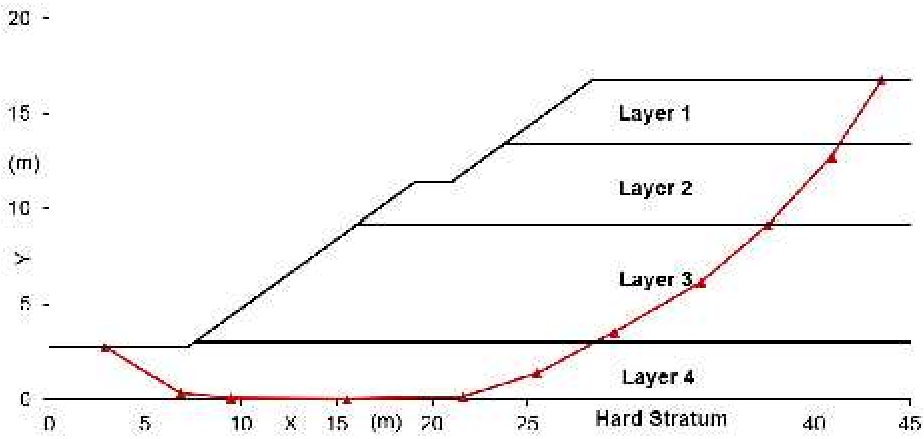

Fig. 2. Critical Slip Surface by Morgenstern and Price Method for Example 1 (K_h = 0.10)

Analysis II: Critical Slip Surface by Spencer Method

Based on the Spencer Based on the Spencer (1973) method, the critical slip surfaces and the associated minimum factors of safety (F_{min}) have been determined for the seismic coefficient K_h ranging from 0.0 to 0.2. The values of F_{min} are presented in Table 2 (column 3). A typical critical slip surface, for the seismic coefficients K_h = 0.10, is shown in Fig. 3.

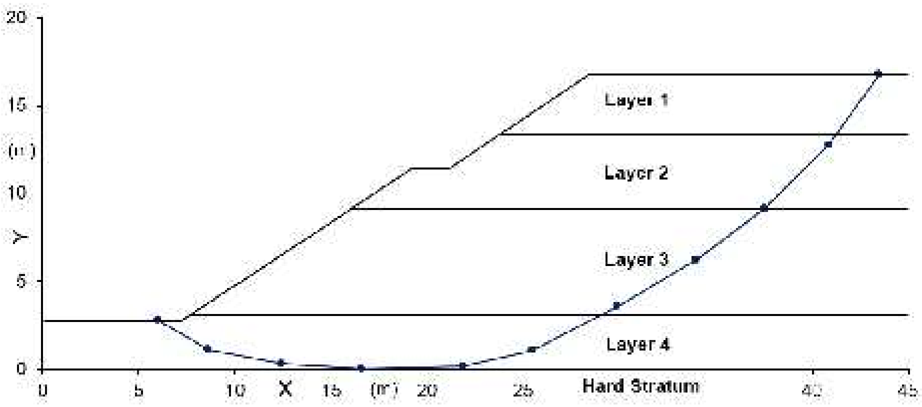

Fig. 3. Critical Slip Surface by Spencer Method for Example 1 (K_h = 0.10).

Analysis III: Evaluation of both the Critical Slip Surfaces by Sarma Method

Based on the Sarma method, the critical slip surfaces obtained based on the Morgenstern and Price method and the Spencer method have been re-evaluated, using the software GEO5. The results are summarized in Table 2 and discussed in Section 7.

Table 2. Summary of Results for Example 1.

K_h	Values of F_{min} by the Morgenstern and Price method (1)	Values of F by the Sarma method corresponding to col. (1) (2)	Values of F_{min} by the Spencer method (3)	Values of F by the Sarma method corresponding to col. (3) (4)
0.00	1.04	1.24	1.06	1.22
0.05	0.96	1.14	0.99	1.11
0.10	0.92	1.02	0.92	1.02
0.15	0.80	0.94	0.85	0.94
0.20	0.74	0.87	0.78	0.85

5 Example 2: A Multilayered Slope (Zhu and Lee, 2002)

5.1 Description

Example 2 concerns another multilayered slope comprising four layers. The slope section is shown in Fig. 4 which is similar to that considered by Zhu and Lee (2002) for purposes of illustrating the application of a new limit equilibrium method of analysis proposed by the authors. The soil properties are as given in Table 3.

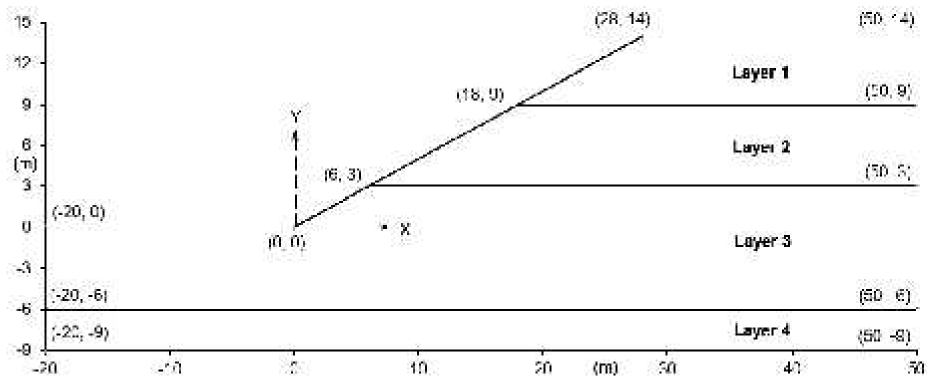

Fig. 4. Slope Section in Example 2 [Similar to the Slope section in Zhu and Lee (2002)].

Table 3. Soil Properties for Example 2 [Zhu and Lee (2002)]

Layer	Parameter	Unit	Value	Layer	Parameter	Unit	Value
	c_1	kN/m^2	20.00		c_3	kN/m^2	25.00
1	ϕ_1	degree	18.00	3	ϕ_3	degree	26.00
	γ_1	kN/m^2	18.80		γ_3	kN/m^2	18.4
	c_2	kN/m^2	40.00	4	c_4	kN/m^2	10.00
2	ϕ_2	degree	22.00		ϕ_4	degree	12.00
	γ_2	kN/m^2	18.50		γ_4	kN/m^2	18.00

Analysis I: Critical Slip Surface by Morgenstern & Price Method
Based on the Morgenstern and Price (1965) method, the critical slip surfaces and the associated minimum factors of safety (F_{min}) have been determined for the seismic coefficient K_h ranging from 0.0 to 0.2 using the software GEO5. The total number of slices considered in the analysis is 10. The values of F_{min} are presented in Table 4. A typical critical slip surface, for the seismic coefficients K_h = 0.2 is shown in Fig. 5.

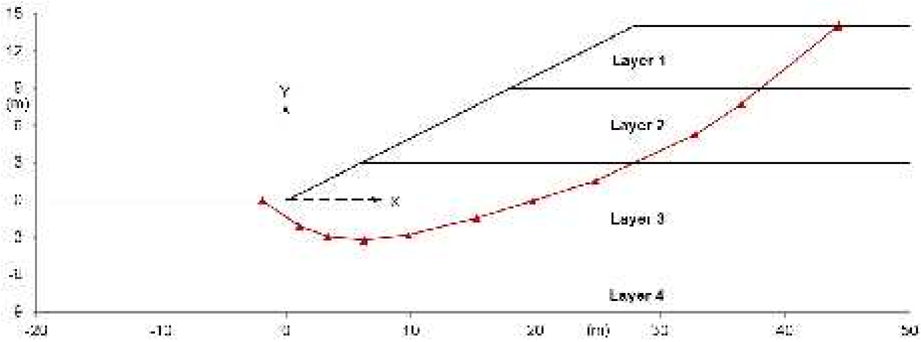

Fig. 5. Critical Slip Surface by Morgenstern & Price Method for Example 2 (K_h = 0.20)

Analysis II: Critical Slip Surface by Spencer Method
Based on the Spencer Based on the Spencer (1973) method, the critical slip surfaces and the associated minimum factors of safety (F_{min}) have been determined for the seismic coefficient K_h ranging from 0.0 to 0.2. The values of F_{min} are presented in Table 4 (column 3). A typical critical slip surface, for the seismic coefficients K_h = 0.20, is shown in Fig. 6.

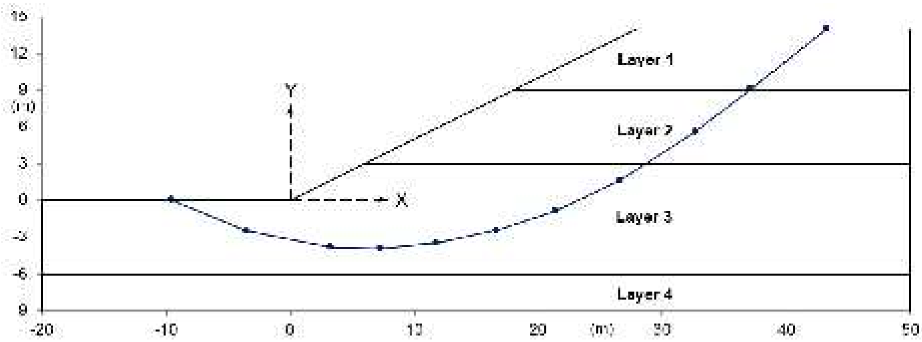

Fig. 6. Critical Slip Surface by Spencer Method for Example 2 (K_h = 0.20)

Analysis III: Evaluation of both the Critical Slip Surfaces by Sarma Metho
Based on the Sarma method, the critical slip surfaces obtained based on the Morgenstern and Price method and the Spencer method have been re-evaluated, using the software GEO5. The results are summarized in Table 4 and discussed in Section 7.

Fig. 7(a) and Fig. 7(b) show the slice configurations corresponding to the re-analysis by Sarma method of the critical slip surfaces for the Spencer method and the Morgenstern method respectively.

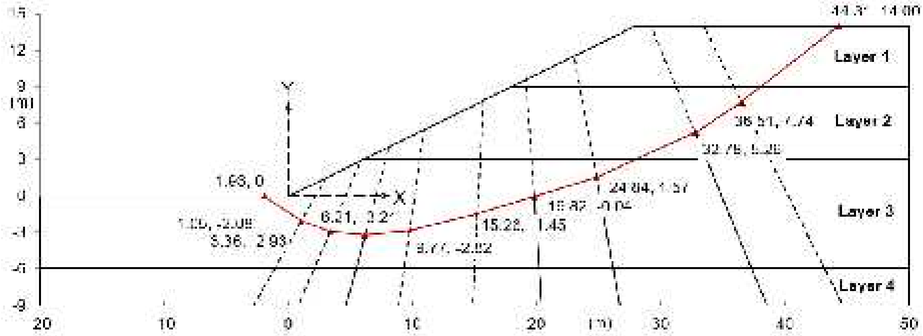

Fig. 7(a). Slice configuration in Sarma method of analysis of the critical slip surface based on Morgenstern and Price method

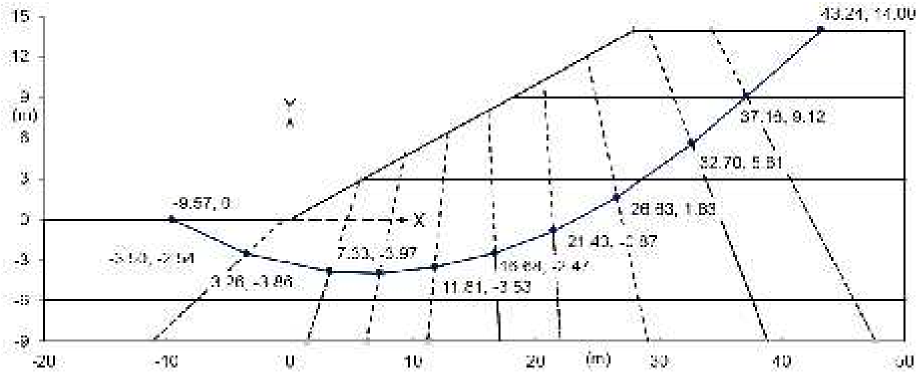

Fig. 7(b). Slice configuration in Sarma method of analysis of the critical slip surface based on Spencer method

Table 4. Summary of Results for Example 2.

K_h	Values of F_{min} by the Morgenstern and Price method (1)	Values of F by the Sarma method corresponding to column (1) (2)	Values of F_{min} by the Spencer method (3)	Values of F by the Sarma method corresponding to column (3) (4)
0.00	1.80	1.98	2.10	2.15
0.05	1.79	1.88	1.90	1.94
0.10	1.66	1.78	1.74	1.77
0.15	1.53	1.62	1.64	1.64
0.20	1.41	1.48	1.48	1.48

6 Example 3: An Embankment on Soft Clay (Low, 1989)

6.1 Description

Low (1989) presented a semi-analytical procedure to calculate the factor of safety of embankments founded on soft clay using the stability numbers defined for the embankment and its foundation. Application of the proposed method was illustrated with the help of a couple of examples. The slope section in example 3, shown in Fig. 8, is similar to one of those considered by Low (1989). In this example, there are three horizontal clay layers, having 5 m, 4 m and 5 m thickness. Values of the undrained shear strength c_u for the upper, middle, and lower strata are 30, 20, and 150 kPa respectively. The soil unit weight is 18 kN/m^3. As shown in Fig. 8, a cut is excavated with side slope of 1V:3H to a depth of 6 m.

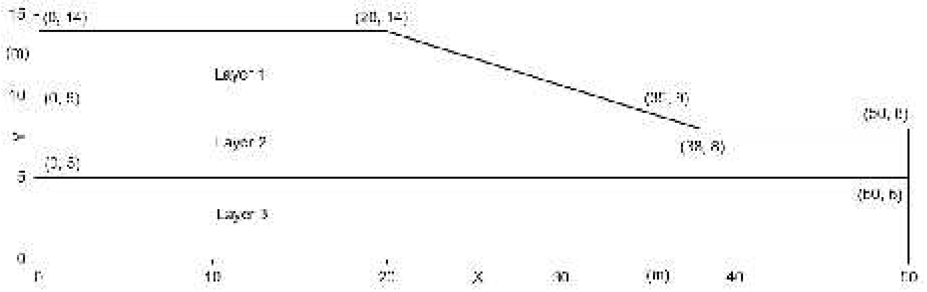

Fig. 8. Slope Section in Example 3 [Similar to one of the slope sections in Low (1989)].

Analysis I: Critical Slip Surface by Morgenstern & Price Method
Based on the Morgenstern and Price (1965) method, the critical slip surfaces and the associated minimum factors of safety (F_{min}) have been determined for the seismic coefficient K_h ranging from 0.0 to 0.15 using the software GEO5. The total number of slices considered in the analysis is 10. The values of F_{min} are presented in Table 5. A typical critical slip surface, for the seismic coefficients K_h = 0.1 is shown in Fig. 9.

Analysis II: Critical Slip Surface by Spencer Method
Based on the Spencer (1973) method, the critical slip surfaces and the associated minimum factors of safety (F_{min}) have been determined for the seismic coefficient K_h ranging from 0.0 to 0.15. The values of F_{min} are presented in Table 5 (column 3). A typical critical slip surface, for the seismic coefficients K_h = 0.1, is shown in Fig. 10.

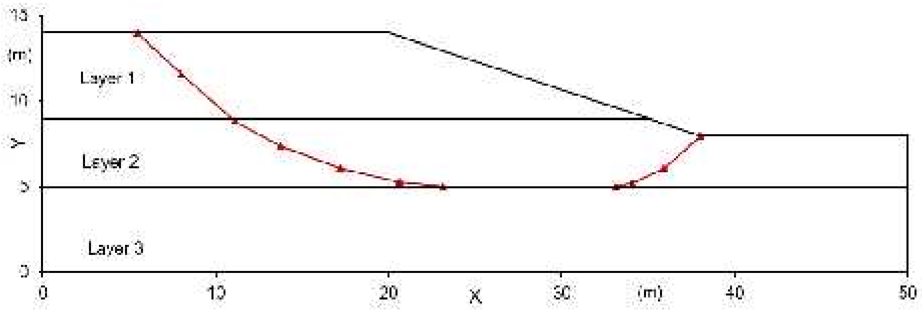

Fig. 9. Critical Slip Surface by Morgenstern & Price Method for Example 3 (K_h=0.10)

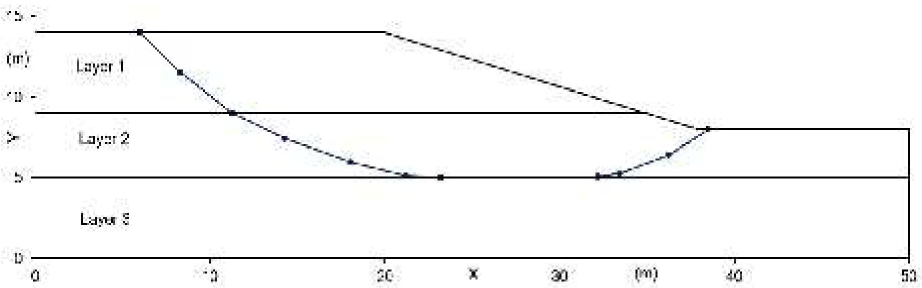

Fig. 10. Critical Slip Surface by Spencer Method for Example 3 (K_h=0.10)

Analysis III: Evaluation of both the Critical Slip Surfaces by Sarma Method
Based on the Sarma method, the critical slip surfaces obtained based on the Morgenstern and Price method and the Spencer method have been re-evaluated, using the software GEO5. The results are summarized in Table 5 and discussed in Section 7.

Table 5. Summary of Results for Example 3.

K_h	Values of F_{min} by the Morgenstern and Price method (1)	Values of F by the Sarma method corresponding to column (1) (2)	Values of F_{min} by the Spencer method (3)	Values of F by the Sarma method corresponding to column (3) (4)
0.00	1.41	1.76	1.42	1.75
0.05	1.22	1.49	1.24	1.49
0.10	1.05	1.26	1.07	1.27
0.15	0.91	1.09	0.92	1.10

7 Concluding Remarks

Stability assessment of multi-layered slopes of complex geometry based on rigorous limit equilibrium methods of slices such as the Morgenstern and Price method (1965) and the Spencer method (1973) valid for general slip surfaces involves lot of computational effort which is further enhanced when such slopes are analyzed under seismic condition by introducing pseudo-static earthquake forces. On the other hand, the Sarma method (1979) of non-vertical slices has always attracted the attention of both researchers and practitioners in geotechnical engineering, yet its application in complex layered slopes has been limited because of requirement of additional computing effort in the iterations involved in finding the critical inclinations of the slices. In recent times, however, making use of commercially available sophisticated slope analysis softwares, it has been practicable to attempt solving complex slope problems based on the Sarma method as well. Keeping the above in view, in the present study such an attempt has been made to use GEO5—a slope analysis software to solve three complex slope examples selected from the literature based on all the three rigorous methods of slices mentioned above and try to draw a comparison of the values of factors of safety so obtained. In view of the additional computations required in the Sarma method, no effort has been made to determine the critical slip surface based on this method. For purposes of comparison, the critical slip surfaces determined based on the other two methods have been re-analyzed based on the Sarma method.

Within the limited scope of the present study based on the analysis of three slope examples, a comparison of the values of factor of safety based on the three rigorous methods of slices has revealed the following:

1. Compared to both the Morgenstern and Price method (1965) and the Spencer method (1973), the Sarma method (1979) yields, for characteristic slip surfaces, values of pseudo-static factor of safety which are on the higher side. The magnitude of the difference, of course, varies widely with the method of analysis, and the value of seismic coefficient, K_h.

2. For instance, considering the specific slip surface as the critical slip surface based on the Morgenstern and Price method, the difference in the values of factor of safety obtained by the Sarma method and the Morgenstern and Price method ranges from 10% to 25% for the static case ($K_h = 0.0$), and from 5% to 22% for the seismic cases (K_h varying from 0.05 to 0.20 for examples 1 and 2, and from 0.05 to 0.15 for example 3).

3. Again, considering the specific slip surface as the critical slip surface based on the Spencer method, the difference in the values of factor of safety obtained by the Sarma method and the Spencer method ranges from 2% to 23% for the static case ($K_h = 0.0$), and from 0% to 20% for the seismic cases (K_h varying from 0.05 to 0.20 for examples 1 and 2, and from 0.05 to 0.15 for example 3).

4. The above observations are based on a total of 3 examples and 10 slices used in each analysis. Thus, it gives a crude estimate of the difference in the values of factor of safety. For purposes of a rigorous comparison, however, analyses of more numbers of examples should be carried out and, also, the effect of increasing the number of slices should be examined (Metya and Bhattacharya, 2016a, b).

5. For a more meaningful comparison of the solutions obtained from the rigorous methods of slices, attempts may be made to determine the critical slip surfaces based on the Sarma method, though it is known to be an extremely tedious job.

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