# Seismic Earth Pressure Coefficients for Vertical Wall using Force-Displacement Curves

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**Abstract.** Determination of soil pressure under seismic condition has always been an issue in designing of earth retaining structures such as abutments and retaining walls. Estimation of lateral earth pressure is often carried out using limit equilibrium approach. This approach was first proposed by Coulomb and further enriched by various researchers, like Rankine, Mononobe-Okabe, Caquot, Sokolovski, for seismic earth pressure using pseudo-static, pseudo-dynamic and modified pseudo-dynamic method. Despite the abundance of research, there is lack of a consensus approach as the previous studies are based on assumptions like linear soil wedge failure surface which limits them to a subset of real earth pressure problems.

The aim of present study is to provide seismic earth pressure coefficients considering the classic system of vertical wall retaining horizontal backfill. The finite element models are developed using ABAQUS [1], with rigid wall and purely frictional soil properties. Soil mass is modeled using plane-strain quadratic quadrilateral (CPE8R) elements with Mohr-Coulomb yield criterion. The seismic inertial force is incorporated using pseudo-static approach in terms of horizontal seismic coefficient,  $\alpha_h$ . Sufficiently large lateral boundary is provided to include failure surface and avoid boundary effects. Effect of various governing parameters, such as backfill soil friction angle,  $\phi$ , with different backfillwall interface friction angle,  $\delta$ , and horizontal seismic coefficient,  $\alpha_h$ , on seismic earth pressure coefficient is explored in detail. The distribution and magnitude of the active and passive earth pressure are compared with those obtained from the classical methods and given in design codes.

Keywords: Earth pressure; Retaining wall; Pseudo-static; Seismic; Finite Element

#### 1 Introduction

Seismic earth pressure value and its distribution along the wall surface has always been of concern for (1) the estimation of seismic bearing capacity of shallow foundation in case of skirt foundations and well foundation; (2) analysis of plate and block

anchors; and (3) calculation of forces on bridge abutments and earth retaining structures.

In the past, several researchers have contributed in development of earth pressure theory and solid design methodology to estimate the magnitude and distribution of earth pressure and its point of application. Extensive literature is available for estimation of seismic earth pressure based on various numerical analysis methods, such as (a) pseudo-static method (method of characteristics [2-3], finite element method [4-5], limit analysis [6-10] and limit equilibrium [11-15]), (b) pseudo-dynamic method [16-18] and (c) modified pseudo-dynamic method [19-20].

In pseudo-static analysis, an equivalent static approach is used to take into account the inertial forces of the system induced during a seismic event. Seismic earth pressure coefficients are calculated for retaining wall with cohesion-less backfill using a linear failure surface, resulting in overestimation of passive and underestimation of active earth pressure. However, assumption of curved failure surface was found more suitable for estimation of passive and active earth pressure [15, 21-23]. In pseudo dynamic approach the phase change of ground motion is taken into consideration, while ignoring the damping characteristics of backfill. In recent year, Pain et al. [19] and Rajesh and Choudhury [20] have presented modified pseudo dynamic methodology to account for the damping of backfill. The major shortcomings of the above mentioned studies is the assumption of a pre-defined failure surface, except in some studies [24-26] where the seismic earth pressure coefficients were estimated using upper and lower bound limit analysis (FELA).

In the present study, an attempt has been made to determine the static and seismic earth pressure coefficients using displacement based FE analysis. Further, the variation of key parameters affecting the seismic earth pressure coefficients are studied in detail.

#### 1.1 Problem Statement

A simple case of a vertical wall retaining horizontal backfill has been considered. 2D plane strain finite element models (FEM) have been developed in ABAQUS [1]. The study involves deduction of active and passive pressure coefficients using pseudo-static approach. The passive and active coefficients have been calculated by laterally pushing and pulling the wall towards and away from the backfill, respectively.

The seismic case involves computation of active and passive coefficients ( $K_a^+, K_a^-, K_p^+, K_p^-$ ), distinguished on the basis of the wall movement relative to horizontal seismic coefficient,  $\alpha_h$ . In case of passive earth pressure coefficient, positive superscript defines the movement of wall opposite to the direction of acceleration,  $K_p^+$ , and negative superscript designate movement of the wall towards the direction of acceleration,  $K_p^-$ . Similarly, in case of active earth pressure coefficient, positive superscript represents wall movement in direction of acceleration,  $K_a^+$ , and negative superscript is used for wall movement in opposite direction of acceleration,  $K_a^-$ .

### 2 Finite Element Modelling

2D finite element models (Fig.1) have been developed in ABAQUS [1] using plane strain elements with quadratic geometric order (CPE8R). The retaining wall has been modelled using linear elastic beam element. Whereas, backfill cohesionless soil has been modelled using Mohr-Coulomb failure criteria.



Fig. 1. ABAQUS [1] model of the retaining wall and backfill.

Two different types of elements (total number of elements equal to 3690) have been used in FE modelling of wall-soil system, as discussed below:

- 1. Beam element, B21 (a 2-node linear beam in a plane) has been used for modelling of wall (50 elements).
- 2. Plane strain element, CPE8R (an 8-node biquadratic plane strain quadrilateral, reduced integration) element has been used to model backfill soil (3640 elements).

Parameter	Value	Unit
Young's modulus (E)	437.4	MPa
Poisson's ratio ( $\mu$ )	0.35	-
Internal friction angle $(\phi)$	10° - 45°	Degree
Cohesion ( <i>c</i> )	0.1	kPa
Unit Weight (γ)	18	kN/m <sup>3</sup>
Ratio of wall friction angle to internal friction angle $(\delta/\phi)$	0-1	-

Table 1. Soil Properties used for Numerical Analysis.

The base of FE model has been kept fixed in all direction, whereas only vertical movement has been allowed at lateral boundaries. The dimension of developed FE model has been taken sufficiently large to incorporate the failure wedge within considered domain and avoid any boundary affect. The soil properties used in the study are given in Table 1.

### **3** Computation of Earth Pressure Coefficients

The earth pressure coefficients have been computed by providing a uniform displacement to the wall towards and away from the backfill for passive and active earth pressure condition, respectively. The total load (acting on the wall due to soil movement, in either dither direction) vs displacement curves has been estimated by adding load vs displacement curves of individual nodes of wall. Typical individual load at each node of wall and total passive pressure (for wall movement opposite to the direction of acceleration) vs displacement curves for wall-backfill system (with wall height, H = 5m,  $\phi = 35^\circ$ ,  $\alpha_h = 0.05$  and  $\delta/\phi = 0.33$ ) are shown in Fig. 2 (a) and 2(b), respectively. The threshold value of earth pressure has been obtained from the total pressure-displacement curve (Fig.2 (b)) and used for estimation of earth pressure coefficients. The triangular distribution of this earth pressure has been considered behind the wall [11], as given by Equation 1 and 2:

$$P_a = \frac{1}{2}\gamma H^2 K_a \tag{1}$$

$$P_p = \frac{1}{2}\gamma H^2 K_p \tag{2}$$

where,  $P_{a}$ ,  $P_{p}$  are active and passive earth pressures and  $\gamma$  is the unit weight of soil.



Fig. 2. Load-displacement curve for (a) individual nodes (b) sum of all nodes of wall.

## 4 Comparison with a Past Study

In the past, several researchers have obtained earth pressure coefficients using (1) plasticity theory; (2) method of stress characteristics; (3) upper and lower bound limit analyses; and (4) upper and lower bound finite element limit analysis (FELA). In the

present study, pressure-displacement curves generated from reaction on the wall has been used to calculate earth pressure coefficients. The earth pressure coefficients obtained from the present study are compared with corresponding values of Krabbenhoft [25]. The study presented by Krabbenhoft [25], based on limit analysis approach, provides earth pressure coefficients with rigorous computation of upper and lower bounds on the collapse load of structures of rigid-plastic material. However, the present study provides the pressure coefficients based on force-displacement curves, which allow a more reliable estimation of pressure coefficients, even when the earth pressure is not completely mobilised or when the soil behind the wall has already yielded. Moreover, the present procedure of estimation of earth pressure coefficients using force-displacement curves can be extended for materials with non-associative flow rule, whereas the upper and lower bound theorems of limit analysis can be applied only to those materials which obey associative flow rule. Figure 3 shows the variation of passive earth pressure  $K_p$  with wall-backfill interface friction angle for static case i.e.  $\alpha_h = 0$ . Similarly, Fig. 4(a) shows the active earth pressure coefficient  $(K_p)$  varying with friction angle of soil and Fig. 4(b) shows the seismic passive earth pressure coefficient  $(K_p^+)$  obtained through pseudostatic analysis with  $\alpha_h = 0.15$ .



**Fig. 3.** Comparison of static passive pressure coefficient ( $\alpha_h = 0$ ).



**Fig. 4.** Comparison of earth pressure coefficient obtained in present study with Krabbenhoft [25] for: (a) active static case ( $\alpha_h = 0$ ); and (b) passive seismic case ( $\alpha_h = 0.15$ ).

As can be observed from the comparison, the results obtained in the present study have been found to be slightly higher for passive earth pressure and slightly lower for active earth pressure coefficients. This slight discrepancy (practically insignificant) is attributed to the fact that the coefficients provided by Krabbenhoft [25] were obtained through FELA taking the average of coefficients obtained from upper and lower bound analysis, whereas the present study utilize the pressure-displacement curve obtained from finite element analysis to obtain these coefficients.

#### 5 Results and Discussion

The effect of different governing factors, friction angle of soil,  $\phi$ , wall-backfill interface angle,  $\delta$ , and horizontal seismic coefficient,  $\alpha_h$ , on earth pressure coefficient has been studied in detail. For this purpose, extensive numerical analysis has been performed by varying  $\phi$  (10° - 45°),  $\delta/\phi$  (0 - 1) and  $\alpha_h$  (0 - 0.3). The results of the parametric study have been presented in terms of earth pressure coefficient  $K_p^+$ ,  $K_p^-$ ,  $K_a^+$ and  $K_a^-$  (as function of  $\phi$ ,  $\delta/\phi$  and  $\alpha_h$ ) in Figs. 5, 6, 7 and 8, respectively. The following observation can be made from the results:

- 1. It has been observed that passive seismic earth pressure coefficient,  $K_p^+$  corresponding to wall movement opposite to the direction of acceleration, gradually increases and passive seismic earth pressure coefficient,  $K_p^-$  corresponding to wall movement in the direction of acceleration, gradually decreases with the increase in horizontal seismic coefficient.
- 2. It has also been observed that active earth pressure coefficients,  $K_a^+$  for wall movement in the direction of acceleration, gradually increases and active earth pressure coefficients,  $K_a^+$ , for wall movement in the opposite direction of acceleration, gradually decreases with the increase in horizontal seismic coefficient.
- 3. It has been found that with increase in soil friction angle,  $\phi$ , and backfill-wall interface angle,  $\delta$ , both passive pressure coefficients ( $K_p^+$  and  $K_p^-$ ) increase.
- 4. It has also been found that with increase in soil friction angle,  $\phi$ , and backfill-wall interface angle,  $\delta$ , both active pressure coefficients ( $K_a^+$  and  $K_a^-$ ) decrease.

# 6 Conclusions

Seismic earth pressure problem has been considered for a vertical wall retaining a horizontal cohesionless backfill. Using displacement based FE analyses, seismic earth pressure coefficients have been derived. The influence of varying horizontal seismic coefficient and wall-soil interface angle on earth pressure coefficients have been studied in detail. It has been found that, passive coefficient  $K_p^+$  increases and  $K_p^-$  decreases with increase in horizontal seismic coefficient, while both increases with increase in soil friction angle,  $\phi$ , and wall-backfill interface angle,  $\delta$ . Active coefficient, while both decreases with increase in soil angle,  $\phi$ , and wall-backfill interface angle,  $\delta$ .



**Fig. 5.** Passive earth pressure coefficient  $K_p^+$  for wall movement opposite to the direction of acceleration with friction angle,  $\phi$ , varying from 10° to 45° for (a)  $\delta'\phi = 0$ ; (b)  $\delta'\phi = 1/3$ ; (c)  $\delta'\phi = 1/2$ ; (d)  $\delta'\phi = 2/3$ ; and (e)  $\delta'\phi = 1$ .



**Fig. 6.** Passive earth pressure coefficient  $K_p$  for wall movement in the direction of acceleration with friction angle,  $\phi$ , varying from 10° to 45° for (a)  $\delta'\phi = 0$ ; (b)  $\delta'\phi = 1/3$ ; (c)  $\delta'\phi = 1/2$ ; (d)  $\delta'\phi = 2/3$ ; and (e)  $\delta'\phi = 1$ .



**Fig. 7.** Passive earth pressure coefficient  $K_a^+$  for wall movement in the direction of acceleration with friction angle,  $\phi$ , varying from 10° to 45° for (a)  $\delta/\phi = 0$ ; (b)  $\delta/\phi = 1/3$ ; (c)  $\delta/\phi = 1/2$ ; (d)  $\delta/\phi = 2/3$ ; and (e)  $\delta/\phi = 1$ .



**Fig. 8.** Passive earth pressure coefficient  $K_a$  for wall movement opposite to the direction of acceleration with friction angle,  $\phi$ , varying from 10° to 45° for (a)  $\delta/\phi = 0$ ; (b)  $\delta/\phi = 1/3$ ; (c)  $\delta/\phi = 1/2$ ; (d)  $\delta/\phi = 2/3$ ; and (e)  $\delta/\phi = 1$ .

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