Cyclic Triaxial Test to Measure Strain-Dependent Dynamic Properties-A Comprehensive Study

Arpit Jain[1], Abhijit Chakraborty[2], Sukanta Das[3] and Satyendra Mittal[4]

[1,2,4] Department of Civil engineering, IIT Roorkee, Uttarakhand, India
[3] Earthquake Engineering Department, IIT Roorkee, Uttarakhand, India
arpitjaintkg@gmail.com

Abstract. A review on dynamic properties of soil using cyclic triaxial test has been proposed in the present study. Several researchers have studied the dynamic properties of soil at large strain level using cyclic triaxial test and proposed either analytical solution or charts for shear modulus degradation curve and damping curve of soil. The dynamic behaviors of soil depend on various factors such as soil type, loading condition, relative density, confining pressure, void ratio, over-consolidation ratio etc. as reported by researchers. The dynamic properties, especially the shear modulus degradation curve and damping curve are important for designing of any geotechnical structures. In research purpose, it helps to simulate numerically to any geotechnical problems. A summarized review has been presented with their methodology and used parameters from the past literature. The locally available Solani River Sand is collected from Roorkee region, tested in laboratory. The obtained test results from cyclic triaxial test have been validated with past literature.

Keywords: Cyclic Triaxial test; Shear modulus; Damping Ratio; Shear Strain.

1 Introduction

Recent few major earthquakes including 2015 Nepal EQ, 2005 Kashmir EQ, 2001 Bhuj EQ, 1999 Chamoli EQ and 1991 Uttarkashi EQ etc. have shown the vulnerability of residential building, bridges, earth retaining structures, foundations, landslides and others important geotechnical failures. The engineers are needed to reduce the probability of damages of important structures during such devastating events. From past few decades the researchers are more concerned about developing in geotechnical earthquake engineering field. In geotechnical engineering, the dynamic soil properties such as Shear Modulus (G), Damping Ratio (¶), Shear wave velocity (V_s), Poisson's Ratio («) and Density (°) are essential for knowing the response of soil subjected to any loading conditions. The several laboratory and field based equipment are available to measure those dynamic properties of the soil. At large strain level, the laboratory based Cyclic Triaxial Test (CTT) is widely used for estimation of dynamic properties as well as liquefaction study. In the present study, past development of dynamic behavior of soil in large strain level using CTT has been proposed.

Seed and Lee [1] performed cyclic triaxial test under consolidated undrain condition with constant amplitude of cyclic axial stress. The shear modulus of any soil shows a nonlinear behavior with strain level [2]. The maximum shear modulus or initial modulus has been observed at very small strain level within 10^{-4}-10^{-5} in % [3]. Many researchers proposed empirical correlations or analytical solutions for estimating the maximum shear modulus and damping ratio for that strain level [2, 4].

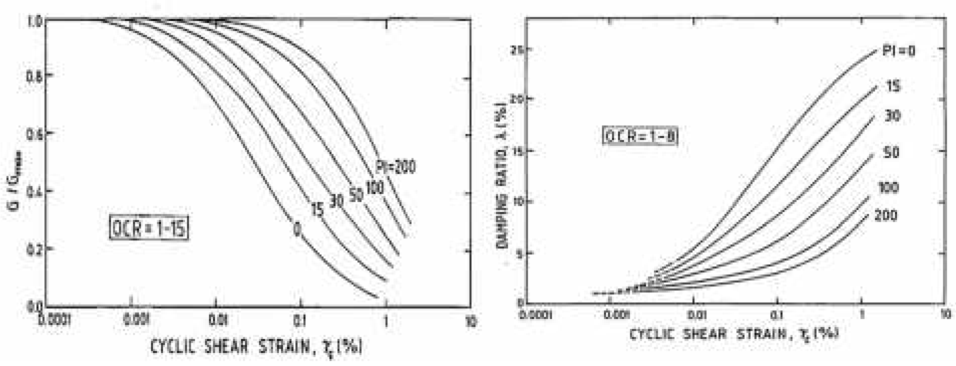
The shear modulus is commonly presented in a normalized form by considering a ratio between shear modulus and maximum shear modulus at small strain which is known as reduction modulus curve. The effect of different influencing parameter such as plasticity, grain size, void ratio, pore pressure and atmospheric pressure on dynamic behavior of sand has been studied by many researchers [5-10]. In past, many experimental studies for determination of the dynamic property of different soil like marine clay, soft clay, plastic and non-plastic soil has been performed [11-14]. Few numerical investigations on dynamic behavior of soil has been conducted [8, 15-17].

However, a literature review has been presented in the in a summarized way and future scopes has been defined based on past literature.

2 Literature Review on Cyclic Behavior of Soils

A summarized literature review has been carried out from past studies to know the dynamic behaviors of soil at large strain level using CTT apparatus. Several researchers have been performed experiments on different soil and proposed empirical equations or charts for estimation of shear modulus and damping ratio [18].

Kousho (1980) performed cyclic loading tests on clean sand to obtain modulus and damping values at a wide strain range. Effect of friction from loading piston, traditional transducers has been studied on *G* vs log γ curve. Vucetic et al. [5] produced curves Fig. 1, a and b on the basis of experimental data of normally consolidated and over consolidated clays (OCR =1-15), as well as for sands in which influence of plasticity index (PI) on cyclic stress-strain parameters of soils under saturated conditions. These charts presented the impact of PI on the location of modulus reduction curve (G/G_{max}) against cyclic shear strain γ_c and on the damping ratio of material (λ) vs cyclic shear strain (γ_c) curve on sandy soils and sandy soils. Fig. 1 shows that with an increase in PI value G/G_{max} increases, while there is a reduction in the value of λ (Fig. 1, b). This phenomenon is valid for both normally and over consolidated soils. The charts presented are suggested to be used in seismic site response evaluation and microzonation.



(a) (b)

Fig. 1. (a) Relation between G/G*max* vs. Cyclic Shear Strain (%), (b) Damping Ratio vs. Cyclic shear strain (%) Curves and Soil Plasticity [PI] for Normally and Over Consolidated Soils.

Kiku et al. (2000) performed cyclic loading tests on sand at large strain. They obtained shear modulus vs. shear strain curve, and then converted to a shear stress-strain relationship and finally resembled with the monotonic loading. Firstly, the peak value of strain is observed as 0.1% which again decreases and further increases at a strain rate of 3%. The same behavior is obtained by monotonic loading tests where softening and hardening phenomenon come up due to negative and positive dilatancy effect. Concluding remarks have been reported showing relation between shear modulus and shear strain can also be obtained from single monotonic tests. It has been found that there is a degradation of the stress-strain curve in each loading due to the generation of pore water pressure. The presence of quasi-liquefaction state at this strain range is also observed.

Park and Stewart (2001) suggested empirical equations for the damping ratio of plastic and non-plastic soils based on previous studies. It was found that the plasticity index (PI) has significant effects on both normalized shear modulus and damping ratio. A correlation between the normalized shear modulus ($\frac{G}{G_{max}}$) and damping ratio (D) for sandy soil was developed as (with a coefficient of determination of 0.918), where G is shear modulus.

$$D(\%)=32.8[0.54(\frac{G}{G_{max}})^{2}-1.53(\frac{G}{G_{max}})+1 \qquad (1)$$

Similarly, an empirical equation was developed for clayey soil between damping ratio and normalized shear modulus with a coefficient of determination of 0.844 as:

$$D(\%)=17.83[0.56(\frac{G}{G_{max}})^{2}-1.39\,(\frac{G}{G_{max}})+1 \qquad (2)$$

These empirical correlations can be used to determine approximate value of damping ratio when the G/G_{max} at any shear strain is known.

Vardanega et al. (2011) used simple soil parameters for the evaluation of strain-dependent stiffness of fine-grained soil through published literature. Regression analysis is performed

along with design charts by considering the corrections on rate effect. Here, the use of reference strain γ_{ref} to normalized shear strain values in relation with modulus reduction is presented. Regression Predictions and design charts have been presented in this study.
For plasticity index this study projected the shortcomings of a chart produced by Vucetic and Dobry [5] like there was an absence of mathematical formulation for degradation curve,
Reference strain is formulated as given below based on PI:

$$\gamma_{ref} = 2.17(I_p)/1000 \quad (3)$$

By using dfferent equations in this study a figure was drawn to predict the behavior of reference strain using plasticity index. The R^2 for this correlation is very good with 96 percent of the variation being explained by the model. the standard error (*S.E.*) is low and the probability of a correlation not existing (*p*) is less than 1 in 1000.

Bahar et al. (2012) where data used from the field tests through geophysical and geotechnical investigations and FLAC is used for the numerical study. The nonlinear elastic model proposed by Ramberg Osgood [20] and limited by the Mohr-coulomb criterion was proposed in their study, which shows good agreement with the field results on dynamic behavior of Algerian soil.This study can form the preliminary basis to give curves of shear degradation modulus and damping, when no experimental data is available or to be used as the starting point for cases where geotechnical measurements are not sufficient.

Neagu et al. (2012) applied cyclic load on a clay sample from Bucharest, Romania in such a way that over a number of cycles, inertial effects do not occur. The results obtained from the experiments conducted are presented in the form of G-γ and D- γ curves. All these results were compared with previous studies. The curve of normalized shear modulus reduction for Bucharest clay was compared with the curves provided by vucetic and Dobry [5] for clays with plasticity index (PI) between 0 to 50. The author suggested the importance of sampling process, transport of soil samples, storage conditions, test sample preparation and setup required for the test.

Tian et al. (2012) performed strain controlled cyclic triaxial tests on undisturbed marine silty clay which is located more than 100m depth of the Qiongzhou Strait seabed by the hollow cylinder apparatus. The study was carried out, to find out the effect of effective confining pressure and plastic index on the dynamic properties. An empirical formula, which is suitable for calculating the maximum dynamic shear modulus of deep-seabed silty clay, is established based on a plasticity index, confining pressure and void ratio. It has been observed that with increasing plasticity index, dynamic shear modulus decreases when all other conditions are kept remains constant. The test was carried out on 14 undisturbed samples of marine silty clay. By a hyperbolic skeleton curve, the relation between dynamic shear modulus and dynamic shear strain is characterized.
Arefi et al. (2012) proposed a simple model to produce the desired stiffness and hysteretic damping at a certain strain level through laboratory testing. A relationship is developed at large strain level with unloading – reloading for total stress seismic site response analysis. Modulus reduction and damping curves measured correctly due to the modification of the extended Masing unloading-reloading relation in which constitutive model uses a hyperbolic equation as backbone curve. Model’s performance is demonstrated by a quasi-static cyclic loading of

increasing amplitude. As per the study carried out by Hardin and Drnevich [2], Darendeli [21], Phillips and Hashash [22] a normalized modulus reduction curve was obtained.

The experimental study is correlated with the above hyperbolic equation in order to simulate modulus reduction curves; the least square error method is used in order to estimate the required parameter of the hyperbolic model.

Subramaniam et al. (2013) presented paper deals with a hyperbolic–hysteretic undrained soil model based on Masing rule [23] with a simple correction for the calculation of damping ratio. Moreover, the efficiency of the model to predict the variation of shear modulus and damping ratio for a wide strain range was done. The corrected damping ratios for various types of soils with Algerian soils, plasticity index were compared with published experimental results and a good agreement was found. Modulus reduction is considered and a function is introduced in their study.

Alimohammadi et. al (2013) developed shear modulus degradation and damping ratio curves based on the nonlinearity of soil from hysteresis loops. Plasticity index and effective confining pressure are the most important factors governing the nonlinear behaviour of soil. The results of this study illustrated the influence of the plasticity index on the shear strength of soil and the site response. Equivalent linear analysis is done by using the Kelvin-Voigt model and non-linear behaviour is estimated by using a hysteresis loop based on the Masing rule [23] and numerical integration of the equation of motion.

Gaitán-Serrano et. al (2013) developed shear modulus reduction curves of Guayuriba sands by cyclic triaxial and bender element tests using Guayuriba River (Colombia) Sand Laboratory experimental programs based on cyclic triaxial and bender element test conducted to find out the cyclic behavior of the above-mentioned soil at different confining pressures. Moreover, the effect of grain size on cyclic behavior was also studied.

Nie. et al (2016) performed numerical modeling of Cyclic Triaxial experiments on Nuozhadu granular soil obtained from the Nuozhadurockfill dam project in China, which was set between the rock fill and core wall of the dam. Constitutive model based on the Bouc–Wen model and endochronic theory was introduced to investigate the dynamic stress-strain relationship and pore pressure of granular soil under cyclic triaxial stress state. Material relative density was kept to be 80%. Densification law with the basic concept of intrinsic time is introduced to get the plastic strain. Moreover, the dilatancy flow law is also used to obtain the incremental plastic shear strain value. For calculating pore pressure, Bryne's model [25] is used.

Chattaraj and Sengupta (2016) found out strain dependent dynamic properties of soil such as damping ratio and shear modulus which are evaluated after performing resonant column tests and undrained cyclic triaxial tests on Kasai river sand. A correlation has been developed, corresponding to the dynamic shear damping (D_s) and maximum dynamic shear modulus (G_{max}) for the sand at the small strain. Proposed relationships are validated using experimental data, along with the previously obtained relationships for the other sand. Estimation of liquefaction potential has been carried out at large strain level for sand at different relative density and damping characteristics. An empirical correlation was developed between G_{max}, e (void ratio), σ_0 (confining pressure) and P_a (atmospheric pressure) based on the test data.

Ghayoomi et al. (2017) conducted suction controlled cyclic triaxial test by using the axis-translation technique at a relative density of 45% and sample prepared by dry pluviation (sand raining) method. Materials used were clean sand with various degrees of saturation, F-75 Ottawa sand, fine-grained, uniformly distributed silica sand. A suction (c) dependent shear modulus relation derived from his study. The results indicated that in unsaturated sand, the shear modulus increased with an increase in the suction level, regardless of the induced strain level. With an increase in the shear strain in the specimens with similar suction values, the modulus decreased nonlinearly. Partially saturated conditions led to a shift in the shear modulus reduction curves as a result of the higher effective stress and suction.

Shaw et al. (2018) studied the dynamic properties and liquefaction potential of Barak river sand. They used resonant column and cyclic triaxial test. They investigated liquefaction potential and dynamic behaviour for different condition at wide strain range. The obtained test results were validated with different dynamic properties of sand.

Table 1. Past studies on dynamic properties of soil using CTT Apparatus.

Literature	Soil Type & Methodology	Important Outcomes
Kokusho (1980)	Cyclic loading tests are performed on clean sand.	Effect of friction from loading piston& traditional transducers on modulus reduction curve has been studied.
Vucetic and Dobry (1991)	OCR (1 -15), sand soil with different PI content.	Charts are prepared on the basis of experimental data presented the impact of PI.
Kiku and Yoshida (2000)	Toyoura Sand under Cyclic and monotonic loading.	Shear modulus-shear strain curve obtained from Cyclic loading tests is converted to shear stress-strain relationship and finally resembled with the monotonic loading.
Park andStewart (2001)	Clayey and sandy soil.	Empirical correlations damping ratio and normalized shear modulus. $D(\%)=32.8[0.54(\frac{G}{G_{max}})^2-1.53(\frac{G}{G_{max}})+1]$ $D(\%)=17.83[0.56(\frac{G}{G_{max}})^2-1.39(\frac{G}{G_{max}})+1]$
Vardanega and Bolton (2011)	Fine-grained silt and clay.	Regression analysis is performed along with design charts by considering the corrections on rate effect.
Bahar et al. (2012)	Algerian soils (marl gravel, pebble, and sand enveloped in silt matrix) Field tests conducted for geophysical and geotechnical investigations. FLAC is used for the numerical study.	Developed an equations as follows $\gamma-\gamma_c=\frac{1}{G_{max}}[1+\alpha(\frac{\lvert\tau-\tau_c\rvert}{n\tau_y})^{r-1}](\tau-\tau_c)$ $G=\frac{d\tau}{d\gamma}=\frac{G_{max}}{1+\alpha(\frac{\lvert\tau-\tau_c\rvert}{n\tau_y})^{r-1}}$ $D=\frac{\Delta W}{4W_e\pi}=\frac{2}{\pi}(\frac{r-1}{r+1})(1-\frac{G_s}{G_{max}})$

Neagu and Arion (2012)	Clay soil. The cyclic load is applied on clay in such a way that inertial effects do not occur.	The author suggested the importance of sampling process, transport of soil samples, storage conditions, test sample preparation and setup required for the test.
Tian et al.(2012)	Deep-seabed marine silty-clay. Cyclic triaxial tests conducted, the empirical equation also derived for the calculation of dynamic shear modulus.	The empirical equation: $G_d = \frac{1}{m+\lambda\gamma_d}$ $G_{dmax} = \frac{152.7-6.7 I_p}{0.3+0.9e}\sigma'_o$ $\lambda = \lambda_p + a(1+b\gamma^{-c})^{-d}$
Arefi (2012)	Sandy soil. The model proposed to produce the desired stiffness and hysteretic damping, its performance is demonstrated by a quasi-static cyclic loading of increasing amplitude.	The normalized modulus reduction curve obtained as: $\frac{G}{G_{max}} = 1/(1+\beta\gamma/\gamma_r\|^{\alpha})$
Subramaniam and Banerjee (2013)	Clayey Soil. Hyperbolic–hysteretic undrained soil model based on the masing rule.	Modulus reduction proposed as: $G/G_{max} = (1/(1+R\varepsilon_r^{m}))^{P}$ Damping ration formulation= $D_{masing} = [\frac{2q_f\varepsilon_r-(\frac{2G_{max}}{R^2})\log_e(1+R\varepsilon_r)}{\varepsilon_r[q_f-G_{max}/(R+R^2\varepsilon_r)]} - 1]$ Corrected Damping ratio $D_{corrected} = D_{min} + C\,D_{original}$ $C = A(\frac{G}{G_{max}})^{\log B}$
Alimohammadi et al. (2013)	Clayey and sandy soil**.** Curves are developed, equivalent linear analysis has been carried out using the Kelvin-Voigt model	Shear modulus degradation curve proposed as: $G/G_{max} = k(\gamma, I_p)\,\sigma_o^{-m(\gamma,I_p)-m_o}$ $k(\gamma,I_p) = 0.5\{1+\tanh[\ln(\frac{\{0.000102+n\ln(I_p)\}^{0.492}}{\gamma})]\}$ $m(\gamma,I_p)-m_o = 0.272[1-\tanh[\ln((\frac{0.000556}{\gamma})^{0.4})]]\,e^{-.00145 I_p^{1.3}}$ Damping Ratio as: $\xi = (\frac{0.233(1+e^{-.0145 I_p^{1.3}})}{2})\{0.586(\frac{G}{G_{max}})^2 - 1.547(\frac{G}{G_{max}}) + 1\}$

Gaitán-Serrano et al. (2013)	Guayuriba River (Colombia) sand consists mostly of quartzite of the group quetame (Paleozoic shale formation). Performed tests are cyclic triaxial and bender element	Emperical equations: $\frac{G}{G_0}=1$ for $\gamma^*<10^{-2}$ $\frac{G}{G_0}=\frac{1-\tanh[0.48\ln(\frac{\gamma^*}{1.9})]}{2}$ for $\gamma^*>10^{-2}$ $\frac{G}{G_0}=1$ for $\gamma^*<10^{-1}$ $\frac{G}{G_0}=\frac{1-\tanh[0.46\ln(\frac{\gamma^*-0.1}{3.4})]}{2}$ $\gamma^*>10^{-1}$
Nie et al. (2016)	Nuozhadu granular soil (between the rockfill and core wall of the dam) Constitutive model based on the Bouc–Wen model and endochronic theory.	Proposed equation: $G=G_{max}/\eta=\frac{1}{1+\eta_0 E_d}kp_a(\frac{p}{p_a})^m$
Chattaraj and Sengupta (2016)	Sandy soil Resonant column, cyclic triaxial tests are performed and correlations are developed for dynamic shear damping, shear modulus	Solutions for Shear Modulus : $G_{max}=\frac{611.58*(P_a)^{0.332}*(\sigma_0)^{0.468}}{0.3+0.7e^2}$ Damping Ratio: $D_s=41.17\,(\frac{\sigma_0}{P_a})^{-0.29}(\gamma^{0.715})$ For 1% axial strain: $G_{max}=\frac{5219.58}{0.3+0.7e^2}(\sigma_0)^{0.426}(\sigma_d)^{0.088}$
Cami et al. (2016)	Silty soil, Clay and sand Resonant column tests, cyclic triaxial and standard triaxial tests are performed and combined to construct a 2D shear modulus reduction model.	Shear modulus reduction of soils on site are represented using a hyperbolic model as: $$\frac{G}{G_{max}}=\frac{1}{1+a(\frac{\gamma}{\gamma_r})^b}$$
Ghayoomi et al. (2017)	Clean sand with various degrees of saturation Ottawa sand, a fine-grained, uniformly distributed silica sand. Suction controlled cyclic triaxial are performed test using the axis-translation technique	Developed Equation for Shear Modulus: $G=A(OCR)^k f(e)\,\sigma'^{0.5}_m f(\gamma)\,f(\psi)$

Note: OCR= Over Consolidation Ratio, PI= Plasticity Index, FLAC= Finite Difference Method based Software (Fast Lagrangian Analysis of Continua), e = Void ratio, σ'_o= Effective stress, E_d = dissipation energy, η = linearly increasing function of E_d in model, P_a = atmospheric pressure, k and m are reduction and increase functions of cyclic shear strain amplitude, respectively.

Present experimental results and validation with past study. The index properties of Solani River sand have been determined as per Indian standard from the basic laboratory tests, presented in Table 2 (IS2720 [28] and ASTM [29-30]). The soil can be characterized as poorly graded sand (SP) according to the Unified soil classification system using the grain size distribution curve as shown in Fig. 2.

Table 2. Basic properties of Solani River sand.

Description		Value
Uniformity Coefficient (C_u)		2.51
Curvature Coefficient (C_c)		1.44
Specific Gravity		2.53
Maximum Void Ratio (e_{max})		0.79
Minimum Void Ratio (e_{min})		0.56
Grain Size	D_{10}	0.13
	D_{50}	0.24
	D_{60}	0.32

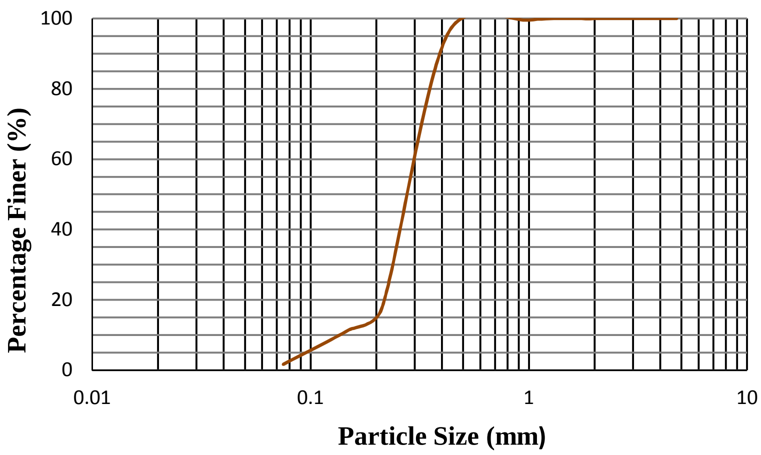


Fig. 2. Grain size distribution curve of Solani River sand.

3.1 Sample preparation and test procedure

All the tests were performed using cyclic cum static triaxial test system with a load cell of 10kN capacity. The linear variable displacement transducer (LDVT) with a range of ±10mm was connected to the top of the specimen. The specimens of diameter 70mm and height 140mm were prepared using dry pluviation method (ASTM D 5311-11). The sand was poured into the split mould through a long funnel to maintaining a constant height. Three successive layers are made and compacted uniformly for achieving the particular relative density. The sampling process is repeated for all tests. A 10kPa suction pressure is applied to the specimen before removing the split mould in order prevents bulging effect. After that, the triaxial cell is placed for the application of cell pressure surrounding the soil specimen.

After sample preparation, a confining pressure of 50kPa is applied isotopically. Tests were performed under UU (unconsolidated-undrained) condition with sinusoidal harmonic excitations. Strain-controlled cyclic triaxial tests were conducted on Solani sand at amplitude of 0.05 %, 0.1% 0.2%, 0.5%, and 1%. In the loading stage, the sinusoidal wave was applied at a prescribed strain rate with a frequency of 1Hz. Total of 100 cycles was considered in the present experiment.

3.2 Comparison with past test results

The test was conducted at 45% relative density and under effective confining pressure of 50kPa. The same relative density and confining pressure was studied by Hardin [2], Bolton [31]. The Fig. 3 shows the comparison of the current study with past studies as the plot of shear modulus ratio and shear strain which is showing a well agreement. Figure 3 shows the plot corresponding to 0.5% longitudinal strain. In the first cycle of loading, the shear modulus value (kPa) of Solani sand was observed as 10.098Mpa which ultimately diminishes to 0.196MPa as the test completed, it shows appreciable degradation in strength. The variation of shear strain (%) with number of cycle has been shown in Fig. 4.

Shear modulus were calculated at different strain amplitudes. Max shear modulus (G_{max}) is considered as the slope of shear stress vs shear strain plot at zero cyclic strain amplitude. In this study G/ G_{max} was calculated and compared with earlier study. Shear modulus (G) was obtained as:

$$G = \frac{E}{2(1+\mu)}$$

Where *E*= modulus of elasticity, μ = poisson's ratio

E is the slope of deviatoric stress vs strain plot. Deviatoric stress is obtained as a ratio of load and cross sectional area (38.465 cm^2) whereas strain is the ratio of deformation and length of the specimen (14 cm). At a strain rate of 1%, max value of E was obtained as 30.294 MPa which further diminished to 0.588 MPa due to higher deviatoric stresses. Also, the maximum cyclic stress ratio obtained for 0.1 % strain was 0.08 which rises to a value of 0.25 at 1% strain. Cyclic stess ratio is calculated as the ratio of maximum cyclic shear stress to the initial effective confining pressure.

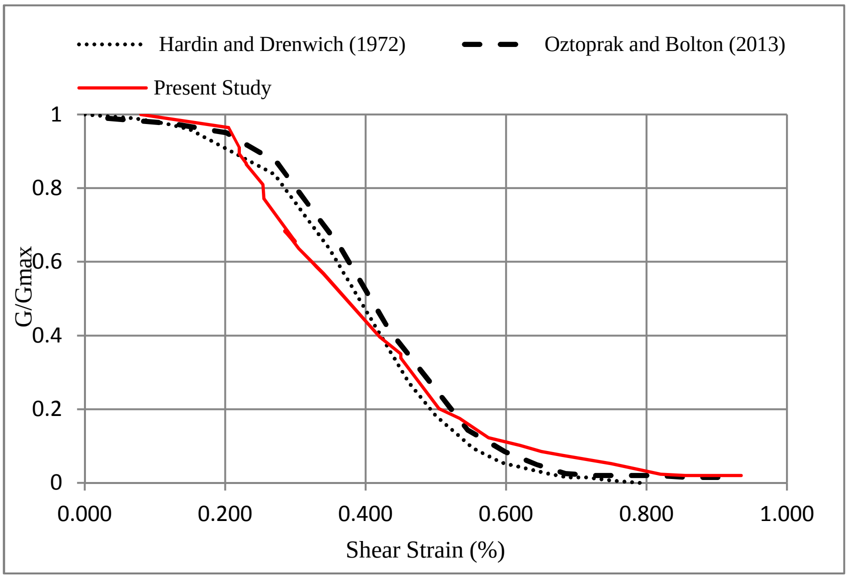


Fig. 3. Variation of G/G_{max} with Shear Strain and comparison with past experimental results.

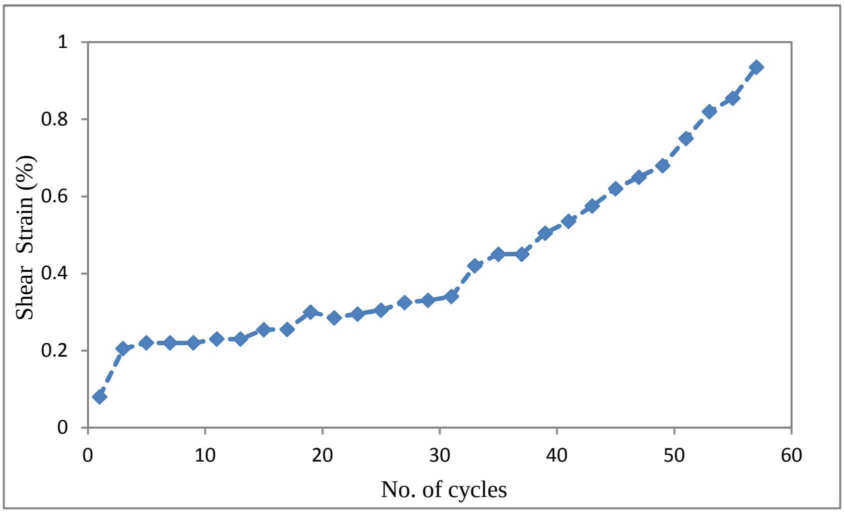


Fig. 4. Variation of Shear Strain with the number of cycles.

3 Conclusions

Many researchers performed experiments using CTT to obtained dynamic properties (mostly shear modulus and damping ratios). From the above literature study and small experimental study, the following conclusions can be drawn:

1. In general, the shear modulus increase with relative density, confining pressure, PI of the soil. Whereas the damping ratio decreases with those factors. Therefore, it can be suggested that the available test results can be used for numerical simulation of any structure placed on soil.
2. Type of loading should be varied (irregular, real earthquake motion etc.) as most of the study reported only for sinusoidal waves subjected to the soil mass.
3. Dynamic behavior and the numerical study of clayey soil is quite less compared to experimental study on sand.
4. The effect of different contaminated water on dynamic behavior of soil at large strain needed to be explored. Moreover, in few studies recycled materials are used for liquefaction mitigation. Such waste material which is more polluted can be used for the liquefaction mitigation study.

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