DYNAMIC ANALYSIS OF BEAMS ON VLASOV FOUNDATION

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Abstract. In the present study, a workable approach for dynamic analysis of beams on Vlasov foundation under various types of load like stepped load, suddenly applied load, harmonic load etc are considered. Closed form solution for simply supported boundary condition is developed and solutions are also obtained by finite element formulation. The modulus of elasticity and Poisson ratio of the soil is assumed constant from top surface to assumed rigid base to evaluate the two soil parameter k and 2t.

Keywords: Soil-structure interaction; Stepped load; Vlasov foundation; Shear modulus.

Introduction

Numerous studies have been performed to estimate the dynamic response of the beams resting on Vlasov foundations. Several researchers had tried to improve the Winkler model by considering the shear strain energy in the soil in addition to the strain energy for normal strains as used in the Winkler model. Two-parameter foundation models account for the displacement continuity of the foundation which is the major defect of the Winkler foundation by the introduction of a second parameter. The two-parameter foundation models derived by Filonenko Borodich [4], Hetenyi [5] and Pasternak [9] provide for the displacement continuity of the soil medium by the adding of a second spring which interacts with the first spring of the Winkler model and Vlasov and Leontíev [14] who made simplifying assumptions to the formulation of elastic continuum foundations by introducing functions for the distribution of displacements in the soil medium. Of all models, a two-parameter model by Vlasov using a variational method has attracted the attention of many engineers. The Vlasov model accounts for the effect of the neglected shear-strain energy in the soil and the shear forces on the beam edges that come from the surrounding soil. To measure the value of the vertical deformation parameter within the subsoil, Vallabhan and Das [11 - 13] developed an iterative technique to solve problems of beams on elastic foundations by introducing a modified Vlasov model. Ayvaz and Ozgan [2] considered the modified Vlasov model to analyze the free vibration of beams resting on elastic foundations. Kim and M. S. Kim [7] have considered vibration of beams with general restrained boundary conditions. Yongjun Lei, Michael I. Friswell and Sondipon Adhikari [8] have considered vibration of beams, with a nonlocal viscoelastic foundation model using the finite element method. Alkim Deniz Senalp,

AytacArikoglu, Ibrahim Ozkol,and Vedat Ziya Dogan [1], have considered dynamic response of Euler-bernoulli beam on linear and nonlinear viscoelastic Winkler foundations to a concentrated moving force. Hizal and Çatal [6] study the dynamic analysis of axially loaded beams on modified Vlasov foundation. Thambiratnam and Zhuge [4] have formulated a basic model to study the dynamic behavior of beam on a flexible foundation under moving load condition.
Dynamic analysis of beams on Vlasov foundation is vast topic. In this study, an efficient method is introduced for the analysis of the free and as well as forced vibration behavior of Euler-Bernoulli beams on an elastic foundation.

List of symbols

L	Length of beam
b	Width of beam
E_s, v_s	Young's modulus and Poisson's ratio of soil
p (x,t) or f (x,t)	Load per unit length
2t, k	Foundation parameter
V(x)	Shear force
M(x)	Bending moment
EI(x)	Bending stiffness
l	Length of beam element.
w	Displacement
P	Point load
M_0, M_A	Point moment
N	Shape function
T	Time
M	Modal mass
K	Modal stiffness
m_1	Mass per unit length of the beam.
m_0	Equivalent mass of soil participating in vibration.
γ	Unit weight of beam material.
g	Acceleration due to gravity.
ξ	Damping ratio
ω	Circular Frequency

Mathematical formulation

Vlasov derived the equation for the bending of the beam using Euler's assumptions. Assuming a displacement variable, w in the vertical direction describing the lateral displacement of a uniform beam as the primary parameter, the soil continuum is assumed to have a finite depth with zero displacements at the bottom. In other words, he assumed that the deformable soil is resting on a rigid rock.

For simple bending theory $EI\frac{\partial^2 w}{\partial x^2} = -M \text{ and } \frac{\partial M}{\partial x} = V$

$$\therefore EI\frac{\partial^4 w}{\partial x^4} = -\frac{\partial^2 M}{\partial x^2} = -\frac{\partial V}{\partial x}$$

$$EI\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} = f(x,t)$$

for free vibration $f(x,t) = 0$

$\therefore$ For a simply supported beam $\emptyset(x) = C_2 sin\beta x = sin\frac{n\pi x}{l}$

The value of $C_2 = 1$ make that max^m value of $\emptyset(x) = 1$

$$\therefore \left(\frac{n\pi}{l}\right)^4 = \frac{\omega^2 m}{EI} \ or \ \omega = \frac{n^2\pi^2}{l^2}\sqrt{\frac{EI}{m}}$$

$$\therefore w(x,t) = \sum_{n=1}^{\infty} sin\frac{n\pi x}{l}(Acos\omega_n t + Bsin\omega_n t)$$

Dynamically deformed state

Force equilibrium in the y -direction:

$$V - V - dV - p(x,t)dx = 0 \quad \therefore \frac{\partial V}{\partial x} = -p(x,t)$$

$$\frac{\partial V}{\partial x} = -EI(x)\frac{\partial^4 w}{\partial x^4} + \frac{\partial}{\partial x}\left(2t\frac{\partial w}{\partial x}\right)$$

$$f(x,t) - kw + \frac{\partial V}{\partial x} - c\frac{\partial w}{\partial t} = m\frac{\partial^2 w}{\partial t^2}$$

$$\therefore EI(x)\frac{\partial^4 w}{\partial x^4} + m\frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} + kw - \frac{\partial}{\partial x}\left(2t\frac{\partial w}{\partial x}\right) = f(x,t)$$

For a uniform cross sectional beam the governing equation is

$$EI\frac{\partial^4 w}{\partial x^4} - 2t\frac{\partial^2 w}{\partial x^2} + c\frac{\partial w}{\partial t} + kw + m\frac{\partial^2 w}{\partial t^2} = f(x,t)$$

PARAMETER

$$k = \frac{E_0}{(1-v_0^2)}\int_0^H \left(\frac{d\varphi}{dz}\right)^2 dz; \ 2t = \frac{E_0}{2(1+v_0)}\int_0^H \varphi^2(z)\,dz;$$

$$m_0 = \rho_s\int_0^H \varphi^2(z)\,dz; \ E_0 = \frac{E}{(1-v^2)} \ \& \ v_0 = \frac{v}{(1-v)}$$

E, v is the Young's modulus of elasticity and poison's ratio of soil

where ρ_s is the mass density of soil.

E_0, v_0 is the effective modulus of elasticity and effective poison's ratio of soil

H is the effective finite depth of the foundation that is dynamically activated.

Vallabhan and Das (1988) has been shown that

For thick layer variation of

$$\varphi(z) = \frac{sinh\gamma(1-\frac{z}{H})}{sinh\gamma}$$

$$k = \frac{E(1-v)\gamma}{(1+v)(1-2v)H}\left(\frac{sinh\gamma cosh\gamma + \gamma}{2sinh^2\gamma}\right);$$

$$2t = \frac{EH}{2\gamma(1+v)}\left(\frac{sinh\gamma cosh\gamma - \gamma}{2sinh^2\gamma}\right); m_0 = \frac{\rho_s H}{\gamma}\left(\frac{sinh\gamma cosh\gamma - \gamma}{2sinh^2\gamma}\right)$$

For thin layer linear variation of

$$\varphi(z) = \left(1 - \frac{z}{H}\right); k = \frac{E(1-v)}{H(1+v)(1-2v)}; 2t = \frac{EH}{6(1+v)}; m_0 = \frac{\rho_s H}{3}$$

γ is the parameter denotes the vertical deformation within subsoil.
Now how obtained the γ parameter

$$\left(\frac{\gamma}{H}\right)^2 = \frac{1-2v}{2(1-v)} \frac{\int_{-\infty}^{\infty} \left(\frac{dw}{dx}\right)^2 dx}{\int_{-\infty}^{\infty} (w)^2 dx}$$

Solution of the governing equation of beams on elasticfoundation is dependent on the vertical deformation profile φ(z), which in turn depends upon verticaldeformation parameter within the subsoil.
Taking beams has simply supported end conditions.

$$w(x,t) = \sum_{n=1}^{\infty} \frac{2}{l} W sin \frac{n\pi x}{l}$$

For a step force or a point load at distance'd' from left support.

$$\therefore W = e^{-\omega_n \xi t}(A cos\omega_d t + B sin\omega_d t) + \frac{P}{m{\omega_n}^2} sin \frac{n\pi d}{l}$$

Using initial condition, $w = u_0 ; \frac{dw}{dt} = v_0 ;$ at $t = 0;$ solve for A and B.
Hence velocity, acceleration and other responses.

Finite element formulation

Beam element stiffness matrix

$$K_b = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix};$$

Matrix for first foundation parameter

$$[K_f] = \begin{bmatrix} \frac{13bkl}{35} & \frac{11bkl^2}{210} & \frac{9bkl}{70} & -\frac{13bkl^2}{420} \\ \frac{11bkl^2}{210} & \frac{bkl^3}{105} & \frac{13bkl^2}{420} & -\frac{bkl^3}{140} \\ \frac{9bkl}{70} & \frac{13bkl^2}{420} & \frac{13bkl}{35} & -\frac{11bkl^2}{210} \\ -\frac{13bkl^2}{420} & -\frac{bkl^3}{140} & -\frac{11bkl^2}{210} & \frac{bkl^3}{105} \end{bmatrix}$$

Matrix for second foundation parameter

$$[K_e] = \frac{2tb}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & l^2 \\ -36 & -3l & 36 & -3l \\ 3l & l^2 & -3l & 4l^2 \end{bmatrix}$$

Kinetic energy $,T=\frac{1}{2}\rho A\int_0^l\left(\frac{\partial w}{\partial t}\right)^2 dx=\frac{1}{2}\{d'\}^T\rho A\int_0^l [N]^T [N]dx\{d'\}$

For mass matrix

$$[m] = \rho A\int_0^l [N]^T [N]dx$$

$$[m] = \frac{\rho Al}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

Stiffness matrix $[K] = [K_b] + [K_f] + [K_e]$

Hence obtain $[x] = [K]^{-1}\{f\}$

Boundary conditions need to be applied before solving equation of system and applying the boundary condition at x = ∞ and x = - ∞ one can get the boundary force √(2kt)w(0) and √(2kt)w(L) so the axial stiffness at start and end of the beam is √(2kt) in the general boundary conditions with proper sign.

For calculation of damping

$$\frac{1}{2}\begin{bmatrix} \frac{1}{\omega_i} & \omega_i \\ \frac{1}{\omega_j} & \omega_j \end{bmatrix}\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{bmatrix} \xi_i \\ \xi_j \end{bmatrix}$$

Results and discussion

This chapter starts with some comparisons with similar studies done by other researchers are made. The present method can be applied to analyze the dynamic response of a beam with various boundary conditions, including free-free, subjected to all types of loadings.

An example has been chosen from the study done by M. I. Friswell, S. Adhikari and Y. Lei [8] beam on Winkler foundation. The material properties for the beam, foundation and load are presented in Table 1

Table 1

Length of the beam	6.096 m.
I of the beam	0.001439 m^4
E of the beam	24.82×10^6 KN/m^2
Mass per meter of the beam	446.3 kg/m
Foundation parameter, k	16550KN/m^2
Foundation parameter,Gb/2t	0

The natural frequencies (Hz) of vibration for theSimple beam, modeled with 10 finite elements, on an elastic foundation.

Table 2

Mode No	**Analytical**	**M.I. Friswell**	**Present Study**
1	32.898	32.898	32.8984
2	56.808	56.812	56.8119
3	111.90	111.95	111.9536
4	193.76	194.08	194.0755

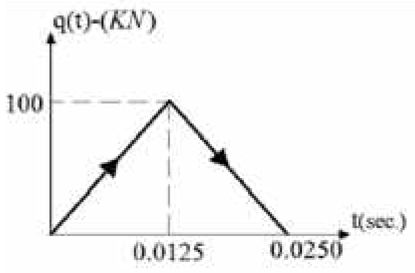

Fig. 1. Dynamic external load

The above problem has been chosen for forced vibration and dynamic external load which was applied to this beam midpoint shown in figure 1 above.

Table 3

	Sapountzakis[10]	**Hizal and Catal [6]**	**Present Study**
Max.	2.630	2.635	2.635
Min.	-2.500	-2.482	-2.489

The above problem has been chosen on Vlasov foundation with following modification.

Table 4

The vertical deformation parameter within the subsoil, γ	1
E of the Soil	20000KN/m^2
Mass density of the soil	1700 kg/m^3
Poison’s ratio of the soil	0.25
Depth of rigid base	5 m.

The natural frequencies (Hz) of vibration for theSimple beam, modeled with 10 finite elements, on an elastic foundation.

Table 5

Mode No	**Analytical**	**Present Study**
1	10.0760	10.0759
2	29.5349	29.5332
3	63.5815	63.5639
4	111.5388	111.4276

Mid-point displacement response

Table 6

in mm.	Present Study
Max.	4.83
Min.	0

The above problem has been chosen on Vlasov foundation with a full UDL of 50 KN/m and free-free boundary condition.

Mid-point displacement response

Table 7

in mm.	Present Study
Max.	43.22
Min.	0

A finite simply supported beam is considered.

The material properties for the beam, foundation and load are presented in Table 8

Table 8

Length of the beam	5.0 m.
Width of beam	0.5 m
Depth of beam	0.4 m
Where response is required from left end	2.50 m
E of the beam	2.1×10^5KN/m^2
Damping ratio	0.05
Initial displacement	0.0 m
Initial velocity	0.0 m/s
Mass density of beam material	7850 kg/m^3
Concentrated load	500.00 KN
Position of load from left end	2.5 m
The vertical deformation parameter within the subsoil, γ	1.988
E of the Soil	20000KN/m^2
Mass density of the soil	1700 kg/m^3
Poison's ratio of the soil	0.25
Depth of rigid base from top surface	5 m.

The results are presented below for the above beam and beam midpoint response in figure 2 and figure 3

Max^m static deflection=2.2048 mm.

Max^m Dynamic deflection =4.4003 mm.

Max^m Dynamic Velocity =0.46 m/s.

Max^m Dynamic Acceleration=1921.2 m/s^2.

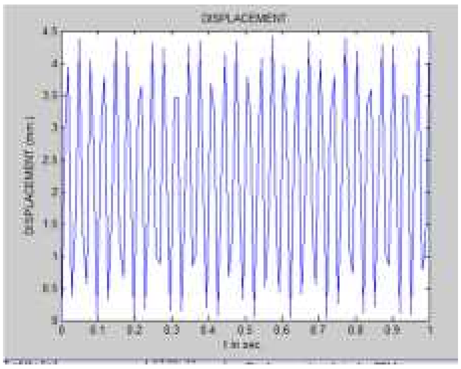

Fig. 2. Time response of beam midpoint (Displacement)

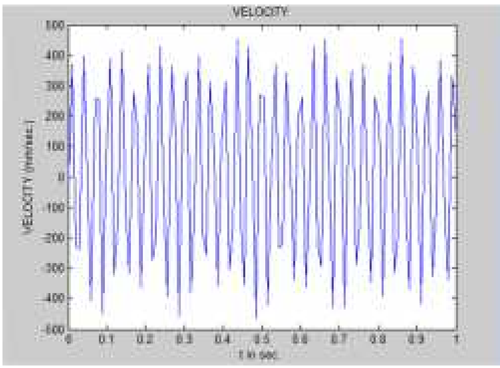

Fig. 3. Time response of beam midpoint (Velocity)

A finite free-free beam is considered.

The material properties for the beam, foundation and load are presented in Table 9

Table 9

Length of the beam	5.0 m.
Width of beam	0.5 m
Depth of beam	0.4 m
Where response is required from left end	2.50 m
E of the beam	2.1×10^5KN/m^2
Damping ratio	0.05
Initial displacement	0.0 m
Initial velocity	0.0 m/s
Mass density of beam material	7850 kg/m^3
Concentrated load	500.00 KN
Position of load from left end	2.5 m
The vertical deformation parameter within the subsoil, γ	0.875
E of the Soil	20000KN/m^2

Mass density of the soil	1700 kg/m^3
Poison's ratio of the soil	0.25
Depth of rigid base	5 m.

The results are presented below for the above beam and beam midpoint response in figure 4

Max^m static deflection=26.105 mm.
Max^m Dynamic deflection =52.05 mm.
Max^m Dynamic Velocity =1.1 m/s.

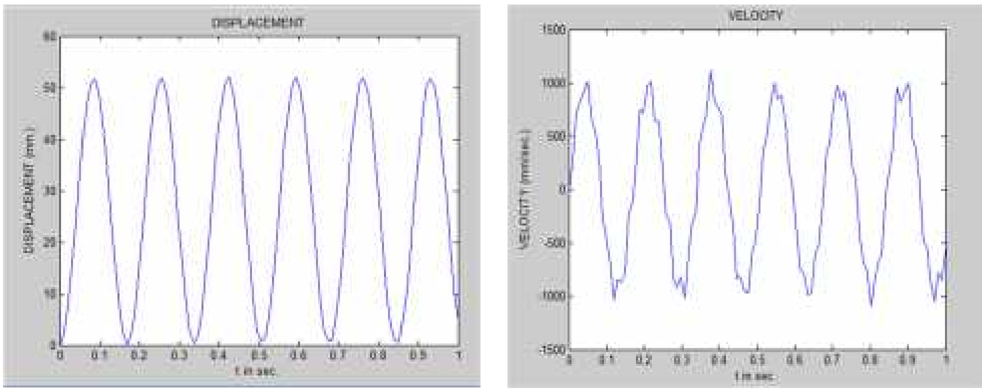

Fig. 4. Time response of beam midpoint (Displacement and velocity)

Conclusions

In this study the effect of the shear deformation of the beam on the dynamic response were investigated on Vlasov foundation.

- The maximum deflection of a beam resting on two-parameter Vlasov foundation is smaller than that of the beam on the Winkler foundation.
- The dynamic displacement response is symmetric with respect to the location acted by the suddenly applied load.
- It is also found that the dynamic deflection of the beam on a two-parameter Vlasov foundation decreases with the increased sub-grade shear modulus.

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