Seismic Stability of Unsupported Conical Excavation in Clayey Ground

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Abstract. Conical excavations without support are often made in some field projects, such as foundations for piers, oil and water tanks, etc. The seismic stability of these excavations has been given least importance in the research field. The instability of these unsupported excavations might increase during a seismic event. Hence, a realistic estimation of the seismic stability of unsupported excavations is a prominent issue, especially for the perpetual purpose. In the present study, seismic stability number for unsupported excavations has been proposed using the Finite Element (FE) analyses. Due to symmetry in the geometry about the vertical axis, axisymmetric FE models have been developed using OptumG2 software based on the finite element limit analysis approach. The soil (clay, with isotropic and constant undrained shear strength property) has been modelled as a perfectly plastic Tresca material following an associated flow rule. Boundary conditions were considered as per the field conditions and seismic loads were considered as pseudo-static force applied in terms of horizontal seismic coefficient, α_h. Adaptive meshing technique was adopted to predict the results reasonably closer to exact solutions. The seismic stability of these conical excavations are reported in terms of stability number. Moreover, the effects of depth of excavation, inclination angle, intensity of seismic loading, etc. on the seismic stability number are explored in detail. For deeper insights and influence of various affecting parameters, non-dimensional design charts are presented. Further, it has been observed that the considered parameters have significant effects on the seismic stability of conical excavations.

Keywords: Seismic stability, Conical excavation, Clay, Axisymmetric, Finite Element Limit Analyses.

1 Introduction

Generally, unsupported excavations are found as primary stage in many projects related to construction of footings, piers, columns, underground water tanks, etc., (as shown in Fig. 1). For cohesive soils, unsupported open vertical or inclined cut, can be achieved by excavating from the ground surface to the final depth of these structures, is still one of the most practical and economical methods for temporary excavations to construct the foundations of the structures. To minimize the ground movement, a temporary retaining wall, such as a braced or cantilever sheet pile wall, is usually employed for the open excavation of such constructions, but this requires the additional expense of installing a retaining wall. In the recent years, unsupported excavation problems has gained attention of most of the geotechnical projects due to its practical importance. A series of detailed investigations has been carried out on assessing the stability of these unsupported excavations. Evaluating the stability of unsupported excavation is one of the most important aspect to ensure safety in both short and long span.

Fig. 1. Unsupported excavation adjacent to an existing building.

Initially, remarkable contributions to estimate the stability of unsupported vertical circular excavations in both cohesive and cohesive-frictional soils. The most notable contributions, associated with the stability of vertical circular excavations, have been made by [1-7]. Based on the limit equilibrium method, the stability of cylindrical excavations formed in cohesive-frictional soils was examined [2,3]. However, these solutions were approximate in nature because of an inherent assumption associated with the geometry of the failure mechanism. With the help of an upper bound rigid block mechanism, the stability of cylindrical excavations in clays whose shear strength increases linearly with depth was evaluated [6,8]. These analyses also require an assumption with reference to the geometry of the collapse mechanism. By using an elastoplastic finite-element analysis, the stability numbers for a cylindrical excavation formed in cohesive-frictional soil was computed [9]. By using an axisymmetric lower bound limit analysis and three dimensional limit analysis, in conjunction with finite elements and linear/nonlinear programming, a series of studies are available in the literature to determine the stability numbers for an unsupported vertical cylindrical excavation [5,10,1,11,12]. Among these computational studies for cylindrical excavations formed in homogeneous cohesive soils [5,10,11]. On the other hand, an unsupported cylindrical excavation made in cohesive frictional soils was also explored [1,12]. By using the three-dimensional finite-element limit analysis and nonlinear programming provided an upper bound solution for this excavation stability problem [1]. On account of the inherent difficulties involved in carrying out a three-dimensional analysis [1,13], it remains, however, always a difficult task to obtain the corresponding three-dimensional solution. Based on an axisymmetric lower bound formulation, the stability numbers for an unsupported vertical cylindrical excavation in clays whose undrained shear strength increases linearly with the depth was obtained [11]. Stability of unsupported axisymmetric excavation has been explored in the past for homogenous clays using lower bound (LB) and upper bound (UB) analyses by [6] and [10]. Later on, the stability of unsupported vertical excavation was obtained using LB and UB finite element limit analysis (FELA) considering linearly increasing cohesion of soil with depth [11,14] and cohesive frictional soils [12,14]. Recently, detailed parametric study using FELA has been carried out to estimate the stability of unsupported conical excavation in homogeneous and nonhomogeneous clays [15] and cohesive frictional soils [16]. From existing literature, it has been found that stability of unsupported conical excavations in clay subjected to seismic loading is mostly lacking and needs to be studied in detail.

The main objective of the present study is to estimate the seismic stability number, N_c, of unsupported conical excavations in homogenous clay subjected to seismic loading (in terms of horizontal seismic coefficient, α_h) through FELA using 2D axisymmetric finite element models developed in OptumG2. The parametric study has been perform by varying dimensions of conical excavation, shear strength parameter and seismic loading. Based on this study, the design charts are developed to estimate the seismic stability number, N_c, of unsupported conical excavations.

2 Finite Element Modelling

In geotechnical engineering, limit analysis and FELA have been extensively used in the past to study various complex stability problems [17,18]. FELA, combination of limit analysis with finite element discretization, is widely used to bracket the exact limit load by upper-bound (UB) and lower-bound (LB) solutions of limit analysis (assuming a rigid-perfectly plastic material with an associated flow rule) for handling complex problems in geotechnical engineering with irregular geometries, varying soil properties, loadings and boundary conditions [19,20]. Under plane strain and axisymmetric condition, both LB and UB problems (in OtumG2) are formulated using second-order cone programming (SOCP) [21,22] to solve geotechnical stability issues. The details of the numerical formulation of FELA in OptumG2 has been elaborated [23].

The procedure adopted for obtaining the seismic stability number, N_c, of unsupported conical excavations in homogenous clay through FELA using OptumG2 is summarized below. Taking advantage of axis-symmetry, 2D axisymmetric finite element models of the conical excavations for seismic condition has been modelled with its geometric parameters (excavation height, H, radius of excavation bottom, b, and slope inclination, β), as represented in Fig. 2. The extents for FE model has been chosen based on sensitivity study so that the failure shear zone does not intersect the right and bottom boundary, and have insignificant effect on the computed results. Based on sensitivity study, the optimal sizes of the FE model along horizontal and vertical directions has been limited to 8H and 3H, respectively.

The soil mass as homogeneous clay (with undrained shear strength, S_u, and unit weight, γ) has been modelled as the rigid-perfectly plastic Tresca material with the associated flow rule. The soil mass has been discretize with 6-noded triangular elements in UB analysis, where each element (with its own unique nodes) has continuous normal and shear stresses at nodes and stresses jump are allowed across shared edges of adjacent elements. However, unknown velocity components (with a quadratic function within the element) are continuous across adjacent elements. In optumG2, UB problem is formulated as SOCP, which satisfies the kinematically admissible velocity constraints, generated from velocity boundary conditions and compatibility equations with associated flow rule on average for 6-noded triangular elements. For the conical excavation problem, the objective function of UB SOCP corresponds to the minimization of the soil unit weight.

In case of LB analysis, the soil mass has been discretized into 3-noded triangular elements. In optumG2, the LB problem is formulated into SOCP satisfying the statically admissible stress constraints, generated from equilibrium equations at centroid of each triangular element and along the stress discontinuities, stress boundary conditions with no violation of the yield criterion for all nodes. For the conical excavation problem, the objective function of LB SOCP corresponds to maximization of soil unit weight.

As this conical excavation problem is formulated in axis symmetry plane and seismic load (as shown in Fig. 2), applied on the entire soil mass as pseudo-static forces in terms of horizontal seismic coefficient, α_h, is non-axis symmetric. To simulate the seismic effect, non-axis symmetric load needs to be converted to axis symmetric. Generally, loading applied to a solid or shell of revolution can be described as the sum of its series components [24]. Let $\alpha_{h,\,3D}$ represent seismic load load in general, such as normal pressure, line load, or temperature in thermal stress analysis. A Fourier series representation of $\alpha_{h,\,3D}$ is presented in Eq. 1 where, $\alpha_h{}^c$ and $\alpha_h{}^S$ are load amplitudes that depend on n (but not on θ). Here n is an integer that represents the harmonic number. Loads described by $\alpha_{h,\,3D}$ can be radial, axial, or circumferential.

$$\alpha_{h,3D} = \sum_{n=0}^{\infty} \alpha_h^c \cos n\theta + \sum_{n=0}^{\infty} \alpha_h^s \sin n\theta \qquad (1)$$

By locating the θ = 0 plane appropriately, it often happens that only one of the two series is needed. Typical problems are solved accurately enough by using only the first few terms of the load series. When a radial load $\alpha_{h,\,3D}$, partly outward and partly inward but otherwise uniform, is represented by the sine series. Cosine terms of Eq.1 are not needed (although the loading could be described by cosine terms alone if the θ = 0 plane were placed 90° to the position shown). A uniformly distributed load, alternately inward and outward, and its representation by a Fourier series (Eq. 2). Thus, by computing the value of α_h in radial, axial or circumferential model the appropriate value of α_h in 3D can be obtained by using Eq. 2:

$$\alpha_{h,3D} = \frac{4\alpha_h}{\pi}\sin\theta \tag{2}$$

Automatic mesh adaptivity was also employed for both lower and upper bound analyses to determine colse upper and lower bound solutions. Adaptive meshing with five iterations was used for all the analyses where the number of elements were increased from 7,000 to 10,000 to obtain results closer to exact solutions. Using adaptive mesh technique gives the advantage of predicting the failure surface exactly which is evident through the small element size adjacent to the rupture area and large elements elsewhere. The seismic stability number, N_c estimated using Eq. 3, is dependent on excavation height, H/b; slope angle, β; and seismic coefficient, α_h.

$$N_c = \frac{\gamma H}{S_u} = f\left(\frac{H}{b}, \beta, \alpha_h\right) \tag{3}$$

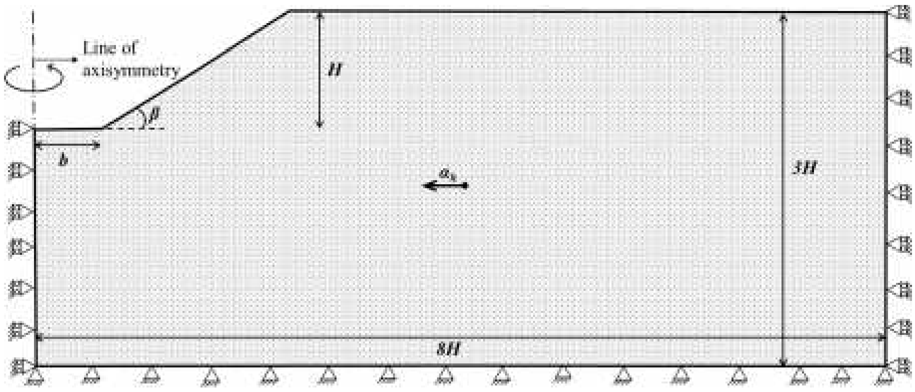

Fig. 2. Axisymmetric numerical model of unsupported conical excavation.

3 Model Validation and Comparison with Past Studies

The developed model is validated for vertical excavation (β = 90°) subjected to static loading (α_h = 0 g) with both lower [11,12,25,16,7] and upper [6,9,14,16,1] bound solutions as shown in Fig. 3 (a) and (b) respectively. The developed FE model predicts the results closer to the results reported in literature while employing lower bound solutions. Whereas, when employing upper bound solution the predicted results are quite far away especially with the results reported based on rigid block mechanism [6] and elastoplastic FE analysis [9]. Further, in this study the average of both the upper and lower bound solutions have been used for interpretation in the following sections.

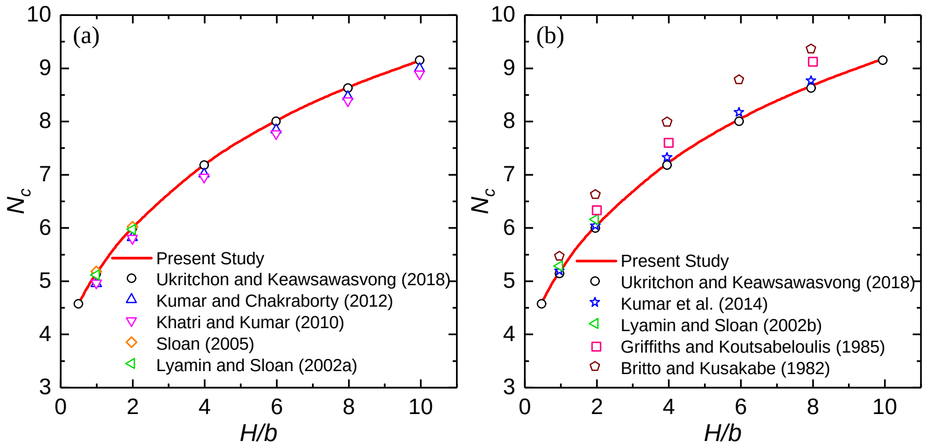

Fig. 3. Validation of the developed model using (a) LB and (b) UB solutions.

4 Results and Discussion

An extensive numerical study has been performed to obtain the seismic stability number, N_c for unsupported conical excavations. The results have been presented for β = 45 - 90°, H/b = 0.5 - 10 and α_h = 0 - 0.2g. The variation of N_c with H/b subjected to increasing loading (α_h = 0, 0.05, 0.10, 0.15 and 0.20 g) for four different excavation angles (β = 90°, 75°, 60° and 45°) has been presented in Fig. 4. It is interesting to note that N_c decreases with increase in α_h at all excavation angles, but the decrease is more prominent at lower β values. In all considered cases, N_c increases with increase in H/b.

The variation of failure surface for vertical (β = 90°) and conical (β = 75°, 60° and 45°) excavations with H/b = 1, 2 and 4 subjected to α_h = 0.10 g has been presented in Fig. 5. With the increase in H/b the soil mass contributing to failure decreases at a particular value of β. However, this decrease is significant at lower β values (i.e. β = 60° and 45°). For a considered H/b value, with the decrease in β the failure surface widens leading to increase in the soil mass participation.

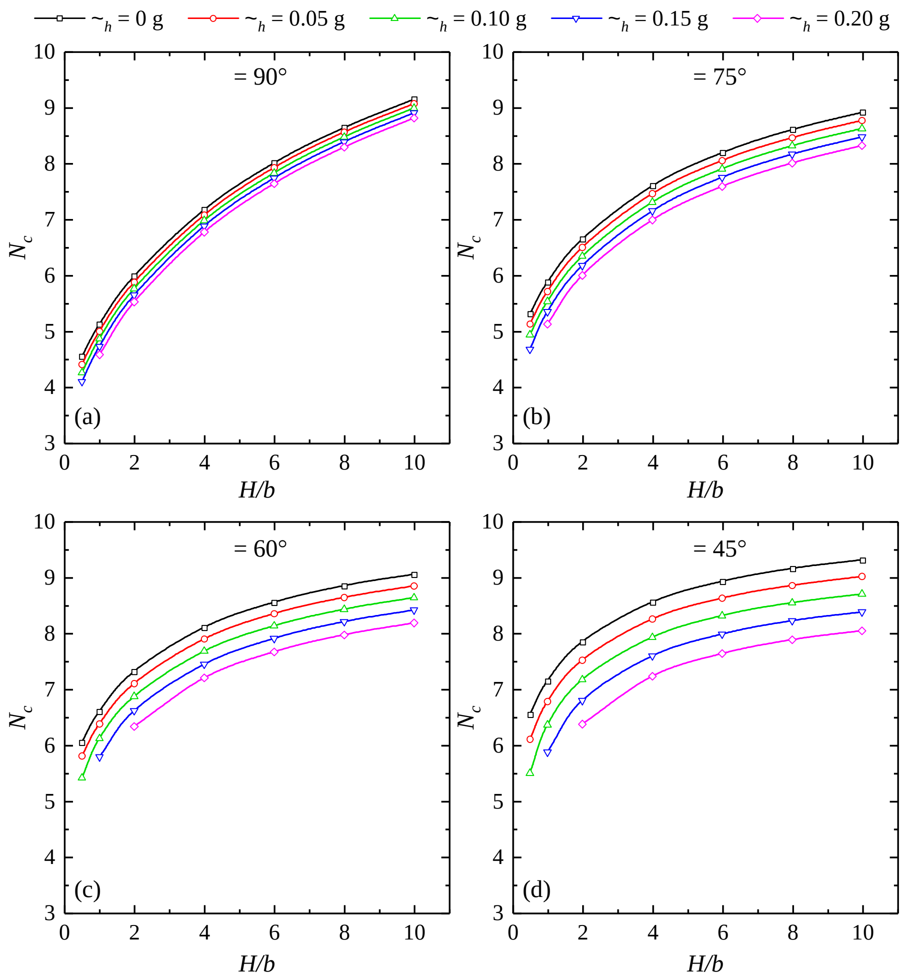

Fig. 4. Variation of N_c with H/b for different α_h and (a) β = 90°, (b) β = 75°, (c) β = 60°, and (d) β = 45°.

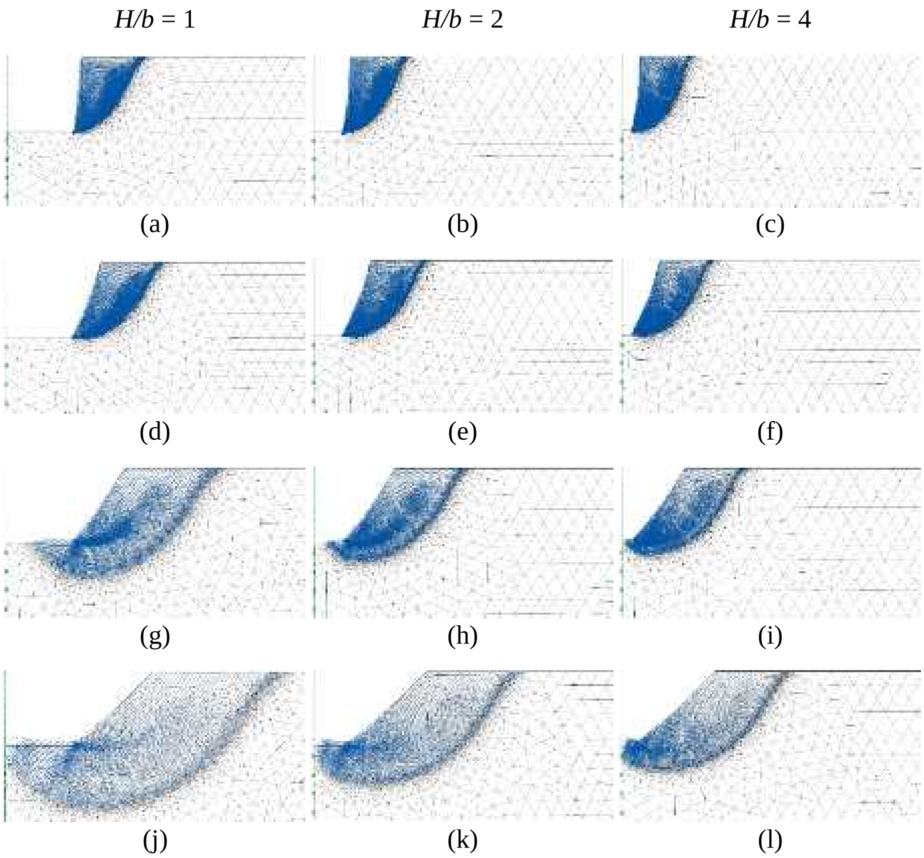

Fig. 5. Variation of failure pattern with *H*/*b* subjected to α_h = 0.10 g for (a-c) β = 90°, (d-f) β = 75°, (g-i) β = 60° and (j-l) β = 45°.

The variation of N_c with varying α_h and *H/b* (0.5, 1, 2, 4, 6, 8 and 10) for four different excavation angles (β = 90°, 75°, 60° and 45°) has been presented in Fig. 6. It can be observed that at a particular β, N_c decreases with the increase in α_h. This decrease in N_c is more rapid at lower excavation angle, β. In all the considered cases, it has been observed that with the increase in *H/b*, N_c also increases. However, this increase is insignificant towards higher *H/b* for lower excavation angle, β (i.e., 60° and 45°).

The variation of failure surface for vertical (β = 90°) and conical (β = 75°, 60° and 45°) excavations with *H/b* = 2 and subjected to α_h = 0, 0.10 and 0.20 g has been presented in Fig. 7. It can be observed that for a β value, with the increase in seismic load intensity the soil mass contributing to failure also increases. For a particular α_h value, with the decrease in β the failure surface widens leading to increase in the soil mass participation.

In all cases, it is interesting to note that (in Fig. 5 and 7) the failure surface mostly passes through either toe (for β = 75° and 90°) or base (β = 45° and 60°) of the conical excavation.

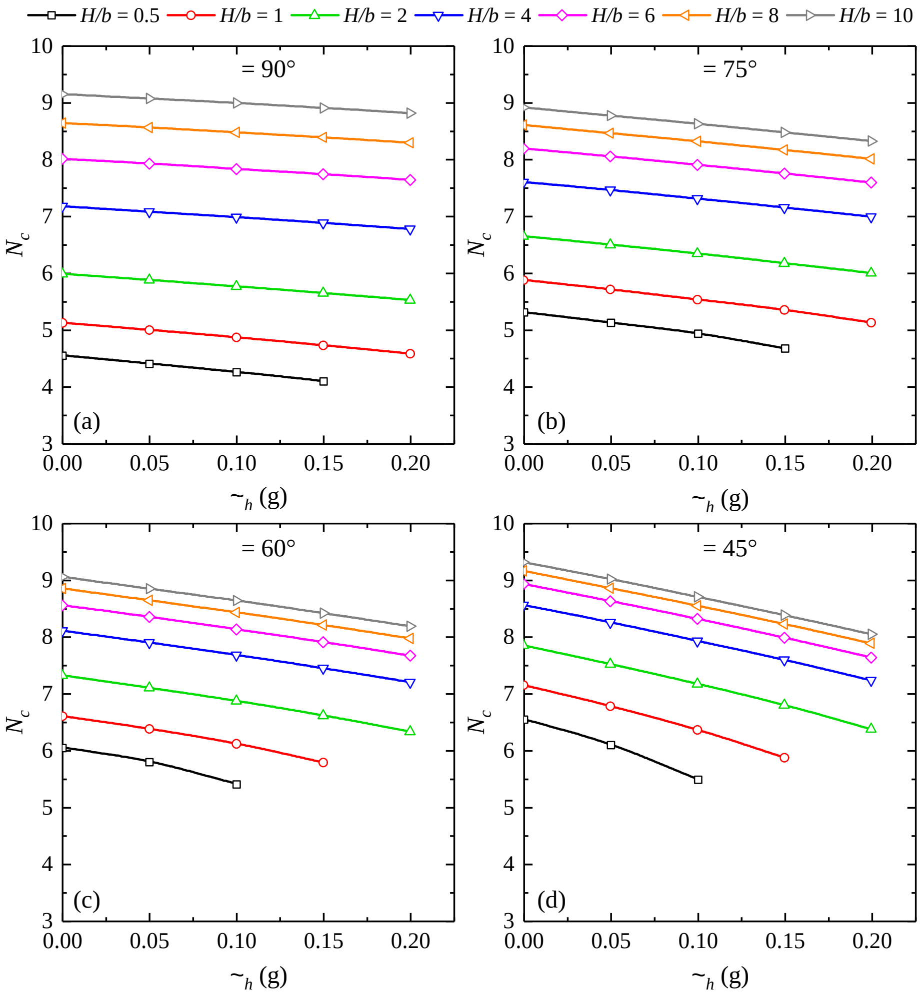

Fig. 6. Variation of N_c for α_h for (a) β = 90°, (b) β = 75°, (c) β = 60° and (d) β = 45°.

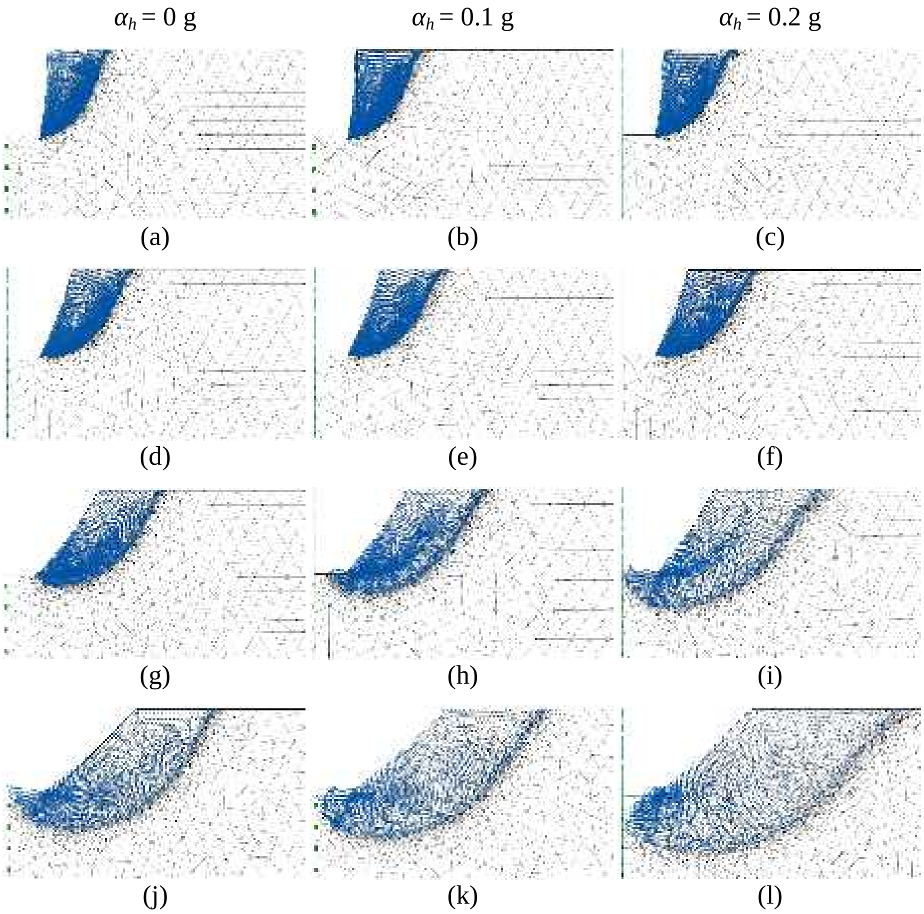

Fig. 7. Variation of failure pattern for *H/b* = 2 with α_h (a-c) β = 90°, (d-f) β = 75°, (g-i) β = 60° and (j-l) β = 45°.

5 Conclusions

A numerical study has been performed to estimate seismic stability number of undrained conical excavation subjected to seismic action by employing UB and LB solutions of FELA. It has been observed that for a particular β, the soil mass contributing to failure also increases with the increase in seismic load intensity. For a particular α_h value, the failure surface widens leading to increase in soil mass participation with decrease in β. It has also been found that, the failure surface for steep cut (β = 75° and 90°) passes through toe of conical excavation. However, in case of less steep cut (β = 45° and 60°) failure surface always passes through base of conical excavation.

It has been observed that N_c decreases with increase in α_h at all β, but the decrease is more prominent at lower β values. In all considered cases, it has been observed that with the increase in *H/b*, N_c also increases. However, this increase is insignificant towards higher *H/b* for lower excavation angle, β (i.e., 60° and 45°).

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