An Observation of Methods of Pile Analysis Subjected to Different Load and Soil Condition

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Abstract: Design of Structures, subjected to lateral loads are very much dependent upon lateral response of pile foundations. The lateral load acting on foundations may be sustained as earth pressure in a retaining wall, or alternated, as reciprocating machinery, as pulsated, from the traffic loading on a bridge pier. The response of pile which is laterally loaded is considered complex soil-structure interaction problem; because deflection of pile is dependent on soil reaction and soil reaction influences the pile deflection. A good number of methods have been developed for the analysis of laterally loaded piles like chin method, p-y relationships, p-y multiplier, modified p-y curves and strain wedge method etc. An observation of a few methods is presented here with their limitations and capabilities of analysis of piles in different conditions of load application and soil type.

Keywords: *Analysis of Piles, Chin Method, P-Y Curves, Strain Wedge Method*

1 Introduction

Prediction of performance of pile under load can be calculated from many methods including very simple to complex with some where finite element methods are also used. In his Rankine Lecture, Poulos (1989) mentioned list of methods [8]. He not only tried to focus on variety of methods but also stressed on importance of input information on results, as they are dependent on quality of input. It is very difficult to obtain easily interpretable input data from routine procedure of site investigation. So a simpler approach is required which can correlate general properties of soil with site experience and which are easy to under stand by users.

The hyperbolic method used by Chin was to plot the behavior of footing and piles. This method is used to determine ultimate loads and is very popular though it does not link with the soil parameters.

Fellenius (1980) and others have found that the Chin method appears to over predict when they compare the results of analysis of Chin method and other methods for defining ultimate loads. However, there is little doubt that in most cases, according to the plotting method, linear functions represent pile performance very well.

Chin (1970, 1972) expressed his method by $A/P = mA + C$, where A is settlement of pile head, P is numerical value of applied load and C is a constant. Thus if A/P is plotted against an abscissa of A, a linear plot is obtained and the inverse slope l/m gives an asymptotic limiting value of P. This is true for piles, where most of the load is carried out by shaft friction and also for footings and piles where end bearing carries the most of the load as explained by Chin. A typical relationship between pile head settlement A and settlement divided by load A/P is shown in the Figure. It has been suggested by Chin & Vail and has often been accepted where (A) of the relationship represents shaft friction while the second part (B) represents total load. This cannot be true for every case because of the nature of hyperbolic functions, but it can easily be accepted that individually shaft and base performance are of hyperbolic form.

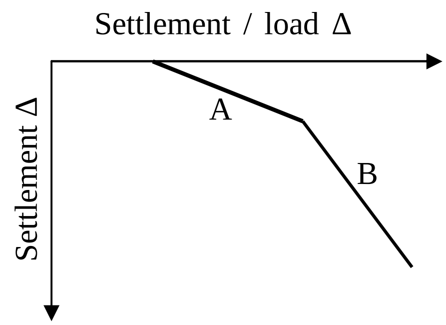

Fig. 1: Relationship of settlement and settlement / load

Chin [6] suggests that mobilization of stress in soil with increase of strain is a function of increasing number of effective soil contacts rather than of a general increase of inters granular stress on a constant number of grain contacts. He suggests that inter granular stress in flocculated clay, for example, is virtually constant and independent of the applied or effective stress. On this basis of hyperbolic function of stress-strain relationship developed by him, it may be understood that when soil is under compressive stress, the load is transmitted by internal column shaped grain structures. As these reaches limiting loads number of columns begin to support this each having nearly similar yield load.

While laterally loaded piles are designed, lateral bearing capacity is calculated at failure, with deflection and moment of a pile under working load. Sometimes, while considering deformation controlling and serviceability requirement, the deflection and bending moment along the shaft is of greater importance in that. The over deformation and overstress make the moment overshoot the allowable moment of the pile materials and induce additional deformation for superstructures.

Active research have been carried out to find working load behavior of laterally loaded pile for many years and many approaches were proposed for load deformation analysis of piles subjected to lateral loads, such as Classical Winkler Model (subgrade reaction method), p-y Curve Method and Continuum Analysis proposed by Xu et al. In continuum analysis, the soil is considered as a three-dimensional continuum and numerical techniques such as boundary element methods, integral equations or finite element methods are used to give numerical solutions for laterally loaded pile. The others keep treating the pile as a flexible material and surrounding soil as number of horizontal linear springs for Winkler and discrete nonlinear springs for p-y curve method. Because of its simplicity in comparison to continuum analysis, beam on foundation model was more acceptable to practicing engineers due to accuracy and precise prediction of responses of laterally loaded piles. Vesic's expression for Winkler Model is more in use in comparison to lot of various expressions developed by research because it is simple and it includes properties of both soil and beam.

2 Chin Method:

A number of methods for prediction of settlement of single plies and pile groups and its analysis are available including load-transfer method, empirical methods, elastic continuum method, the 'hybrid' type of approach etc. Guo et al. presented an infinite layer approach for single pile and Cheung et al., presented the same for pile groups.

2.1 Validity of Chin's Method:

To check Chin's method, 50 loading tests were carried out by Serge Borel et al. [15] under MLT procedure which was limited to head settlement greater than 10 % of dia. of pile and each load step was maintained for 60 minutes of time. Every test carried out with utmost care and all readings were digital. Load cells were used to check load and linear potentiometers to measure settlements. Measurement of shaft and toe resistances was taken by removable extensometers. In all 50 tests, various pile techniques and variety of soils were used. With pile diameters ranging from 200 mm to 1000mm, load applied were from 675 kN to 10,000 kN with pile length vary from 5m to 45 m. During tests, four piles were loaded in tension. Out of 50 piles tested, bored (6), impact driven steel and concrete (17), CFA (12), vibratory driven (5) and screwed displacement auger (10) were tested. There were 20 piles in clay, 15 in sand, 8 in calcareous and 7 in mixed soil.

Two different approaches were used to check the performance of Chin method: 1) complete load settlement curve and 2) partial load settlement curve taken into account, which was obtained by gradually removing the last points as if the test was stopped at lower loading step. To determine the result of ultimate resistance from the complete curve, each test result was plotted in hyperbolic coordinates to find part B of load settlement curve which is useful to calculate Chin parameters. Except in five cases, the Chin method estimated very satisfactory ultimate resistance. The five cases where plunging failure were observed which was not anticipated by Chin method, ultimate resistance was 25% more than estimated. As per the second approach, the load settlement curves were altered by removing last point one after another to check sensitivity of Chin method for head settlement to the number of loading steps. A graphical representation is shown for the test carried out at Mittersheim site. To obtain a shortened load settlement curve, last two points out of 13 were removed. The maximum settlement observed is 3.3 % of pile diameter against 92 % of measured ultimate resistance. The same is repeated for other readings and 1.4 % of settlement is observed against 81 % of ultimate resistance.

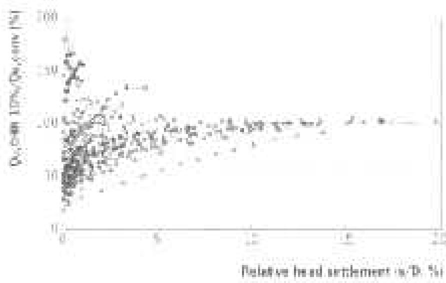

Fig. 2: Performance of Chin Method

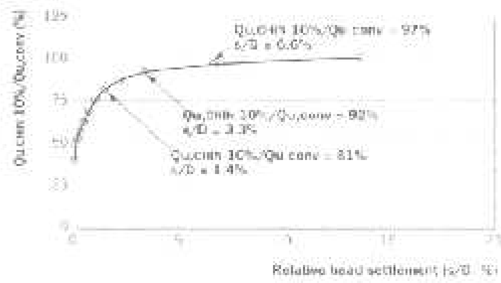

Fig. 3: Prediction of the ultimate resistance

The main conclusions for the Chin method are as following: It is observed that Chin method generally over predicts the shaft resistance. It was also observed that when shaft resistance is more than 55%, predicted shaft friction varies from 120% to 60% of the measured value.

3 P-Y Method

3.1 Introduction:

When a structure is subjected to earthquakes, high winds, ship impacts and wave action, it is essential to design lateral load capacity of pile foundations. For single pile design under static load, many researchers have proposed different methods but for closely spaced pile groups, where spacing is less than six pile diameters, a very few methods are available for reference. It is seen many times that piles in front row show similar kind of load-displacement curve as shown by single pile, but piles in trailing rows show less resistance in comparison with front rows. This phenomenon is known as shadowing. The effect of shadowing becomes less as the spacing between piles increases and becomes unimportant once the spacing exceeds about six times pile diameters center to center. Due to this uncertainty, design of pile groups is very conservative. Some numerical methods [2] and models are available for this but a large number of full scale tests are essential for verification and to provide real information about the ground.

Two popular ways to analyze laterally loaded piles both numerically or analytically are: Continuum based methods and Load transfer methods. The most popular method is ‘p-y’ curve, which is simple and reliable hence widely used. These curves can be produced by measuring soil resistance [1] directly or by measuring strain along the length of pile indirectly. There are some short comings due to differentiation procedure when ‘p-y” curve is generated indirectly as P values are sensitive to pile curvature [3]. Research shows that when a lager diameter shaft is used, stiffer response is measured than that of general curves.

3.2 P-Y Curves for Laterally Loaded Piles in Sand:

Reese

The best available criterion for the analysis of behavior of single vertical pile in sand is subjected to static or cyclic loading suggested by Reese et al. To check the accuracy of prediction of behavior of piles which are laterally loaded they are compared to measured analytical results of load tests [11]. For the purpose of analysis, different assumptions were made in selecting soil properties to correlate with the results of standard penetration test. Hence all soil properties are not exact but were the best available from the information collected.

Arkansas River

Furgo and Associates have carried out a large number of lateral load tests for Corps of Engineers at a site on Arkansas river in Arkansas. Soil properties reported for all test piles were similar but because of differences in loading condition, pile stiffness etc., a number of different analyses were performed. Some of the results are presented here. One boring was selected as representative of soil conditions at the test site which was done in a 20 feet deep test pit prior to testing the piles.

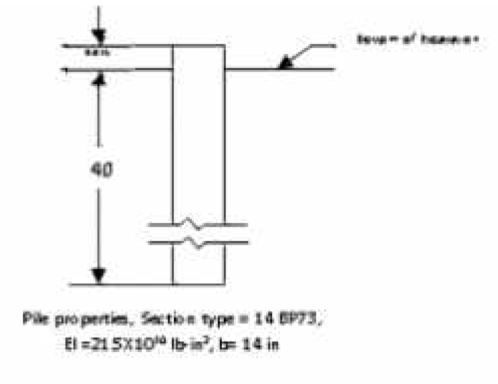

Fig. 4 :Test set up for test pile 6 at Arkansas River

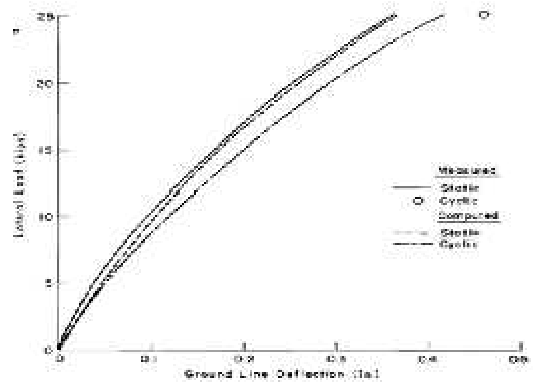

Fig. 5 :Comparison of measured and computed deflection for test pile 6 at Arkansas River

Among number of piles, test pile 6, which was 14BP73, driven vertically, short-term and cyclic loading applied is presented here. The graph representing measured and computed deflection shows good similarities for static loading. For cyclic loading only one deflection was reported. The computed deflection underestimates the measured deflection by 10%.

Apapa

Number of load tests were carried out on Raymond step tapered piles near Apapa, Nigeria. The load applied was short-term static lateral load. Only one analysis was performed for test piles 1 and 2 because of similarities in test conditions. These piles were driven vertically and reinforced concrete was filled in steel shells. The soil profile at site was a layer of soft organic clay below 5 feet thick hydraulically placed sand. The sand layer governs the pile behavior but clay layer is having some influence on it. Laboratory triaxial test and in situ vane test were used to find angle of internal friction and c. A graphical representation is shown to compare measured and computed deflections by load-deflection curves which are dissimilar in shape. This also represents that initial slopes were nearly similar for shorter loads but at lager loads beyond 11 kips, the curves diverge much.

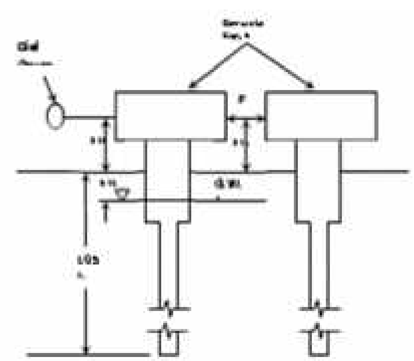

Fig. 6 :Information for the analysis of tests at Appa

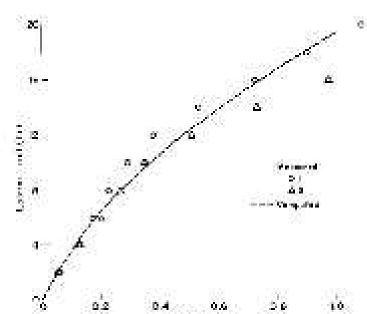

Fig. 7 :Comparison of measured and computed deflection for tests at Appa

Bailly

Bergstrom (1974) reported tests of two 14BP89 piles at a site for proposed nuclear power plant which were driven vertically and loads were applied to free head piles at 1.5 feet above the ground surface. Maximum load applied incrementally was 39 kips, unloaded and then cycled 25 times at 22 kips.

For the test piles TP7 and TP8, a graphical representation is shown. At site, soil was fine sand, loose to moderately dense. As the soil properties were similar for both test piles only one analysis was carried out. It can be seen that computed and measured deflections are very similar other than at 39 kips load point, where computed deflection is 12 % more than measured one. The load-deflection curve for 25 cycles of loading also shows great similarities for measured and computed deflections though curve was developed from results of submerged sand it works equally for soil above water table.

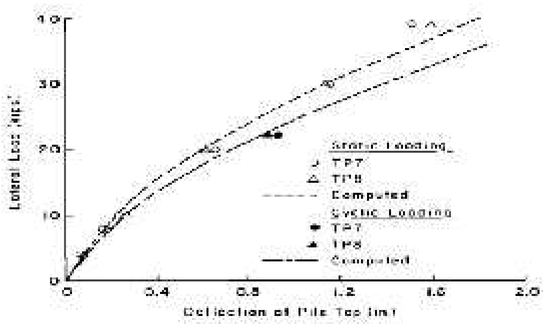

Fig. 8 : Comparison between measured and computed deflection of tests at Bailly

Conclusion for Sand (Reese):

Researchers have shown many measured versus computed deflections analysis from the tests carried out in sand, which were very much in agreement with the criteria adopted for p-y curve method though some results were conservative, but overall the comparison between measured and computed results for piles embedded in sand show excellent to very good for most of the cases at working loads.

Development of p-y curves for sand by other Authors

Khari et al.

In 2014, Khari et al. [12] developed other set of p-y curves for sand with very limited parameters with their experimental model which was for piles in offshore construction. The test set up includes soil tank with dimensions 900mm X 700mm X 65mm, soil thickness below pile tip was at least 6D, two LVDT to measure deflection and seven levels of strain gauges to measure bending moments. The soil samples were collected from Johor Bahru, Malaysia and tested at laboratory temperature. Among many methods of reconstructing sand samples, they have used pluviation and tamping to prepare the samples.

Results of Khari et al.:

Single fixed-head smooth and rough piles in loose and dense sand were chosen for series of tests. T44 and T45 were tests on smooth piles while T48 and T48 were tests on rough piles in loose and dense sand samples. Sand was considered loose when relative density was 30 % and dense when relative density was 75 %. Incremental loads were applied to test piles in all cases. As seen from the figure due to active length of pile, it did not show any deflection at a depth of 26 to 28 cm. Differences in deflection at ground surface is also visible against applied load in all cases of pile type and relative densities. Computed results with integration process and measured by LVDT show similar nature. It is also observed that smooth pile in loose sand demonstrates 200 % larger deflection than in dense piles. For rough piles observed values of deflection were 175 % and 23 % larger for relative densities 75% and 30% compared to smooth pile.

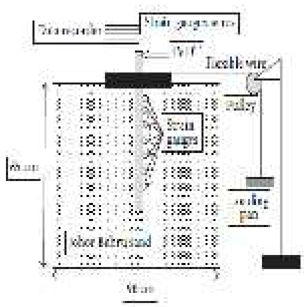

Fig. 9 : Schematic diagram for the test carried out by Khari et al. (2014)

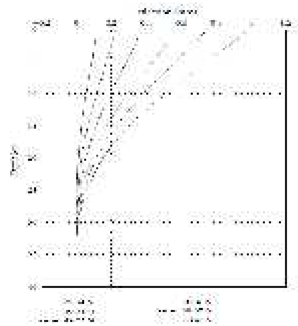

Fig. 10 :Deflection versus Depth, Khari et al., (2014)

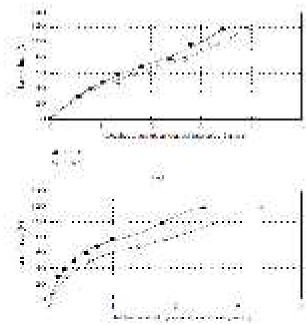

Fig. 11 :Deflection at ground surface versus lateral load (a) Dr =30 %, (b) Dr = 75 %

Conclusion – Khari et al.:

Khari et al., conducted number of experiments on their model to determine and compare their results with popular methods. They have adopted some modification factors to consider density of soil and wall friction. The p-y curves generated using strains recorded along the length of pile as hyperbolic function showed good values in comparison with older graphs available.

3.3 P-Y Curves for Laterally Loaded Piles in Clay

Reese

Numbers of lateral load tests [13] were also carried out on clay. To check the efficiency of p-y curve method for the prediction of behavior of test piles in clay thorough analysis was done with reference to location of site wherever possible.

Test results: Bagnolet

A graphical representation for test piles B1 and B4 in uniform deposit of stiff clay, carried out at Bagnolet, east of Paris, France is shown. As the water table was below the pile tips, Stiff A criteria was used for analysis. Comparison of both test piles shows good results for B1 pile for measured and computed maximum moments, while some scattered results can be seen for B4. For deflections, both piles show fair comparison for measured and computed deflections.

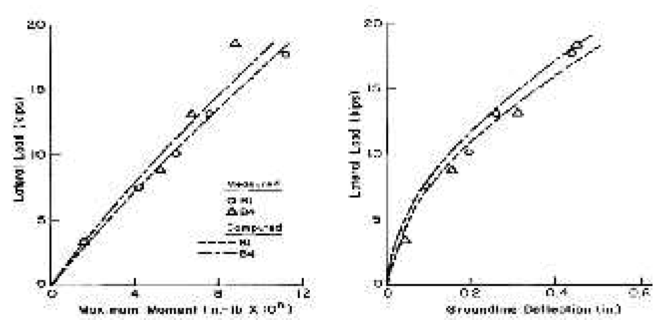

Fig. 12 :Comparison of measured and computed results for test pile B 5 at Bagnolet

Bay Mud

To have different type of soil, site at Bay Mud was chosen for Gill for the purpose of testing. The soil which is classified as CH was composed of insensitive, slightly organic silty clay. Two test conditions were defined and eight tests were carried out. For piles D1, D2, D3, and D4 water table was below ground level at 7.5 feet provided dry soil condition and for these piles Stiff A criteria was used for analysis. For test piles F1, F2, F3 and F4 soil was wet for number of days. Soft B and Unified criteria were used for analysis of these piles. Type of piles used for all tests were same but diameter was from 4.5 to 16 in. As seen in the figure measured deflection for D1 and D2 both are large than computed results on non-conservative side by 33 %. For D3 and D4, measured deflections are also larger.

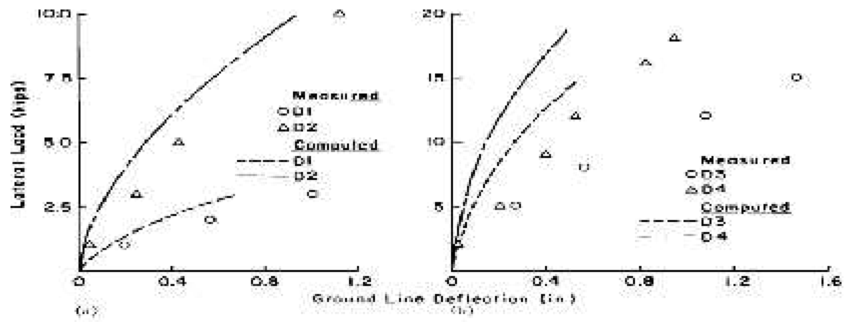

Fig. 13 : Comparison of measured and computed deflection for tests at dry bay mud site (a) for test piles D1 and D2 (b) for test piles D3 and D4

For the piles in flooded site, Comparison of computed deflection with measured deflection by Soft B and Unified criteria show that F1 and F2 show very good results for Unified criteria while F1 shows non-conservative results for Soft B criteria. While for test piles F3 and F4, for both criteria are used, measured deflections are more than computed deflections. For F3 test pile, difference of 53 % in Soft B criteria and a difference of 36 % in Unified approach were reported. Similar kind of results can be seen for F4 as well.

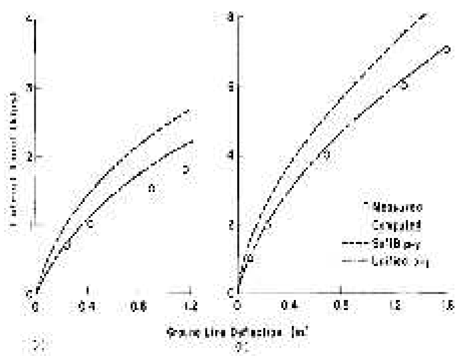

Fig. 14 : Comparison of measured and computed deflection For tests at flooded Bay mud site (a) for test pile F1

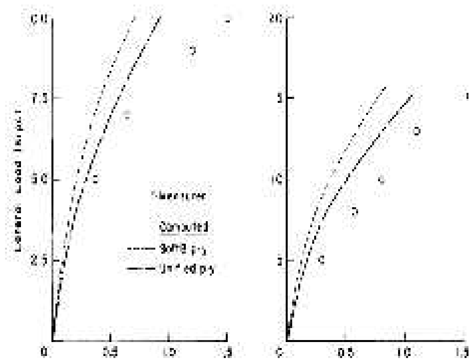

Fig. 15 : Comparison of measured and computed deflection for tests at flooded Bay mud site (a) and (b) for test pile F2 for test pile F3 and (b) for test pile F4

Conclusion (Reese):

As per the results provided by different researchers showed, while measured and computed results were compared relatively good results for piles in clay for most of the tests was seen at working loads. In those cases where there is noticeable difference in results, it was believed that it was due to lack of availability of better soil property results for analysis. It was observed through different cases that p-y method for the analysis of laterally loaded pile offers the best result among the various methods available. Results of full scale pile tests can make this method better and accurate results of soil properties will make this method more faithful in prediction of pile behaviour.

Kyle et al.

A pile group in 3 x 3 patterns with center to center spacing of three times pile diameter was tested by Kyle et al. [14] under static and dynamic lateral loads. It was a full scale test set up at Salt Lake International Airport. This assembly was accompanied by a single pile driven to 1.8 m depth. It was taken care that for both the cases soil properties should be the same. The piles used were closed ended steel pipes. To measure the resistance provided by each pile, strain gauges were attached to tie-rods. A computer program was designed for the analysis of results.

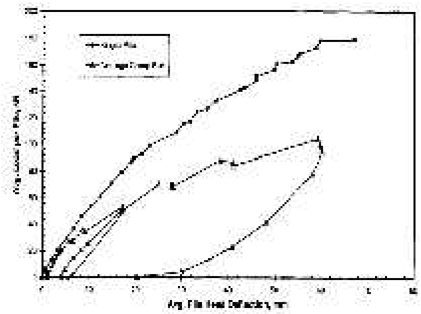

Fig. 16 :Average load versus deflection curve for single Pile test and pile group test

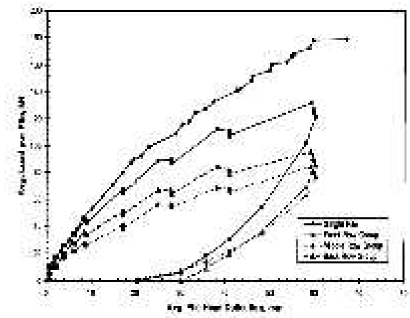

Fig. 17 : Average load versus and single deflection curve for every row of pile group pile

A graphical representation of Load-deflection curve for both single pile and group of piles is presented. For better comparison with single pile, average load carried by each pile of group was calculated by dividing total load carried by group with number of piles. It can be observed that for smaller displacements results are similar but for increased deflection average load carried by each group pile is considerably smaller than carried by single pile. It was also observed that for same pile, load displacement of single pile is noticeably smaller. It was very interesting to observe load-deflection curve of each row of pile group. When compared with single pile load-deflection curve it was concluded that load distribution in pile group is not uniform but depends upon the position of pile row.

Conclusion (Kyle et al.) :

It was concluded that single pile under given average load deflected very less in comparison with closely spaced pile group. It was also noticed that trailing row of pile group carried less load than load carried by front row, proving that load distribution is not similar and depends upon the row position. It was also concluded that single pile carried more load than load carried by piles in all rows.

3.4 Limitation of p-y Method

The basic limitation of p-y method is resistance due to shear deformation of soil because soil resistance against pile movement is not considered [8]. Apart from this, subgrade modulus is considered to be dependent on relative density of soil. Clarification is also required for the use of friction angles to be used in p-y equation required to plot p-y curve.

4 Strain Wedge Method

4.1 Introduction:

With a view to utilize traditional one dimensional Beam on Elastic Foundation concept, the strain wedge approach was developed to predict the response of flexible pile under lateral loading. Initially it was developed to analyze free head pile in uniform soil. Researchers have modified this to analyze pile in multiple soil layers and effect of pile head condition on behaviour of soil and pile both. The cutting edge of this method over other approaches is simplicity of required soil properties. Very common properties of soil are required for the analysis. Soil layers are divided into layers [9] of smaller thickness and each layer is considered as individual soil layer.

Properties Employed for Sand	Properties Employed for Clay
Effective unit weight (total above water table, buoyant below)	Effective unit weight
Void ratio, e, or relative density, Dr	Plasticity index, PI
Angle of internal friction, ø	Effective angle of friction, ø
Soil strain at 50% stress level, ε_{50}	Undrained shear strength, Su
	Soil strain at 50% stress level, ε_{50}

Las Vegas Field Test (Short Shaft)

An 8 feet diameter and 32 feet long reinforced concrete drilled shaft with 1 % longitudinal steel reinforcement was tested at Las Vegas as short shaft. A team of University of Florida incorporated FLPIER/COM624 programs for soil input data and same data were provided to strain wedge model. The strength of concrete at 28 days was assumed to be 5.0 ksi. It is observed from the graph that SW model has predicted very good results which nearly similar to measured results while results obtained from FLPIER/COM624 are very different than actual.

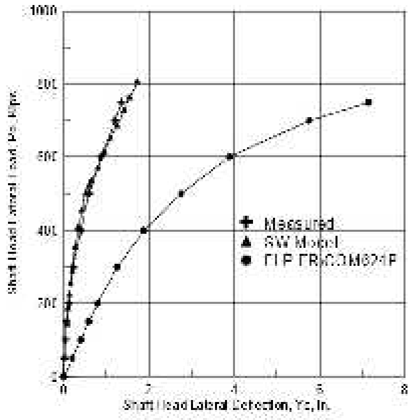

Fig. 18 : Measured and computed shaft response of Las Vegas Test

Full-scale Load Test on Pile in Liquefied Soil

With the help of soil properties reported by Weaver, Ashford and Rollins a number of full scale field tests were carried out at the Treasured Island [4]. The special characteristic of these tests was that they were carried out in liquefied soils. Controlled blasts were used to liquefy the soil without increasing its density. The sand at site contained 5% fines. The piles tested were of 0.61 m dia., long isolated pipe pile filled with concrete and laterally loaded at 1 meter above the ground.

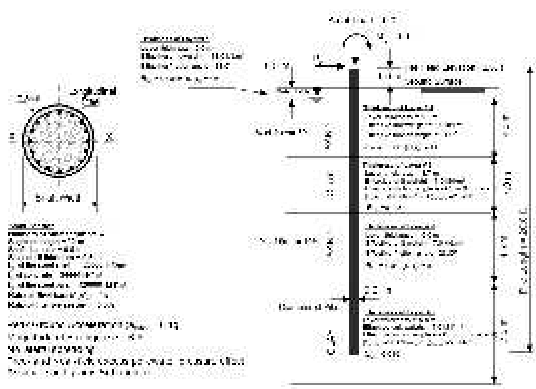

Fig. 19 :Soil and Pile properties for at Treasure Island test

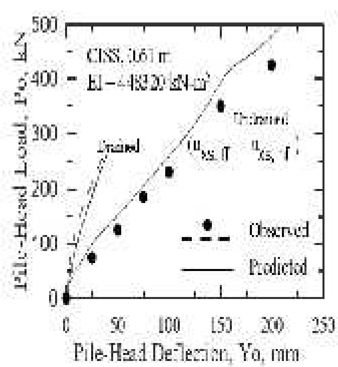

Fig. 20 : Post Liquefaction pile head deflection the test at Treasure Island

The piles tested at Treasure Island were loaded after the soil around it was liquefied for static drained test and same was observed for SW model. Observation of figure shows that SW model predicts the drained response in line with the observed response. The behaviour of pile in undrained post liquefaction status was with the effect of excess pore pressure, both free-field and near field. After providing the first blast at site and maximum ground acceleration assumed as 0.11g, piles were loaded for cyclic load. Figure shows very similar measured and computed undrained response. Analysis carried out with assumed maximum ground acceleration caused an excess pore water pressure in most of the sand which was in line with measured free field excess pore water pressure.

Table 1

Soil Layer Thick (m)	Soil Type	Unit Weight, ($kN/m^{3)}$	$(N_1)_{60}$	Ö (deg.)	$å_{50}$ %	S_u kN/m^2
0.5	Brown, loose sand (SP)	18.0	16	33	0.45	
4.0	Brown, loose sand (SP)	8.0	11	31	0.6	
3.7	Gray clay (CL)	7.0	4		1.5	20
4.5	Gray, loose sand (SP)	7.0	5	28	1.0	
5.5	Gray clay (CL)	7.0	4		1.5	20

The strength of SW model for prediction of p-y curves for piles or shaft in partially or fully liquefied soil can be seen from the comparison shown in graph. The traditional p-y curve method used with reduction multiplier fails to show concave upward movement for p-y curves which are back calculated.

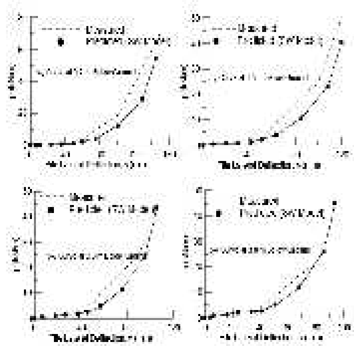

Fig. 21 : Predicted p-y curves using Strain wedge method versus observed ones from Treasure Island Test

After the first blast at site, upper 4.6 m length of pile was almost liquefied and pore water pressure was almost 1.0. This was measured after application of few cycles of load. When maximum ground acceleration was increased to 0.15g for analysis in SW model, soil was completely liquefied and load-deflection curve shows upward concave movement.

Full-scale Load Test on a Bored Pile in Layered Soil

Brown et al., carried out full scale load test on bored pile [4], 1.5meter diameter and 34meter deep below ground surface in Chaiyi, Taiwan. The soil at the site was layered in which both sand and clay were present. FLPIER program was used for analysis which showed very poor prediction of p-y curves for both sand and clay layers [10]. Brown carried out another analysis with FLPIER program with modified soil properties which gave very good correlation with measured test results.

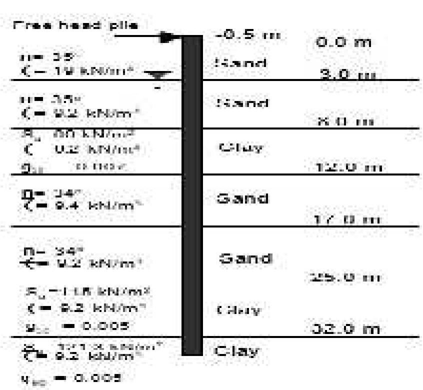

Fig. 22 : Soil profile and pile tested at Chaiyi test, Taiwan

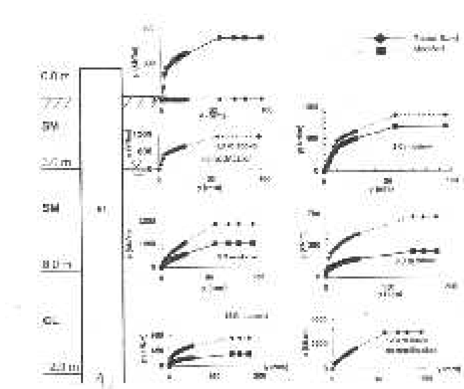

Fig. 23 : Traditional p-y curves modified to obtain good match with field data, Chaiyi Test

To compare the results with SW model, original soil and pile properties were used for analysis without any site specific modification. The response predicted single free head pile was comparative with results obtained through test. It should be noted that SW model uses non linear model for pile material behaviour which plays an important role in prediction of results.

Conclusion (Strain Wedge Method):

It was observed through various comparisons with field test data that SW model can carry out prediction of pile behavior without changing the original soil and pile properties and it is the biggest advantage of this approach. Other approaches like FLPIER/COM 624 etc., requires fairly moderate soil properties for prediction of pile behavior under laterally loaded condition [17].

5 Conclusion:

It was observed that Chin method generally over predicts the shaft resistance.

The p-y curve method which was initially developed by Reese and then Matlock is most widely used method for the analysis of laterally loaded piles. After incorporating the results of some full scale tests this method is more trustworthy as p-y curves can be back calculated. A lot of research is still going on to improve this method using different test instruments/techniques. For soil under test p-y curves are not unique as it is affected by soil and pile properties. This is the biggest drawback of this method. For partially and/or fully liquefied soil it does not show the upward concave movement which can be seen in SW model analysis.

The SW model uses original soil and pile properties and predicts the behavior of pile under lateral load nearly similar to the results obtained on field. The biggest advantage of this method is that it does not require modified soil properties for prediction. The non linear model for pile material is one of the reasons for its successful observations.

The current SW model can be made better by incorporating effect of vertical side shear resistance which has greater impact on lateral response of piles/shafts with large diameter. Apart from this, in many cases, piles with large diameter are designed as long shafts. The behavior of these shafts is similar to intermediate shafts which give softer response in comparison with large shafts. The p-y curve method is having this advantage over SW method that it can be used for all types of shafts/piles.

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