RELIABILITY ANALYSIS OF RETAINING WALL USING ARTIFICIAL NEURAL NETWORK (ANN) AND ADAPTIVE NEURO FUZZY INFERENCE SYSTEM (ANFIS)

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Abstract: Reliability of Geotechnical structures is the main concern of Geotechnical engineers as is clear from previous studies and evaluations. It also helps us to determine probability of failure. First order second moment method (FOSM) helps us to determine the reliability index of geo-structure. This study employs Artificial neural network (ANN) and adaptive neuro fuzzy inference system (ANFIS) for determination of reliability index of retaining wall based on sliding criterion. ANN has played a vital role in the field of geotechnical engineering as it has reduced cumbersome calculations and has increased the precision of result. The strong non-linear relationship between the known random variables and unknown output or result is mapped easily by using ANN. ANN also ascertains the result by removing the uncertainties involved in the problem. ANFIS is an ANN system which uses fuzzy logic in contemplating the data. It works on removing the fuzziness of the values entered (random variables) and gives more realistic values of the output as compared to other approaches. This study adopts ANN and ANFIS as regression techniques. The performance of ANN and ANFIS has been assessed based on different parameters such as coefficient of correlation, root mean square error, mean absolute error, etc. A comparative study has been presented between the FOSM, ANN based FOSM and ANFIS FOSM models. Therefore, this study concludes that ANN and ANFIS is a better alternative to solve for the reliability of the retaining wall.

Keywords: Retaining Wall, Reliability Index, ANN, ANFIS.

1. Introduction

Retaining wall, a geotechnical structure is of sheer importance for the stability of slopes. From geotechnical learning and rapid advancements, it is known that slopes fail due to different mechanisms. For instance, slopes suffer rotational failure, translational failure, compound failure, wedge failure and other failures in the form of flows and spreads. Many remedial measures are followed to avoid the failure of the slope and construction of retaining wall is among one of the remedies. For construction of a retaining wall the soil parameters that influence the bearing capacity of the soil along with the earth pressure are evaluated. Primitive parameters that define the failure are cohesion intercept, angle of shearing resistance, unit weight and angle of wall friction. Considering these parameters, the factor of safety is calculated. Also, to measure the ability to meet requirements under a specified period of time, reliability analysis is performed. For reliability analysis, First Order Second Moment Method (FOSM) is widely used but this technique is quite time consuming [1-2]. This problem has been remedied by the researchers by using certain other methods such as response surface method [3-4], multiple tangent plane surface [5], multi-plane surfaces method etc. which are used to solve the ambiguities of non-linear limit state surface. But these approaches are limited to nonlinear convex or concave surfaces only. This article performed Reliability analysis of retaining wall by using Artificial Neural Network (ANN) [6-7] and Adaptive Neuro Fuzzy Inference System (ANFIS). ANN has made progress in many fields like in medical, geotechnical, defense etc. Applications of ANN in geotechnical engineering are prediction of pile capacity, settlement of foundation, soil properties and behavior, characterization of site, determination of liquefaction potential, evaluation of stability of slopes, prediction of settlement of underground structures such as tunnels and estimation of maximum deflection of earth retaining structures [8]. ANFIS has also covered many areas of geotechnical engineering for example applications employed in triaxial testing, resonant column testing and liquefaction triggering. ANN and ANFIS amalgamates the different probabilities of occurring of events and pops up with accurate results considering all the possibilities as these are trained with set of data which when tested brings in modified correct output. The predicted values of the output are further used for reliability analysis. In this paper back propagation technique is employed in ANN and clustering technique in ANFIS. Also, the reliability index of the results generated from both ANN and ANFIS is calculated and compared thereafter.

2. Artificial Neural Network (ANN) model

Artificial intelligence has left a major mark in the computing field and other fields as well. ANN is a branch of artificial intelligence or precisely machine learning. Neural network is a representation of the human neural system. Networks here are defined using three components- transfer function, architecture of network and learning law. These components depend upon the type of problem to be solved. Using this algorithm machine can be trained to give appropriate result by changing the weights given to the inputs and using certain formulations. One of the renowned neural network is the back propagation network. Although there are several other algorithms as well but back propagation (in ANN) [9-10] is most versatile and robust among all. Back propagation is a concept of machine learning that works on reducing the cost function. After giving the first result and comparing it using the cost function, mechanism propagates backwards changing the weight factors and thereby bringing the change in the result until and unless it reduces the cost function resulting in accurate result. Neural networks trains and tests data like a human mind does. Mainly, back propagation's objective is to change or reduce the error in a quick response of time. Also, it uses partial derivative of cost function for all the weights individually. Cost function (equation 1) is nothing but root mean squared error (RMSE).

$$C = \frac{\sum |y(x) - a|^2}{2n} \tag{1}$$

Here C is the cost function, x is the input from the training set, y(x) is the observed output, n is the total number of input training set and a is the output from the model. Cost function is minimized in order to get results up to the mark.

Back propagation works in two phases. First phase is propagation in which setting and initialization of weights take place. Input is worked upon to generate proper output. Errors are calculated and then output is propagated back in the neural system to generate errors in the output and hidden MLPs. Second phase is concerned with updating weights of connections. In this phase calculation of the gradient of the weight is performed and certain percentage of this gradient (based on the learning) is subtracted from the weight. Each work of this technique is done in the hidden neuron layers where different composition of inputs with different weights is taken. A network can have many hidden neurons in accordance with the need of the problem. The back propagation technique is shown in the figure 1. It can be seen that a primary level neural network has one input layer, at least one hidden layer (there can be many depending upon the complexity of the input) and one output layer and all the connections are given particular weight. The hidden layers are also called perceptron which behave like human neurons. These can be contained in large number in a network to bifurcate large inputs into different possibilities. These Multi-Layer Perceptron (MLPs) are trained to give unbiased and learned results are these are highly capable of data mapping. The weights of the connections are altered accordingly depending upon the error and weight gradient.

This model is fed with four inputs required for calculating the factor of safety of the retaining wall based on sliding criteria i.e. are cohesion intercept (c), angle of shearing resistance (φ), angle of wall friction (δ) and unit weight (γ) and corresponding output to train the data. We have total 80 data out of which 70% is taken for training model and 30% is used for testing.

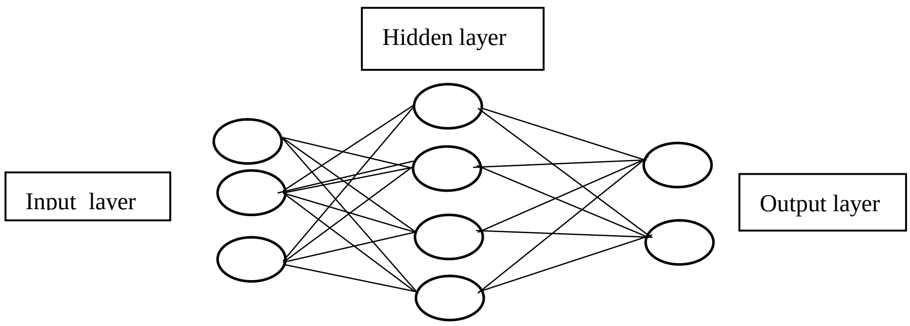

Fig. 1. ANN three-layer network

3. Adaptive Neuro-Fuzzy Interference System (ANFIS) model.

Role and perfection of ANN model is already explained in the previous section but a shortcoming of the ANN model is the complexity of the connection weights of MLPs which cannot be deciphered. Therefore, the rules defining the relation between input and output variable are difficult to quantify. To overcome this drawback, neurofuzzy models are used. These models are trained to provide data mappings. Also, it extracts knowledge about the relationship between model input and corresponding output data. ANFIS has removed the drawbacks of the other models in use and has provided us with accurate results comparatively. Advancement in modeling techniques has led to soft computing, artificial intelligence and fuzzy modeling system. ANFIS is a hybrid technique based on understanding of the researchers. Fuzzy logic works on ‘if then’ rules to establish a qualitative relation between input and output variables. It is a heuristic approach. Concept of clustering is used to resolve the problem. This approach is based on forming the unsupervised group of input and output data based on their similarities and dissimilarities. Neurofuzzy networks employ fuzzy conditional statement i.e. if-then rule. For instance, **If U then V** where U and V are labels of unsupervised fuzzy set. This rule makes us aware of contribution of set of inputs to the output. All the fuzzy logic systems have two components: sets and rules. To determine fuzzy sets linguistic terms are interpreted mathematically as membership functions and variables in the model are fuzzified to be fractional or partial members of the membership functions in the interval of (0,1). For each and every variable, fuzzy sets overlap and necessary range of variation is covered, this process is called fuzzification. Now, as the output of the fuzzified input is fuzzified too therefore defuzzification algorithm such as the mean of maxima ad center of gravity is applied, to get real valued outputs.

Fuzzy interference system is also known as fuzzy rule based system, fuzzy associative memories, fuzzy models or fuzzy controllers. This system is made of five blocks:

- *Rule base* (consists of neurofuzzy if-then rules)
- *Dataset* (defines membership functions of fuzzy sets used in the neurofuzzy rules)
- *Decision making unit* (performs interference operation on rules)
- *Fuzzification interface* (converts real valued inputs into degree of match with linguistic values)
- *Defuzzification interface* (converts fuzzified output into real value output)

Rule base and database are collectively referred to as *Knowledge base*. Neuro fuzzy system enhances the generalization capacity of the network i.e. when data inputted is beyond the training data set, networks inbuilt learning program helps it to extrapolate the result and revert back an appropriate result based on the learning. Learning process is knowledge based but data driven. Being adaptive of the system explains the dependency of nodes to the linked parameters. It supports the learning rule which minimizes the errors by making changes in the linked parameters. Figure 2 shows an adaptive network.

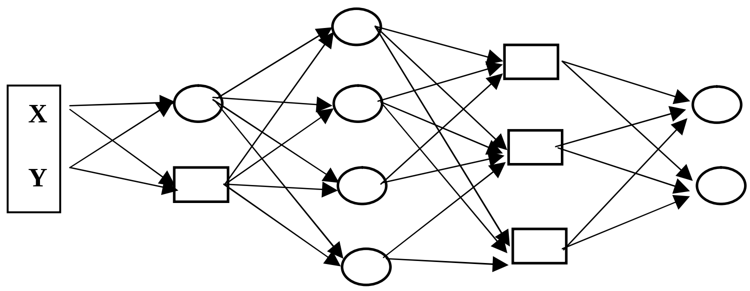

Fig. 2. An Adaptive Network

This model was fed with the same variables as in ANN with 70-30% bifurcation of data for training and testing respectively. Also, certain values are calculated which ascertain the model behavior and its accuracy in producing result. These values compare the results from both the models.

4. Model performance assessment and reliability index

Reliability index was calculated using First Order Second Moment Method which gave us the probability of failure of the retaining wall which is showed using reliability index 'β'. Deducing this value gives an overview of the structural reliability. And as it is a statistical value based on number of variations, when used for different models, function of model and accuracy of its prediction becomes understandable. This particular value along with other values predicts the failure probability of a model and the perfection of the model. As in this paper problem is modelled using ANN and ANFIS, values measuring their extent of perfection are given as follows:

1. Nash-Sutcliffe efficiency (NS) [11] indicates the predictive power of the models. More the NS value closer to 1 more is predictive power.

$$NS = 1 - \frac{\sum_{i=1}^{n}(d_i - y_i)^2}{\sum_{i=1}^{n}(d_i - d_{mean})^2} \quad (2)$$

2. Root Mean Square Error (RMSE) [12] value closer or equal to 0 indicates that the error in prediction is less.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(d_i - y_i)^2} \quad (3)$$

3. Variance Account Factor (VAF) [13-14] value equal to 100% shows model performance gives good result.

$$VAF = \left(1 - \frac{var(d_i - y_i)}{var(d_i)}\right) \times 100 \quad (4)$$

4. R^2 (Coefficient of determination) and Adj. R^2 (adjusted determination coefficient) [15] values should be closer to 1 and also closer to each other shows that the model used most of the variability in soil parameters.

$$R^2 = \frac{\sum_{i=1}^{n}(d_i - d_{mean})^2 - \sum_{i=1}^{n}(d_i - y_i)^2}{\sum_{i=1}^{n}(d_i - d_{mean})^2} \quad (5)$$

$$AdjR^2 = 1 - \frac{(n-1)}{(n-p-1)}(1 - R^2) \quad (6)$$

5. Performance Index (PI) [16] indicates the performance of the models.

$$PI = adj.R^2 + 0.01VAF - RMSE \quad (7)$$

6. Bias Factor is a factor whose value more than unity represents the overestimated model, value of less than unity represents an underestimation model, and a value of unity indicates a prediction, which is unbiased [17].

$$\text{Bias Factor} = \frac{1}{N}\sum_{i=1}^{n}\frac{y_i}{d_i} \quad (8)$$

7. Root mean square error to observation's Standard deviation Ratio (RSR) [18] have the benefit of error index statistics. More the value closer to 0 more the is the prediction power.

$$RSR = \frac{RMSE}{\sqrt{\frac{1}{N}\sum_{i=1}^{n}(d_i - d_{mean})^2}} \quad (91)$$

8. Normalized Mean Bias Error (NMBE) calculates the model's ability to predict a value which is away from the mean value. NMBE equal to 0 indicates perfect model [19].

$$NMBE(\%) = \frac{\frac{1}{N}\sum_{i=1}^{n}(y_i - d_i)}{\frac{1}{N}\sum_{i=1}^{n} d_i} \times 100 \quad (102)$$

9. Mean Absolute Percentage Error (MAPE) [20] value closer to 0 shows high prediction accuracy.

$$MAPE = \frac{1}{N}\sum_{i=1}^{n}\left|\frac{d_i - y_i}{d_i}\right| \quad (11)$$

10. Relative Percentage Difference (RPD) [21] is given in eq. 13

$$RPD = \frac{SD}{RMSE} \quad (12)$$

Table 1. RPD values for evaluating models

RPD	**Model type**
<1	Very poor models
1.0 - 1.4	Poor models
1.4 - 1.8	Fair models
1.8 - 2.0	Good models
2.0 - 2.5	Very good models
> 2.5	Excellent models

11. Willmott's Index of agreement (WI) shows the degree of model prediction error. WI range is from 0 to 1 and WI = 1 shows perfect model [22-23].

$$WI = 1 - \left[\frac{\sum_{i=1}^{N}(d_i - y_i)^2}{\sum_{i=1}^{N}(|y_i - d_{mean}| + |d_i - d_{mean}|)^2}\right] \tag{13}$$

12. Mean Bias Error (MBE) and Mean Absolute Error (MAE) values closer to 0 shows lesser error in prediction [24].

$$MBE = \frac{1}{N}\sum_{i=1}^{N}(y_i - d_i) \tag{14}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|(y_i - d_i)| \tag{15}$$

13. The Range of Legate and McCabe's Index (LMI) is (-∞, 1) [25-26]. Values closer to 1 represent a perfect model. The lesser the value, the more is divergence between observed and predicted values.

$$LMI = 1 - \left[\frac{\sum_{i=1}^{N}|d_i - y_i|}{\sum_{i=1}^{N}|d_i - d_{mean}|}\right] \tag{16}$$

14. Expanded uncertainty (U_{95}) indicates the short-term performance of the model. Smaller the value high the performance of model [27].

$$U_{95} = 1.96(SD^2 + RMSE^2)^{1/2} \tag{17}$$

15. t-statistic smaller value indicates the superior performance of model [28].

$$\text{t-stat} = \sqrt{\frac{(N-1)MBE^2}{RMSE^2 - MBE^2}} \tag{18}$$

16. Global Performance Indicator (GPI) use all the parameters to analyze model in a single parameter. Higher the value of GPI higher is the accuracy of model.

$$\mathrm{GPI} = \mathrm{MBE} \times \mathrm{RMSE} \times U_{95} \times t_{stat} \times (1 - R^2) \tag{19}$$

17. Reliability Index (β) is a parameter to check the reliability of the model. It is calculated using FOSM. Value of reliability index between 3 and 4 indicates good performance of model [29].

$$\beta = \frac{\mu - 1}{\sigma} \tag{20}$$

Here d_i and y_i are the observed and predicted i^{th} value respectively, d_{mean} is the average of observed value, SD is the standard deviation, σ is the standard deviation and µ is the mean of the dataset generated. n is the number of training or testing samples and p is the model input quantity.

5. Result and discussion

Table 2. Performance assessment using various parameters of both the models ANN and ANFIS

S. NO.	ASSESMENT VALUES	ANN (TRAINING)	ANN (TESTING)	ANFIS (TRAINING)	ANFIS (TESTING)
1.	WMAPE	0.0077	0.0201	8.83E-05	0.0224
2.	NS	0.9992	0.989	1	0.9832
3.	RMSE	0.0301	0.0722	0.0004	0.0894
4.	VAF	99.942	99.285	100	98.615
5.	R^2	0.9992	0.989	1	0.9832
6.	$AdjR^2$	0.9992	0.9867	1	0.9797
7.	PI	1.9685	1.9074	1.9996	1.8764
8.	RMSD	0.0301	0.0722	0.0004	0.0894
9.	BIAS FACTOR	0.9947	0.9831	1	0.9819
10.	RSR	0.028	0.1047	0.0004	0.1296
11.	NMBE	-0.5659	-1.739	0.0009	-1.53
12.	MAPE	0.021	0.0492	0.0002	0.0551
13.	RPD	0.8573	0.8073	0.9978	0.9078
14.	WI	0.9998	0.9972	1	0.9959
15.	MAE	0.021	0.0492	0.0002	0.0551
16.	MBE	-0.0155	-0.043	2.53E-05	-0.037
17.	LMI	0.9764	0.9096	0.9997	0.8988
18.	U_{95}	0.0776	0.1819	0.001	0.2366
19.	t-stat	4.4526	3.5057	0.4982	2.216
20.	GPI	-1.26E-07	-2.15E-05	6.16E-19	-2.95E-05
21.	β	1.6134	2.0884	1.6174	1.9751

All the models are analyzed on the basis of various parameters (table 1) VAF, RMSE, R^2, Adj. R^2, MAE, PI, RSR, NS, the bias factor, LMI, NMBE, RPD, MAPE, U95, t-statistic, GPI and β.

From the observation table 2 it is visible that ANFIS is a better model than ANN but with less deviation of assessment values from the desired values for instance bias factor is almost 1 in the ANFIS model and this predicts that model is neither overestimated nor underestimated. Also, RSR is almost zero in ANFIS which is as desired out of the model. RPD value is less than 1 in ANN which makes ANFIS comparatively better than ANN in performance. Also, MAE and MBE when calculated gives result in the favor of ANFIS but ANN shares the same range i.e. its value is near to zero. There are other variables for which values are not distinct for both the models but ANFIS has an upper hand therefore it is suggested to use ANFIS above ANN. But it is observed that training data model is more appropriate as compared to that of testing in case of ANFIS, as there is ample amount of input in training model (70 percent) therefore we can conclude that adaptive networks work good with fuzzy logic in wide range of data. Also, if value of reliability index is compared with the reliability index value of observed dataset (1.6174 for 70% observed data and 2.103 for 30% observed data) it is observed that both models data almost coincide the observed dataset. Overall assessment concludes that ANFIS is better to work with if wide range of data is to be worked upon but ANN cannot be discarded as a model because it works equally well for both training and testing models.

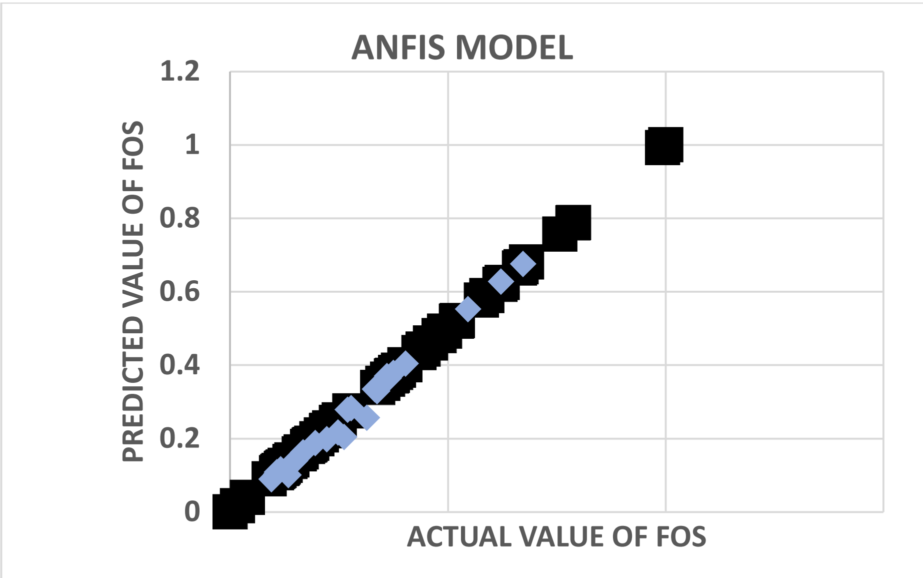

Fig. 3. Actual values vs. Predicted values of FOS using ANN model

Figure 3 and 4 shows the plot of predicted values against actual or observed values of both training and testing data using ANN and ANFIS model. It is observed that data points coincides each other and there is not much difference in the result. Therefore, it can be concluded that these models prediction power is high. But when they are examined with other parameters, overall assessment shows that ANFIS model is better than the ANN model.

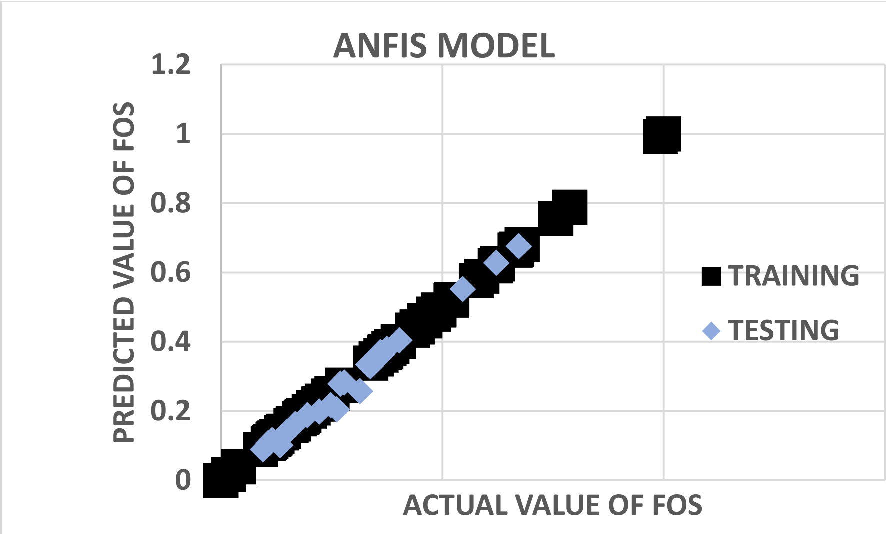

Fig. 4. Actual values vs Predicted values of FOS using ANFIS model

5.1 AOC-REC Curve

Regression Error Characteristics curve (REC) [30] is a probability curve and a metric system to check performance of regression model. Area Over Curve (AOC) is the measure of distinction of predicted data of the model from the actual data.

From the AOC value it is clear that due to less value of AOC of ANFIS training model, it proves to be a better model as compared to the other models. Also, it is observed that AOC value of ANN testing model is less than ANFIS testing model, it is already mentioned in sections above that fuzziness works good with wide range of data.

5.2 Taylor diagram

Taylor diagram [31] is the graphical representation of how closely the pattern (or patterns) matches observation which is quantified in terms of **correlation**, **root mean square error** and **amplitude of their variations (standard deviations).** This diagram evaluates the aspects of different complex models and performs a comparative analysis of these models with the reference data (self-observed data).

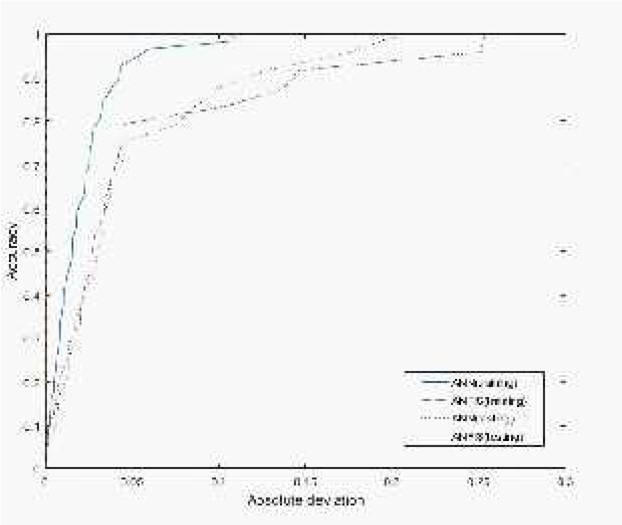

Fig. 5. REC curve plot for ANN and ANFIS (training data set)

Table 3. Area Over REC plot value for ANN and ANFIS models.

MODELS	**AOC**
ANN (Training)	**0.0200**
ANFIS (Training)	**2.2721e-04**
ANN (Testing)	**0.0449**
ANFIS (Testing)	**0.0498**

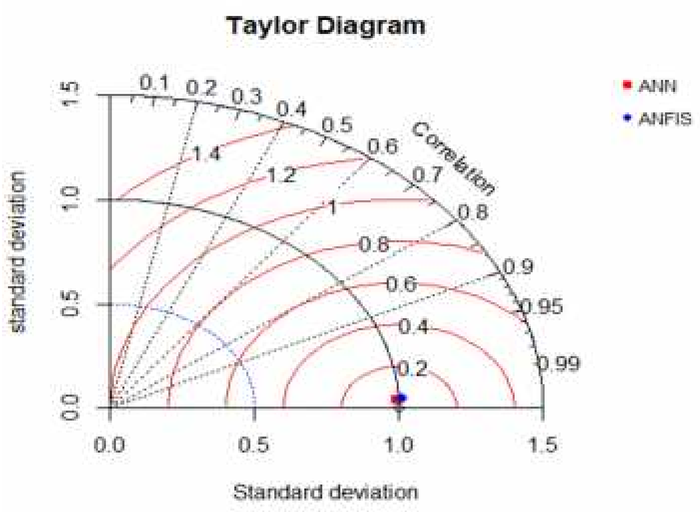

Fig 6. Taylor Diagram for ANN and ANFIS

Figure 6 shows that both models lie near to the observed value and deviation is quite less. Therefore, both these models have a good performance and overall experimentation shows that there is a good agreement between the predicted values generated from the models and actual values calculated.

Figure 7 is the plot of Fp/Fm against cumulative probability for both ANN and ANFIS model respectively. From figure 7 result can be extracted that for ANFIS(training) at P_{50}, value of Fp/Fm is near to 1 i.e., for training set, value of Fp/Fm is 1.0000 and for testing set, value is 0.9916. whereas for ANN model at P_{50} Fp/Fm is 0.9950 and 0.9876 for training and testing data sets respectively. Therefore, ANFIS observation is comparable to ANN but to due to slight change and less deviation from the value 1 ANFIS acts as a better model. But when P_{90} is checked for, it is seen that all the values of all the models are almost one. Therefore, both these models stand a good chance but ANFIS has already proven to be a better model. Figure 8 is the plot of probability density function against Fp/Fm for both ANN and ANFIS training and testing dataset. This plot is lognormal distribution for Fp/Fm and from this plot it is observed that ANFIS functions better in training period than ANN as probability of FOS within 20% accuracy level is concerned under lognormal distribution and it is clear from figure 8 that most of the points lie in the region that is under 20% accuracy level also it is visible that for ANN there is deviation of graph from 1. Therefore, ANFIS is better than ANN. But for testing dataset both ANN and ANFIS show same accuracy levels.

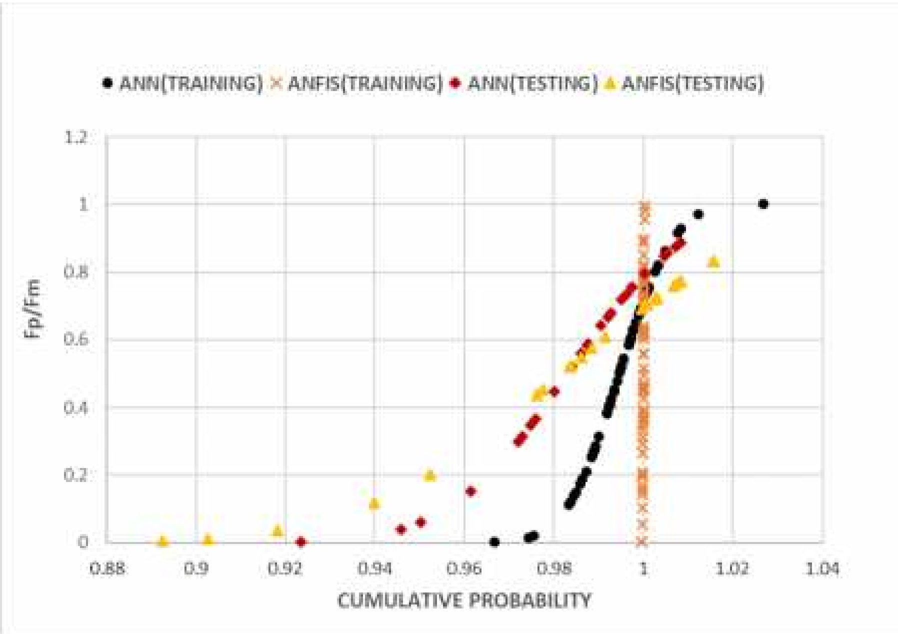

Fig. 7. Cumulative probability plot for Fp/Fm for ANN and ANFIS models

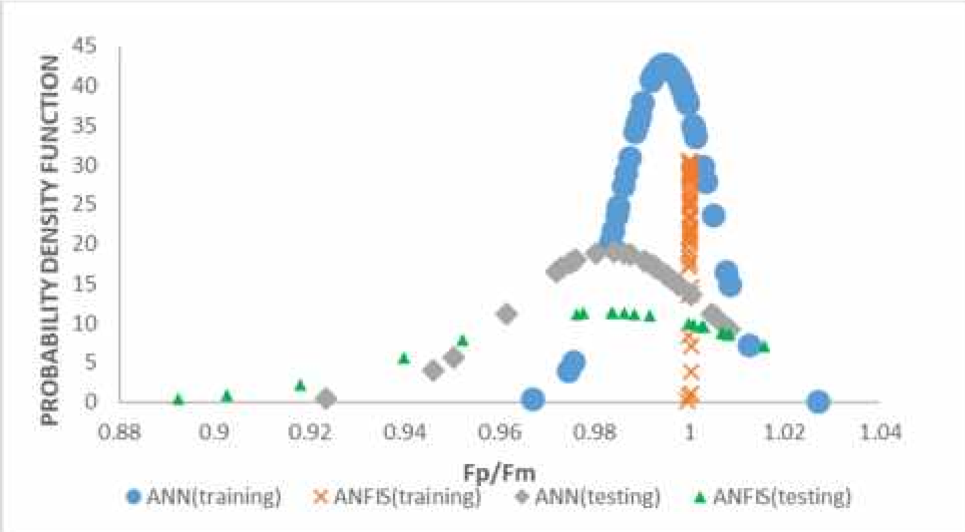

Fig. 8. Lognormal distribution for Fp/Fm for ANN and ANFIS for training and testing data

6. Conclusion

In this paper, reliability index of the retaining wall with c, φ, γ and δ as input parameters along with other performance assessment values were calculated for ANN and ANFIS models. Bothe these models were compared with different parameters and both of them showed equal efficiency. Also, their results almost converged to the same value. But ANFIS with wide range of data outperformed as fuzziness works good with ample data. 70 % of total data was used for training and 30% for testing therefore in training dataset modeling ANFIS performed better but for testing dataset both ANN and ANFIS performed equally well converging to same result or just say FOS value 1. Therefore, both these models can be used as soft computing technique for computing the factor of safety as testing model does and then calculating the reliability of the retaining wall. With collective observations ANFIS can be concluded as the better model amongst both.

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