Bearing Capacity of Strip Footing on Real Slope by Limit Equilibrium and Limit Analysis Based on DLO

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Abstract: Bearing capacity is an important aspect to design a footing located on or near to a slope as the conventional two sided failure mechanism due to shear failure may transit to a one sided slope failure when influence of slope is significant. In contrast with conventional limit equilibrium method postulating a log-spiral failure mechanism, this study adopt a discretization procedure known as Discontinuity Layout Optimization (DLO) technique to generate a kinematically admissible failure mechanism using Limit State GEO. Parameters such as horizontal set-back distance, footing width, slope inclination angle, internal friction angle, cohesion and unit weight of soil are taken into consideration to investigate the effect on bearing capacity. Slope of different regions of India are considered for parametric study. The study shows that with increase in slope angle, bearing capacity decreases but increases with increase in other parameters. The failure mechanism of the footing is changed after a particular set-back distance as influence of slope is negligible.

Keywords: Bearing capacity; Limit equilibrium; Lmit analysis; DLO

1 Introduction

Bearing capacity is an important aspect to design a foundation of all kind of civil engineering structure. In nature, ground surface may not be level always; sometimes it may be sloping one in that case when a foundation is built-up on the sloping ground, as a compulsion both bearing capacity and slope stability are taken into account for designing the foundation. Both the ultimate bearing capacity and failure mechanism are affected if slope stability is dominating in nature. These two parameters can be computed by different approaches like limit equilibrium method, limit analysis method (upper-bound and lower-bound method), method of past characteristics, slip line method, FEM.

Meyerhof (1957) proposed an empirical solution and provide a set of values of bearing capacity factors i.e. N_c and N_γ on the basis of distance from sloping ground and slope angle by using limit equilibrium method. In (1981) Kusakabe et al. calculated the ultimate bearing capacity using by upper bound theory of slopes under strip loads on top surface on purely cohesive soils and for validation model test was done by using Kanto loam. Both limit equilibrium and limit analysis approaches were adopted by Saran et al. (1989) to give an analytical solution in the form of nondimensional charts of the bearing capacity of footings adjacent to slopes. A numerical characteristics and applicability of the log-spiral solutions to practical problems were examined by Narita and Yamaguchi (1990) to obtain the values of bearing capacity factors. Huang et al. (1994) determined the bearing capacity and failure mechanism by conducting plane strain model test on footing on both reinforced and unreinforced sand slope and resulted that using reinforcing strips into the vicinity of active wedge increased the bearing capacity. Huang and Kang (2008) analyzed through limit-equilibrium-based method to evaluate the ultimate bearing capacity of rigid surface footings, got a linear functions of the setback-to footing- width ratio up to a certain threshold value. Castelli and Motta (2009) assumed a circular surface for the evaluation of the seismic bearing capacity by adopting the limit equilibrium method, which considered both the inertial and kinematic effects of the seismic loading. Shiau et al. (2011) quantified the effect of footing roughness and surface surcharge to give the solutions for the ultimate bearing capacity of footings on purely cohesive slopes are obtained by applying finite element upper and lower bound methods. Leshchinsky (2015) observed the failure mechanism and corresponding ultimate bearing capacity for strip footings placed adjacent to slopes of c- φ soils by using upper-bound limit state plasticity failure discretization scheme, known as discontinuity layout optimization (DLO), which uses nonassumptive failure geometry (under translational kinematics) in its formulation. Zhou et al. (2017) investigated the collapse mechanism of a vertically loaded strip footing placed at the top of a native slope containing c- φ soils, particularly for the effect of footing placement on the failure mode. Halder et al. (2017) in this paper, bearing capacity of a surface strip footing on slope is computed through lower bound finite elements limit analysis technique. A non-associated flow is considered to account for the dilation of the soil. Qin and Chian (2017) applied pseudostatic approach of upper-bound theorem to obtain the normalized ultimate bearing capacity and yield seismic coefficient that the slope could withstand without failure under the limit state. Yonggui and Leshchinsky (2017) evaluated both bearing capacity near slopes and coupled slope failure using a parallelized limit equilibrium procedure and compared to numerical analyses. Qin and Chian (2018) predicted the optimum bearing capacity and discretized failure mechanism of a saturated non-uniform slope based on the discretized elements where the total external work rates and internal energy dissipation are obtained through summation. Halder and Chakraborty (2018) computed the bearing capacity of a strip-reinforced footing placed on the top of a cohesionless soil slope with the use of lower-bound finite-element limit analysis. The axial tension that developed along the reinforcement layer because of the footing load was also calculated, and the variation of the axial tension along the length of the reinforcement is presented. Acharyya and Dey (2018) investigated in a finite element framework of a strip footing resting on non-dilatant cohesionless soil to observe the failure mechanism which is manifested in terms of incremental displacement and incremental deviatoric strain patterns. Xiao (2018) presented the bearing capacity of rigid foundations on soil slopes using simple upper bound theorem of kinematical limit analysis with calibration from laboratory model test results.

Although a lot of analysis have done by using limit equilibrium and limit analysis methods but these were assumed either cohesive or cohesionless soil but a few works have been done on the failure mechanism of footing resting on or near to a sloping ground using LimitState:GEO. Though Haizuo Zhou et al. (2017) used the LimitSate:GEO to study the failure mechanism but no comparable study was done. Ben Leshchinsky (2015) analyzed the limit equilibrium method and a comparable study with LimitState:GEO but he did not take any real or existing c-φ soil data which are available. Hence objective of the present study is to analyze the bearing capacity factors of a vertically loaded strip footing resting on or close to a sloping ground by limit equilibrium method and using LimitState:GEO.

2 MATRIALS AND METHODOLOGY

2.1 Materials

In this study cohesionless soil is taken for analysis of both limit equilibrium and DLO approach, in case of Limit State GEO native soils are considered to study the failure behavior and ultimate bearing capacity of a footing, located on or near to a sloping ground.

2.2 METHODOLOGY

Two methods are applied to study

1) Limit equilibrium method
2) Limit State GEO which is based on DLO is an advance part of limit analysis upper bound solution.

Limit Equilibrium Method

This method follows the equation of static to estimate the ultimate load by adopting force and/ or moment equilibrium. Moment due to external forces should be equal to the resisting moment of soil wedge at origin just prior to failure that satisfy the Mohr-Coulomb's yield criteria, which is assumed for this present study.

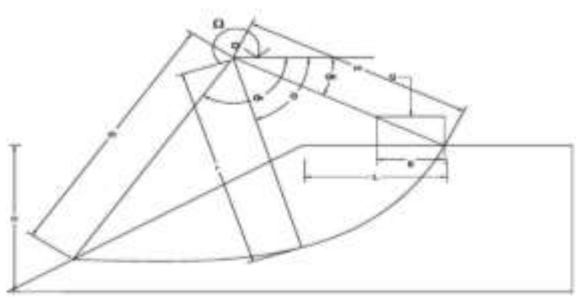


Fig. 1. Log-spiral failure surface of footing rested near a sloping ground

A surface footing under a load of Q having width (B) is assumed to rest near to a simple slope which has an inclination angle β as shown in fig.1. Assume O is the center of the rotation about which the soil mass in the region of ABC rotates as a rigid body creates a log-spiral surface passing through the toe of the slope. This assumed be specified completely by two variables i.e. θ_0 and θ_h which are the angles from the origin to chord OB and OC respectively. The equation of any radial radius with compare to initial one in logarithmic spiral in polar coordinates can be calculated as

$$r = r_0 e^{(\theta-\theta_0)\tan\varphi} \quad (1)$$

Based on the Eq.1 it can be said that the friction angle, φ governed the shape of the logarithmic-spiral. The larger the φ value the greater weight is placed near the toe, and the smaller the overturning moment.

In the same way the length of OC can be expressed as

$$r_h = r_0 e^{(\theta_h-\theta_0)\tan\varphi} \quad (2)$$

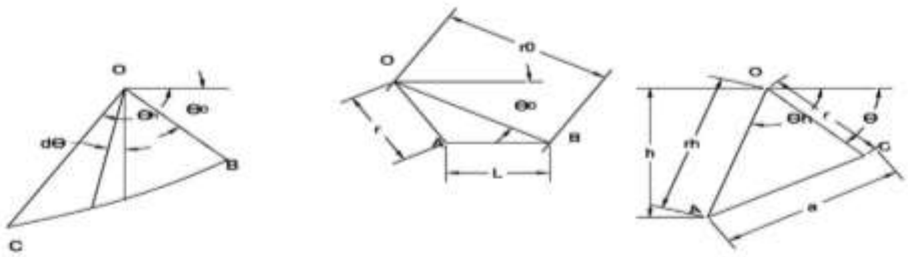


Fig.2a Fig.2b Fig.2c

Fig. 2. Detail parts of above fig.

Method of superposition is applied to overcome the difficulty on finding the moment of the region ABC about the center 'O'. Hence the soil wedge divided into three different parts and their individual moments are assumed to be 'M_1', 'M_2' and 'M_3' respectively as shown in Fig. 2. Then moment of the region ABC simply calculated as M_1- M_2- M_3.

$$M_1 = \gamma r_0^3 f1(\theta_h - \theta_0) \quad (3)$$

$$f_1 = \frac{e^{[3(\theta h-\theta 0)tan\varphi]}(sin\theta_h + 3tan\varphi cos\theta_h) - (sin\theta_0 + 3tan\varphi cos\theta_0)}{3(1+9tan^2\varphi)}$$

$$M_2 = \gamma r_0^3 f_2(\theta_h, \theta_0) \quad (4)$$

$$f_2 = \frac{1}{6}\frac{L}{r_0}\left(2\, cos\theta_0 - \frac{L}{r_0}\right) sin\theta$$

$$M_3 = \gamma {r_0}^3 f_3(\theta_h, \theta_0) \quad (5)$$

$$f_3 = \frac{1}{6sin^2\beta} e^{[(\theta h - \theta 0)tan\varphi]}\left[e^{[(\theta h - \theta 0)tan\varphi]} sin\theta_h - \, sin\theta_0\right] sin(180° - \theta_h - \beta)$$
$$* \, [e^{[(\theta h - \theta 0)tan\varphi]} sin\theta_h - sin\theta_0] sin(180° - \theta_h - \beta)$$

Finally bearing capacity can be obtained as follows:

$$q = \frac{Q}{B}, and \quad Q(r_0\, cos\theta_0 - \frac{B}{2}) = (M_1 - M_2 - M_3) \quad (6)$$

Discontinuity Layout Optimization

At the core of LimitState:GEO; the Discontinuity Layout Optimization (DLO), is a solution engine which uses numerical investigation strategy to discover an answer. The method was created at the University of Sheffield and was first depicted in a paper distributed in the Proceedings of the Royal Society (Smith and Gilbert 2007a). Generally DLO can be utilized to recognize basic translational sliding square displacement instruments, yield in a frame which will be natural to most geotechnical engineers. In the DLO procedure, the matter is fully developed in terms of relative displacements along discontinuities, e.g. changeable that illustrates the connecting slip settlement on that separation can be allocated to each potential line of separation.

3 Result and comparative analysis

The normalized bearing capacity of a foundation is the function of various criteria that can be expressed as:

$$\frac{q}{\gamma B} = f\left(\lambda, \beta, \varphi, \frac{c}{\gamma B}, \frac{H}{B}\right) \quad (7)$$

The bearing capacity factor, N_γ can be expressed as

$$N_\gamma = \frac{q}{\gamma B} \quad (8)$$

3.1 Comparison of Limit Equilibrium Method

The method used in this study to analyze the bearing capacity factor (N_γ) by limit equilibrium analysis, Thereafter the generalized values of N_γ compare with the values that are given by Meyerhof (1957) and Yonggui Xie et al. (2017) with respect to different slope angles on the face of slope taken into account for the validation of proposed method, presented in a graphical format as shown in fig 3.

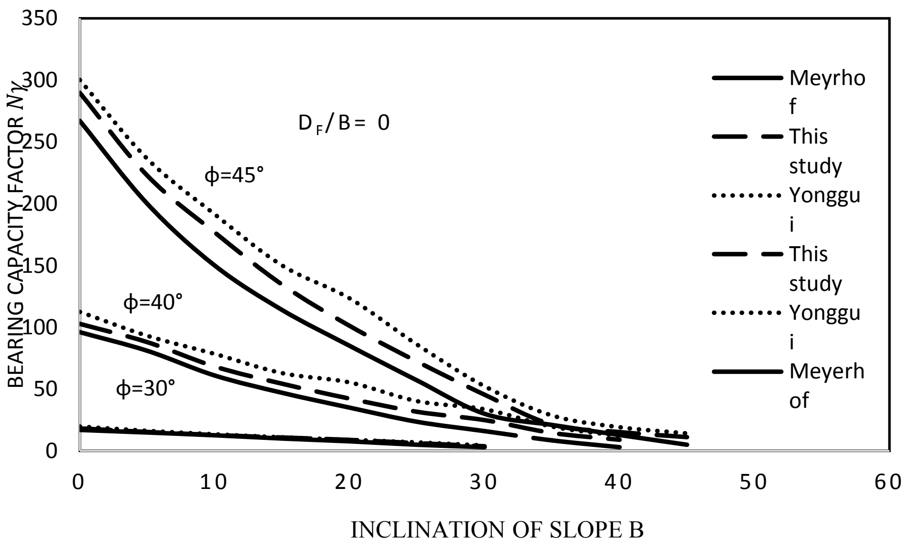


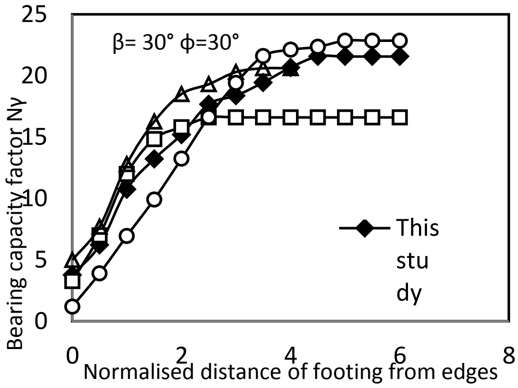
Fig. 3: comparative study between bearing capacity factor and slope inclination of the present study with other literatures

From the result it can be outlined that if the slope angle (β) is very close or equal to soil friction angle (φ), the value of N_γ is very close to zero that means the soil beneath the foundation has no resisting capacity and it can fail at very low load.

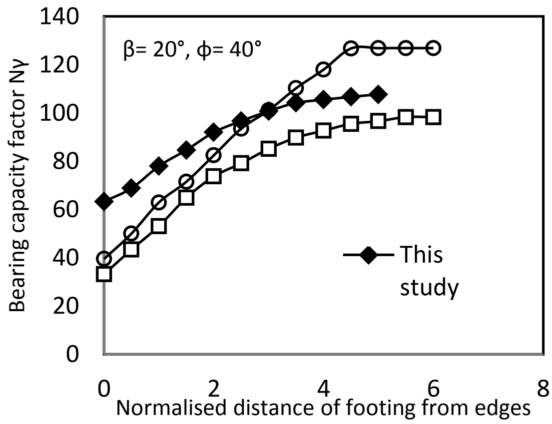
3.2 Validation of DLO Approach

A comparative study is performed to verify the validity of the DLO approach which will use further for the study of failure mode of footing on existing soil slope with the present study and the literature available.

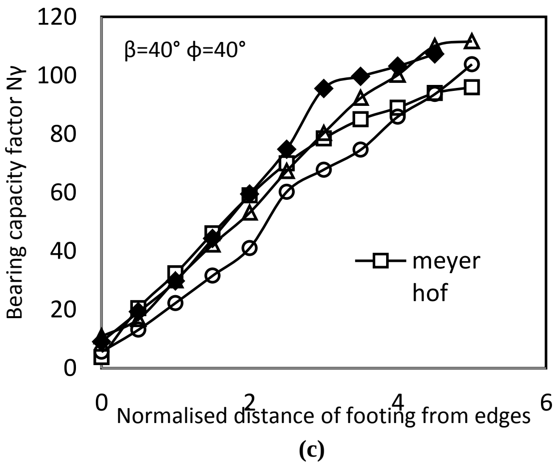
The values of N_γ possess higher values for same internal friction angle of soil when the inclination of slope is low when a footing rested on crest of slope. In both level and sloping ground it ultimately depends on φ values where influence of slope angle (β) is minimized so at a larger distance from the slope these values are quite constant for the above mentioned criteria. In every individual study by the other literature N_γ increases non-linearly and also gives identical results for LimitState:GEO. In all the study, it is shown that at lower slope angle variation of N_γ quite linear near the crest of slope but it continuously increases for β=40° even at a greater set-back distance.


β= 30° ϕ=30°
Bearing capacity factor Nγ
Normalised distance of footing from edges
This study

(a)


β= 20°, ϕ= 40°
Bearing capacity factor Nγ
Normalised distance of footing from edges
This study

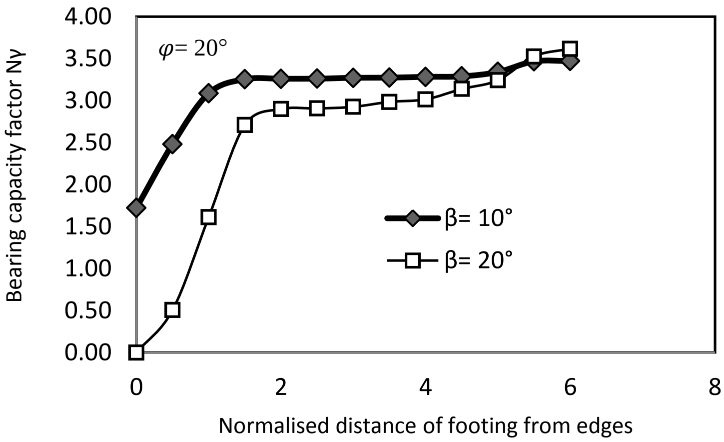
(b)


β=40° ϕ=40°
Bearing capacity factor Nγ
Normalised distance of footing from edges
meyer hof

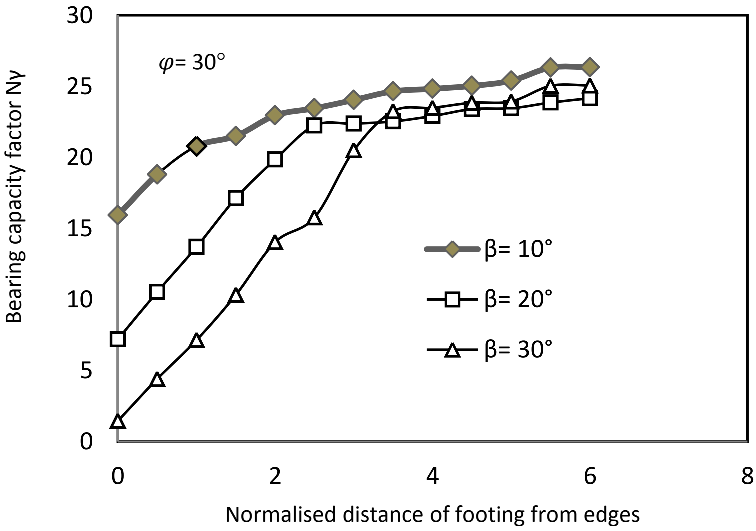
(c)

Fig. 4. Comparison of bearing capacity factors obtained from DLO for different φ angle at different slope angle with existing literature

3.3 Values of N_γ Through DLO Approach



(a)



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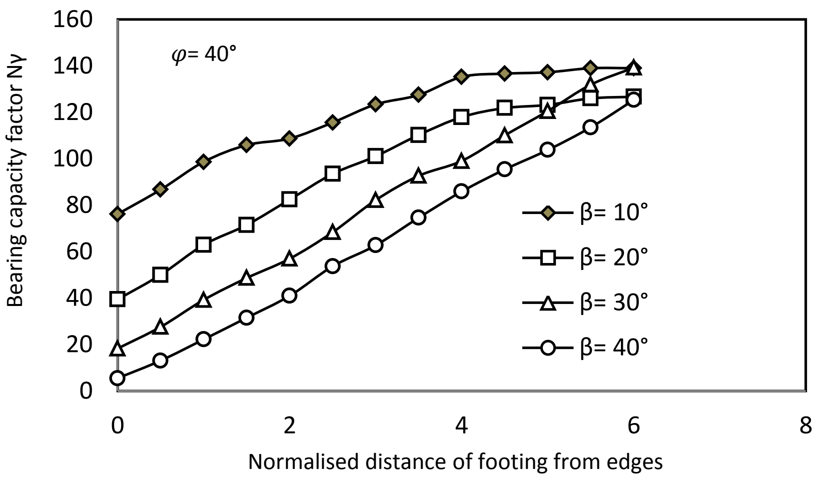

(c)

Fig. 5. comparison of N_γ at different slope angle for different values of φ in DLO

From the above figs. it is clear that the values of N_γ is much lower at crest if the slope inclination angle is more for same φ value. As the set-back distance increases N_γ value increases non-linearly but after a certain normalized distance these values are closed to each other for different slope angle at same internal friction of soil.

The values of N_γ is much greater when β=10° comparison to higher slope angle when footing is placed on or near to the crest of slope. Although these values also change non-linearly, but these variation of N_γ values comparatively low with respect to higher β values with increase in set-back distance.

It is observed that when slope angle equal or close to angle of internal friction of soil, the bearing capacity factor starts from a very low value and increase rapidly. After certain threshold distance of footing location these values almost constant. It is also noticed that if the internal-friction-of-soil (φ) possess lower value the variation of bearing-capacity-factor (N_γ) will comparatively low at lower set-back distance (λ).

3.4 Bearing Capacity of Footing on Existing Slope

Table 1. Soil properties and slope inclination angle of existing slope

PLACE	SOURCE	β in degree	SOIL PROPERTIES		
			C in kPa	Φ in degree	γ in kPa
Guwahati	Das and Saikia (2010)	35	10	31	16.5
Manali	Rentala (2011)	12	21	29	19.25
Mizoram	Panigrahi et.al (2011)	28	10	30	20
Thiruvanta-puram	Lekshmi et.al (2016)	24	22	31	16.3
Uttarakhand	Pandit et.al (2016)	42	12	26.5	18

Table 2. ultimate bearing capacity obtained from DLO of above mentioned slopes

Guwahati		Manali		Mizoram		Thiruvantapuram		Uttarakhand	
(λ)	(q) kN/m^2	(λ)	(q) kN/m^2	(λ)	(q) kN/m^2	(λ)	(q) kN/m^2	(λ)	(q) kN/m^2
0	188.8	0	829.7	0	366.9	0	564.3	0	92.03
2	320	2	976.4	2	539.4	2	793	2	151.9
4	451.2	4	1068	4	700.6	4	1019	4	235.4
6	591	6	1175	6	869.1	6	1261	6	335.2
8	741.6	8	1179	8	1034	8	1475	8	440.1
10	900.6	10	1192	10	1086	10	1620	10	554.1
12	918.5	12	1211	12	1087	12	1653	12	579.4

The value of ultimate bearing capacity depends on the soil properties, slope angle (β) and set back distance (λ). It is observed that in Uttarakhand even though soil possess a high unit weight but ultimate bearing capacity is very low as soil have low cohesion (c) and φ values. And this observation is hold good for other existing slope also.

4 Conclusions

The primary aim of the present study to detect the ultimate bearing capacity of a vertically loaded surface strip footing placed on various distance from the crest of slope in different methods. A detailed study of limit equilibrium method has conducted considering a logarithmic-spiral failure surface in which it is observed f_1, f_2 and f_3 are not only function of θ_0 and θ_h;

inclination of slope (β) is also an important factor upon which f_3 depends. DLO is another approach base on limit analysis; able to derive the ultimate bearing capacity. It is possible to draw the following findings:

- The slope stability mechanism will dominate for small footing setbacks, transitioning to a bearing capacity mechanism as footing setback increases, particularly for larger slope inclination angles. The distances from the edge of a slope required to make bearing capacity independent of slope effects is heavily dependent on both slope inclination and soil shear strength. Particularly, larger dependent on both slope inclination and soil shear strength. Particularly, larger friction angles increase the distance required to avoid slope effects.
- The limit equilibrium method corresponding to this literature overestimates the value of bearing capacity factor as compare to Meyerhof (1957) and underestimates as compare to Yonggui (2017) even though these three methods shows the same variation of N_γ at different slope angle at different φ value and at different normalized distance. In the other hand the outcomes of LimitState:GEO shows a comparatively lower value of N_γ at different β for same φ value when the footing is near to crest.

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6 References:

1. Meyerhof, G. G. "The ultimate bearing capacity of foundations on slopes." Proc., 4th Int. Conf. on Soil Mechanics and Foundation Engineering. Vol. 1. 1957.
2. Kusakabe, Osamu, Tsutomu Kimura, and Hakuju Yamaguchi. "Bearing capacity of slopes under strip loads on the top surfaces." Soils and foundations 21.4 (1981): 29-40.
3. Saran, Swami, V. K. Sud, and S. C. Handa. "Bearing capacity of footings adjacent to slopes." Journal of geotechnical engineering 115.4 (1989): 553-573.
4. Narita, Kumanto, and Hakuhu Yamaguchi. "Bearing capacity analysis of foudations on slopes by use of log-spiral sliding surfaces." Soils and Foundations 30.3 (1990): 144-152.
5. Huang, Ching-Chuan, and Wen-Wei Kang. "The effects of a setback on the bearing capacity of a surface footing near a slope." Journal of GeoEngineering 3.1 (2008): 25-32.

6. Ching chuan huang, fumio tatsuka and Yashiko Sato “failure mechanism of reinforced sand slopes loaded with a footing” SOIL AND FOUNDATION Vol- 34, No 2, 27- 40, 1994
7. Castelli, Francesco, and Ernesto Motta. "Bearing capacity of strip footings near slopes." Geotechnical and Geological Engineering 28.2 (2009): 187-198.
8. Panigrahi, R. K., et al. "Investigation and Design for Restoration of Hill slope in Mizoram." Indian Geotechnical Journal 41.4 (2011): 215-225.
9. Rentala, Vijayalakshmi, and N. Satyam. "Numerical modeling of rock slopes in Siwalik hills near Manali region: a case study." Electron J Geotech Eng 16 (2011): 763-783.
10. Shiau, J. S., et al. "Undrained stability of footings on slopes." International Journal of Geomechanics 11.5 (2011): 381-390.
11. Leshchinsky, Ben. "Bearing capacity of footings placed adjacent to c′-ϕ′ slopes." Journal of geotechnical and geoenvironmental engineering 141.6 (2015): 04015022.
12. Sharma, L. K., et al. "Geotechnical characterization of road cut hill slope forming unconsolidated geo-materials: a case study." Geotechnical and Geological Engineering 35.1 (2017): 503-515.
13. Xie, Yonggui, and Ben Leshchinsky. "Ultimate Bearing Capacity near Slopes: Transition from a Bearing Capacity Problem to a Slope Stability Problem." Geotechnical Frontiers 2017. 2017.
14. Qin, Changbing, and Siau Chen Chian. "Kinematic stability of a two-stage slope in layered soils." International Journal of Geomechanics 17.9 (2017): 06017006.
15. Moayedi, Hossein, and Sajad Hayati. "Modelling and optimization of ultimate bearing capacity of strip footing near a slope by soft computing methods." Applied Soft Computing 66 (2018): 208-219.
16. Qin, Changbing, and Siau Chen Chian. "Bearing capacity analysis of a saturated non-uniform soil slope with discretization-based kinematic analysis." Computers and Geotechnics 96 (2018): 246-257.
17. Xiao, Shiguo. "Limit Analysis of Bearing Capacity of a Rigid Strip Footing on a Soil Slope Based on the Upper Bound Theorem." GeoShanghai International Conference. Springer, Singapore, 2018.
18. Zhou, Haizuo et al. "The bearing capacity and failure mechanism of a vertically loaded strip footing placed on the top of slopes." Computers and Geotechnics 94 (2018): 12-21.