Predictive Models for Estimating the Coefficient of Permeability for Sands

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Abstract. Particularly for granular soils several correlations are available in literature for predicting the coefficient of permeability of soils as a function of particle size, properties of pore fluid, void ratio, shape of particles etc. In this paper a comparative study of some of the existing predictive models is presented to examine how far these are applicable to natural sands. With the above in view coefficients of permeability of two different types of sands (sub-angular Ganga sand and rounded Ennore sand) were found out with varying voids ratio by constant head laboratory test. The laboratory test data is compared with various predictive models available in the literature. This study highlights the fact that there exists no unique relationship that would be the panacea for predicting the coefficient of permeability values for all types of soils. The expressions as given by Kozeny-Carman, Taylor and Chapuis are rational and scientific. These may be used for quick predictions of the coefficient of permeability; but the coefficients and parameters appearing in these equations need to be properly determined specific to the concerned soil.

Keywords: Permeability, Sand, Void ratio, Particle size.

1 Introduction

Permeability is one of the most important soil properties which indicates the relative ease with which fluid can flow through soils. Soil being a particulate system is inherently a multiphase and permits passage of water through their continuous but disordered voids (or pores) whose size, cross-section shape and orientation are highly variable from point to point. Physical characterization of such a disordered system is very difficult. Size of the pore spaces, the nature of the soil particle plays an important role on the flow of water though soils. The path of water through soils is generally tortuous. Therefore, the flow velocity would vary from point to point with variations in pore sizes, and its direction, which at times may even, be in opposite direction of the macro-flow (Leonard; 1962).

Flow of water through most soils mainly follows Darcy's law. The classic experiment conducted by Darcy in 1856 provided the physical basis for the analysis of flow through porous media; based on the experimental findings he proposed a simple law for discharge velocity (v) of water:

$$v = ki \tag{1}$$

i = total head lost per unit length of macroscopic flow path, called hydraulic gradient, k = coefficient of permeability or simply the permeability.

k appearing in the above equation is one of the most important and widely used soil properties in estimating the quantity of water seepage either through the body or foundation of earth structures. For liquid flow at very high velocity Darcy's law becomes invalid. The same is true also for clays with steady state flow. Many experiments that have been performed to study the validity of Darcy's law have been summarized and discussed by Muskat (1946) and Scheidegger (1957). Several empirical and semi-empirical relations for predicting the hydraulic conductivity or the permeability of granular soils are available in literature. Two relations like that of Kozeny-Carman (propsed by Kozeny ; 1927 latter modified by Carman; 1937, 1956) and Taylor (1948) that have been developed for granular soils have a good theoretical basis. Recently a renewed interest in reviewing the various models for predicting the saturated hydraulic conductivity or the coefficient of permeability and checking the utility of the more theoretically sound predictive models to different local soils has been observed [Carrier III (2003), Chapuis (2004)]. By now, how the factors as stated above affect the permeability of clean sand and gravels are well understood. However, there is still a need to have more data base on different local soils to assess how these changing factors affect the actual k values and either check the validity of existing expressions or develop new expressions for estimating quickly the values of k. With the above in view k is determined in the laboratory for two sands namely Ganga sand (a clean river sand) and Ennore sand (a clean beach sand) pouring them at different void ratio. These data are then used to check the validity of a few empirical and semi-empirical relations that are available in literature.

2 Models for Estimation of Coefficient of Permeability

Prior to the development of rational models on mathematical basis few purely empirical correlations have been suggested. One of the earliest efforts based on grain size distribution by Hazen (1911), is for uniform sand ($C_U < 5$) having D_{10} between 0.1 and 3 mm in their loose condition, Darcy's k equals,

$$k(cm/sec) = CD_{10}^2 \tag{2}$$

where D_{10} is in cm.; C = Hazen's coefficient=80 to 120 (a value of 100 is commonly used). The coefficient 100 is the average of many values; individual values ranged from 41 to 146, but most of the values were between 81 and 117 (Hazen; 1911). The pore shape and size are not considered in equation 2. Thus, this formula can be used only as an expression of average conditions for the range as mentioned, and cannot reflect the changing variables as discussed.

Casagrande (1941) proposed a simple relation for the coefficient of permeability for fine to medium clean sand in the following form:

$$k = 1.4e^2 k_{0.85} \text{ cm/sec} \quad (3)$$

where $k_{0.85}$ is the co-efficient of permeability at a void ratio of 0.85.

The above correlation depends on the void ratio only and, as such, many of the limitations of Hazen's equation also apply herein also. The constant 1.4 appearing in the equation may also be not applicable for all types of sands.

The following Kozeny-Carman equation for the permeability of soils takes in to account the specific surface area of the soil, tortuicity of the flow path apart from the other factors:

$$k = \frac{1}{k_0 S^2}\frac{\gamma}{\mu}\frac{e^3}{1+e} \quad \text{cm/sec; at 20°C} \quad (4)$$

k_0= factor depending on pore shape and ratio of length of actual flow path to soil bed thickness; S = specific surface area(1/cm); γ/μ = permeant unit weight / permeant viscosity (1/cm-sec); e = void ratio.

Carrier III (2003) suggested a procedure to estimate the specific surface area (S) form from the particle size distribution. For example, if a soil consists of uniform spheres of diameter D (cm),

$$S = \text{area/volume} = (\pi D^2)/[(\pi D^3/6)] = 6/D \quad (5)$$

If the soil consists of non-uniform spheres, then the effective diameter D_{eff} can be calculated from the particle size distribution

$$D_{eff} = 100\%/\left[\sum(f_i D_{av_i})\right] \quad (6)$$

where f_i =fraction of particles between two sieve sizes; larger [l] and smaller [s] (%); and $D_{ave\ i}$ =average particle size between two sieve sizes (cm) = $D_{li}^{0.404} \times D_{si}^{0.595}$

$$\text{Then, } S = 6/D_{eff} \text{ cm}^{-1} \quad (7)$$

The smaller particles have the most influence on the calculated D_{eff} and therefore on S. Finally, to account for the angularity of the individual soil particles, a shape factor SF can be introduced

$$S = SF/D_{eff} \text{ cm}^{-1} \quad (8)$$

Fair and Hatch (1933) suggested the following values for the shape factor, SF: spherical—6.0; rounded—6.1; worn—6.4; sharp—7.4; and angular—7.7. Loudon (1952) suggested the following values: rounded—6.6; medium angularity—7.5; and angular—8.4.

Considering flow through soil similar to flow through a bundle of capillary tubes of extremely irregular shape Taylor (1948) developed the following equation using Poiseuille's law.

$$k = D_s^2 \frac{\gamma_w}{\mu} \frac{e^3}{1+e} C \quad \text{cm/sec} \tag{9}$$

where, k = the Darcy coefficient of permeability; D_s = some effective particle diameter; C = shape factor. As D_s is the diameter of particle having a specific surface of S, Taylor's equation can be considered simplification of Kozeny-Carman equation.

Chapuis (2004) observed that the extended Hazen's equation and a specific Kozeny-Carman equation for sand and gravel give predictions that vary from good to questionable; and then form the basic considerations developed a better semi-empirical predictive relationship for k in cm/sec;

$$k = 2.4622 \left[D_{10}^2 \frac{e^3}{1+e} \right]^{0.7825} \text{cm/sec} \tag{10}$$

Using the above correlations, it is possible to estimate the permeability values if the constants and the parameters appearing in the equations are known. Some of these aspects are looked into by conducting laboratory tests on local river and beach sands.

3 Experimental Studies

3.1 Materials Used

To meet the objective as enunciated, Ganga sand and Ennore sand were used in the present study. Ganga sand is river sand collected from the flood plains at Kanpur, situated in northern part of India. Ennore sand is Indian standard sand obtained from Tamilnadu, situated in southern part of India. The grains of Ganga sand are predominantly angular with minor amounts of flaky as well as rounded grains; however, the grains of Ennore sand are nearly spherical and highly rounded. Grain size distribution curves for Ganga and Ennore sands as obtained from sieve analysis are presented in Fig. 1.

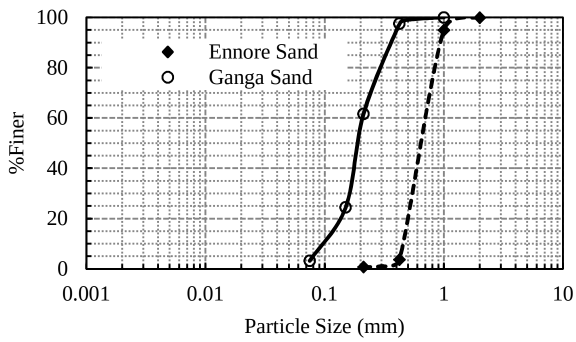

Fig. 1. Particle size distribution for Ganga and Ennore Sands

The results of grain size analysis and other tests for both the sand are presented in Table-1. As per Unified Classification System, both the sands can be classified as poorly graded sand (Group Symbol - SP) but the Ganga sand has better particle size distribution than Ennore sand.

Table 1. Physical properties of Ganga and Ennore Sand

Margin	Ganga Sand	Ennore Sand
D_{10} (mm)	0.09	0.46
D_{30} (mm)	0.17	0.57
D_{60} (mm)	0.2	0.7
C_u	2.22	1.52
C_c	1.445	1.009
G_s	2.67	2.65
e_{max}	0.91	0.82
e_{min}	0.57	0.45
D_{10}, D_{30}, D_{60}, C_u, C_c, $G_{s,}$ e_{max}, e_{min} are standard notations		

3.2 Test Procedure

Coefficient of permeability of Ganga and Ennore sand at different void ratio were determined by constant head permeability testing method as per [IS:2720 (Part 36)]. Samples were prepared by dry pouring of sand into the mould in layers and subsequent tamping. A hydraulic constant gradient equal to 10 was set for all the tests. A downward water flow was applied and steady flow was established for all the samples before taking the measurements. Coefficients of permeability obtained at room temperature were converted to the same at 20°C.

4 Results and Discussions

Various theoretical expressions discussed above for predicting the coefficient of permeability and its variation with voids ratio were obtained for both Ganga and Ennore sands. The theoretical values were compared with the measured values.

Hazen's equation (Eq. 2) when applied to Ganga sand, predicts higher permeability values; however, the permeability values are of the same order (Fig. 2). Predictions from Hazen's equation do not depend on the change in void ratio. But, in fact the permeability values are dependent on the same and it increases as the void ratio is increased.

In case of Ennore sand the predictions were very high as compared to experimental results. The predicted permeability values were as high as 102 cm/sec (Fig. 3). Using

Casagrande equation (Eq. 3) and measured k value for one e value $k_{0.85}$ was found out for both Ganga and Ennore sand. Now using this $k_{0.85}$ value, the variation of permeability with void ratio was calculated for Ganga sand and Ennore sand and plotted in Fig. 2 and 3.

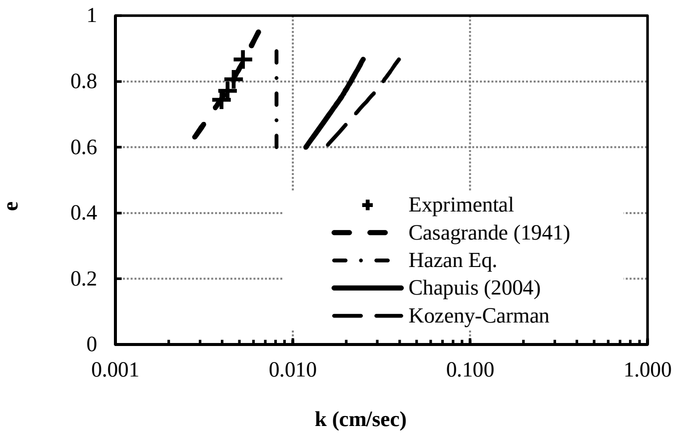

Fig. 2. Void ratio vs. coefficient of permeability–Ganga Sand

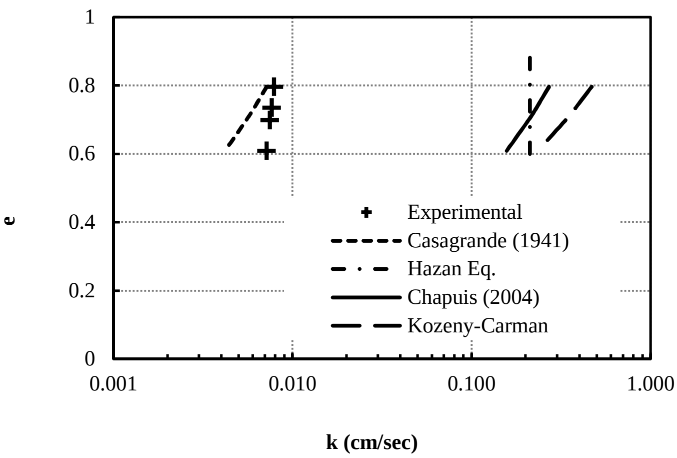

Fig. 3. Void ratio vs. coeff. of permeability–Ennore Sand

From Kozeny-Carman equation (Eq.4) the permeability values of both Ganga sand and Ennore sand were predicted with different void ratio and plotted in Fig. 2 and Fig. 3 respectively. The specific surface areas were predicted by the procedure described above (Carrier III; 2003). It is observed that the predicted values are very high as compared to the experimental values.

The permeability equations (Eq. 4 & 9) indicate that a plot between k and $e^3/(1+e)$ can be represented by straight line. But it is also seen from experimental data (Taylor, 1948) that other relationships between k and $e^3/(1+e)$ or k and e^2 can be represented by linear equations as well. The variations of coefficient of permeability with e^2, $e^2/1+e$ and $e^3/1+e$ for Ganga sand and Ennore sand are shown in Fig. 4 and Fig. 5.

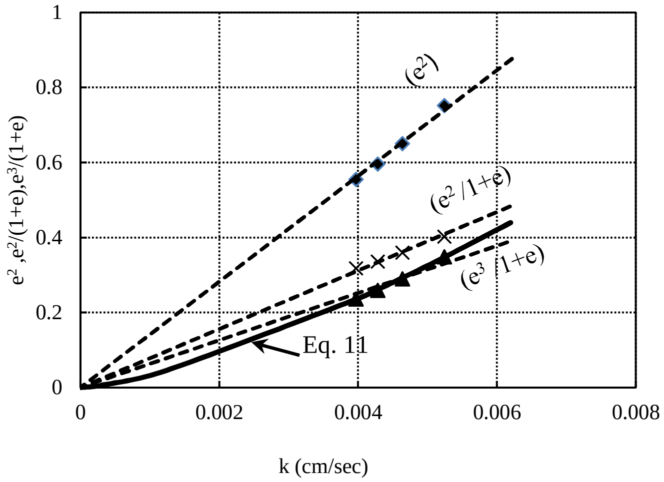

Fig. 4. *k* vs. void ratio functions-Ganga Sand

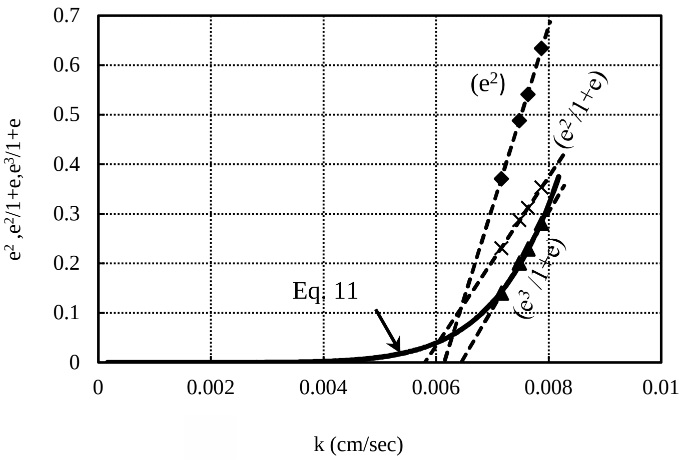

Fig. 5. *k* vs. void ratio functions-Ennore Sand

The test data on Ganga sand shows that the plot of k versus $e^3/1+e$, $e^2/1+e$ and e^2 can be approximated by straight line. Casagrande's equation (Eq. 3) shows that the permeability bears a linear relationship with the square of the void ratio. Similar behavior is observed when the data is plotted as a function of $e^2/1+e$ as was first observed by Taylor (1948). But, straight lines passing through origin cannot be fitted through Ennore sand data. As the linear variation of k vs $e^3/1+e$ is found suitable for Ganga sand, it can be concluded that Taylor's equation (Eq. 9) is applicable to Ganga sand. The expression given by Chapuis (2004) could not predict the correct permeability values for Ganga sand and Ennore sand. Considering the trend of data, a power equation similar to equation 10 was fitted to the experimental data (Fig. 5) and the following equation can be used for sands considered in the present study:

$$k = A\left[D_{10}^2 \frac{e^3}{1+e}\right]^B \quad (11)$$

where, for Ennore sand $A = 0.0116$ and $B = 0.1368$

It is also observed (Fig. 4) that for Ganga sand the variation of k with $e^3/(1+e)$ can be better represented by Eq. 11 (R^2=0.997). Therefore, the constants 'A' and 'B' were also calculated for Ganga sand and it was found that A=0.3406 and B=0.7107 can be used to predict the permeability values of Ganga sand.

5 Conclusions

Hazan equation gives correct order of permeability value for Ganga sand but for Ennore sand none of the permeability predictive models could provide the correct picture. Taylor's equation fitted the permeability data for Ganga sand better and the empirical relation proposed by Casagrande (1941) gives quite accurate variation of permeability with void ratio. Relations as suggested by Kozeny-Carman and Chapuis (2004) could not predict the correct permeability coefficients for both Ganga sand and Ennore sand. For Ganga sand predictions were one order higher and for Ennore sand predicted values were two orders higher than that of experimental values. For both Ganga sand and Ennore sand, equation as proposed by Chapuis (2004) can be used for correct predictions of the permeability values provided the multiplication and power coefficients (A and B) are found from experiments conducted on the respective sands used. The coefficients cannot be universally applied for all types of sand. The limited study shows that no single equations with well-defined parameters are adequate enough to predict the coefficient of permeability for all types of soils. This study also highlights the fact that there exists no unique relationship (whatever may be its rational and scientific basis) that would be the panacea for predicting the coefficient of permeability values for all types of soils.

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