Effect of Relative Density on Elastic properties of sand

Mrs.Arpita V Patel[1 *], Dr. N H Joshi [2]

[1] Applied Mechanics Department, Faculty of Technology and Engineering,M S University of Baroda, Vadodara– 390001
arpsoham@gmail.com
[2] Applied Mechanics Department, Faculty of Technology and Engineering,M S University of Baroda, Vadodara– 390001
nhjoshi-appmech@msubaroda.ac.in

Abstract. This investigation presents the variation in modulus of elasticity and Modulus of Subgrade reaction of dry sand with respect to change in relative density of sand. The sand was collected from Orsang River and Narmada River (Poicha). The effect of the relative density of sands on both elastic properties was carried out at 40%, 60% and 80% relative density. Modulus of Subgrade reaction was obtained by performing model plate load test and modulus of elasticity using simple triaxial test. Correlations between Modulus of Elasticity and Modulus of Subgrade reaction developed by various investigators were found from the literature review and it was observed that the pattern of changing of Modulus of Subgrade reaction with respect to change in the relative density of sand obtained in present study was nearer to correlation developed by Vesic and Selvadurai. It was also observed that as relative density of sand increased the Modulus of Elasticity and Modulus of Subgrade reaction was also increased.

Keywords: modulus of elasticity; Modulus of Subgrade reaction, relative density, gradation curve relative density, gradation curve

1 Introduction

1.1 Elastic Properties of Soil

Modulus of Elasticity (E_s), Poisson's Ratio(μ_s)and Modulus of Subgrade reaction (k_s) are the main Elastic Properties of soil for flexible analysis of foundation.Modulus of elasticity of soil is defined as the slope of the initial tangent or secant tangent to the deviator stress-strain curve of soil sample plotted from triaxial test.The initial tangent modulus is most often used for E_s.This is because the soil is elastic only near the origin, and there is less deviation between all plots in this portion. The ranges of modulus of elasticity are given in Table 1[8]. Poisson's ratio of soil represents the ratio of Lateral strain to linear strain observed on cylindrical soil sample of triaxial test. As the direct determination of poisson's ratio is extremely difficult, the commonly adopted value ranges are given in the Table 2[8] for different types of soils.Both E_s and μ_s are affected by the following.

1. Method of laboratory tests (confined, unconfined, drained, undrained).
2. Over consolidation ratio.

3. Degree of confinement.
4. Density of soil.
5. Strain rate.
6. Sample disturbance.

Modulus of Subgrade Reaction of soil represents a ratio of load intensity (Load per unit area (applied through a centrally loaded rigid body) of horizontal surface of a mass of soil to corresponding settlement of the surface. It is determined as the slope of the secant drawn between the point corresponding to zero settlement and the point of 1.25 mm settlement, of a load-settlement curve obtained from a plate load test on a soil using a 75 cm diameter or smaller loading plate with corrections for size of plate used. Typical range of values for modulus of subgrade reaction k_s (Bowles, 1988) are as shown in Table 3.

Table 1. Typical range of values for the modulus of elasticity Es for sandy soils

Soil	Es (kPa)
Silty Sand	5000– 20000
Loose Sand	10000– 25000
Dense	50000– 81000
Sand and Gravel	
Loose Sand and Gravel	50000– 150000

Table 2. Typical range of values for the Poisson's ratio of sandy soils

Type of soil	Poisson's ratio
Sandy clay	0.2 - 0.3
Silt	0.3 - 0.35
Sand, Gravelly sand	0.1 - 1.0, commonly used 0.3 - 0.4

Table 3. Typical range of values for modulus of subgrade reaction k_s

Soil	k_s(KN/m^2/m)
Loose sand	4800 – 16000
Medium dense sand	9600 – 80000
Dense sand	64000 – 128000
Clayey medium dense sand	32000 – 80000
Silty medium dense sand	24000 – 48000

1.2 Literature Review on Elastic Properties of Soil

Jamshid Sadrekarimi compared different correlation proposed for determination of the coefficient of subgrade reaction, k_s by various authors as listed in Table 4. They had used advanced soil models in Safe and Plaxis software on Tabriz Marl soil. They discovered that for Tabriz Marl, soft soil model is the best governing model and Vesic relation among the methods of determination of k_s gives a negligible error in comparison to the soft soil model. Also, to achieve more accurate results from these methods, they proposed to use mean elasticity modulus which takes into account the effect of geometric and mechanical properties of sub-layers.[1]

Table 4. Common relations suggested for k_s by various investigators[1]

No.	Investigator	Suggested expression
1	Biot	$k_s = 0.95 \frac{E_s}{B(1-\mu_s^2)}\left[\frac{B^4 E_s}{(1-\mu_s^2)EI}\right]^{0.108}$
2	Terzaghi	For sands $k_s = k_{s1}\left(\frac{B+1}{2B}\right)^2$ For clays $k_s = k_{s1}\left(\frac{1}{B}\right)$
3	Vlassov	$k_s = \frac{E_s(1-\mu_s)}{(1+\mu_s)(1-2\mu_s)}\left[\frac{\upsilon}{2B}\right]$
4	Vesic	$k_s = 0.65 \frac{E_s}{B(1-\mu_s^2)}\left[\frac{B^4 E_s}{EI}\right]^{1/12}$
5	Meyerhof and Baike	$k_s = \frac{E_s}{B(1-\mu_s^2)}$
6	Klopple and Glock	$k_s = \frac{2E_s}{B(1+\mu_s)}$
7	Selvadurai	$k_s = 0.65 \frac{E_s}{B(1-\mu_s^2)}$
8	Bowles	$k_s = \frac{E_s}{B'(1-\mu_s^2)\, m I_S I_F}$

in Table 4: Es = modulus of elasticity, μs = Poisson's ratio, B = width of footing, EI = flexural rigidity of footing, ks1 = the coefficient of subgrade reaction for a plate 1 ft wide, υ = non-dimensional soil mass per unit length, B' = least lateral dimension of footing, IS and IF = influence factors which depend on the shape of footing and parameter m takes 1, 2 and 4 for edges, sides and center of footing, respectively. Eq. (1) and (4) are defined for infinite beams resting on an elastic soil continuum (Biot 1937; Vesic 1961), but application of them in mat footings is observed widely in technical literatures (Bowles 1998). Eq. (2) when the quantity of the coefficient of subgrade reaction beneath a plate of 1 ft wide is defined only can be used. This equation is also relevant in analysis of plate load test results by substituting width of loading plate with 1ft, but some of the researchers instead of using these equations in plate load test suggest using of those modified by Arnold (Al-sanad*et al.*1993). Eq. (3) is introduced for beams and plates resting on elastic half space (Elachachi*et al.*2004), but ambiguities of estimating μ makes the problem more complex. Eq. (5), (6) and (7) are proposed for computing the coefficient of horizontal subgrade reaction in buried circular conduits (Okeagu and Abdel-Sayed 1984) and are employed for evaluation of k_s in few limited cases (Elachachi*et al.* 2004). Also, k_s can be determined using the theory of elasticity. By rewriting the relation of settlement of rectangular plates resting on elastic half space, k_s can be expressed as Eq.(8) (Bowles 1998).

R. Ziaie_Moayed& S.A. Naeini had obtained the correlation between SPT results $(N_1)_{60}$ and modulus of subgrade reaction (K_s) as below [2]

$$K_s = 3.143 (N_1)_{60} \quad (1)$$

Where:

$$(N_1)_{60} = C_N * C_S * C_R * C_B * C_E \quad (2)$$

N: measured SPT below counts
C_E: Energy effect coefficient
C_B: Correction factor for borehole diameter
C_R: Correction factor for rod length
C_S: Correction factor for type of samplers
C_N: Effective overburden pressure coefficient obtained from the following relation:
Where: $C_N = \left(\frac{P_a}{\sigma'_{v0}}\right)$
Pa: Atmospheric pressure
σ'_{v0} =Effective vertical pressure at considered depth

DaeSang Kim obtained the relationship between the subgrade reaction modulus and the strain modulus obtained using a plate loading test on railroad subgrade in Korea. They used unrepetitive plate loading test (UPLT) to obtain the subgrade reaction modulus (k_{30}) and repetitive plate loading test (RPLT) to obtain the strain modulus (Ev).

[illegible] (3)

$$E_v = \alpha K_{30}$$
$$E_s = \frac{\pi}{2}\frac{\sigma_0 r}{s}(1-\mu^2) \quad (4)$$

$$E_v = \frac{1.5r}{a_1 + a_2 \sigma_{0max}} \quad (5)$$

Where E_s = elastic modulus; σ_0= average normal stress; r = radius of the plate; s = settlement of the plate associated with the pressure; and μ = Poisson's ratio. , a_1 and a_2 represents the factors (mm/(MN/m^2), mm/(MN^2/m^4), respectively). Here, E_v = the strain modulus in MPa, K_{30} is the subgrade reaction modulus in MN/m^3, and α = the coefficient (ranges between 0.04~0.71 with a best fit of 0.36).
In spite of these shortcomings, use of Es and µs are computational conveniences that generally work with the theory of elasticity equations. The field (in situ) tests such as standard penetration test and cone penetration test can also be used to obtain the modulus of elasticity of soil [3].

This Paper represents the effect of Relative density on Elastic Properties of sand. This study includes determination of Index Properties of sand, Shear strength parameters, Modulus of elasticity and Modulus of Subgrade Reaction of sand at different relative densities. It also represents the comparison of the Modulus of subgrade reaction obtained from Model Plate load test with the theoretical relationships developed by various researchers mentioned in table 4.

2 Material and Test Set up

2.1 Material

Two types of sand were used in this study one was procured from Orsang River and another from Narmada River. The Particle Size Distribution Curve of both types of soils was obtained by performing Dry Sieve Analysis Test as shown in Fig.1, which shows that Orsang Sand (OS) has almost all particle size but less amount of fine sand (less than 0.2mm) while Narmada Sand (NS) has almost all particle sizes between 0.6mm to 0.4mm and few amounts of other sizes. The Index Properties of both types of sands are as listed in table 5. The abbreviations used for Orsang sand at 40%, 60% and 80% relative density are OS-40, OS-60 and OS-80 respectively and for Narmada sand NS-40, NS-60 and NS-80.
Mild Steel Plate having 220mm x 220mm x 25mm dimensions was used to perform Plate load Test.

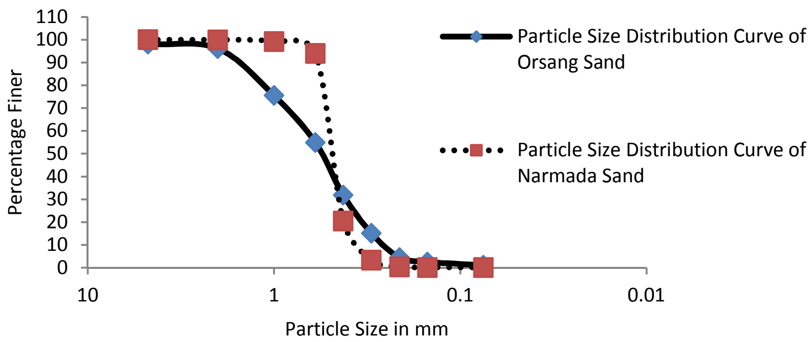

Fig.1. Particle Size Distribution Curve for Orsang Sand and Narmada Sand

2.2 Test Setups

Triaxial Test Setup: To determine Modulus of Elasticity of sand Simple Triaxial Test was performed on both types of sand at 40%, 60% and 80% Relative Density. The dimension of cylindrical sand samples used in this test was 38mm diameter and 81mm height. It was performed as per IS: 2720-11[4]. The detailed procedure of the preparation of sand sample had been adopted as below.
A metal former, split mould of about 38.5mm internal diameter, with vacuum pressure valve was used for preparing the soil sample. A porous stone was placed on the top of the pedestal of the triaxial base, and the pressure connection was attached to a vacuum pump. One end of a membrane was sealed to the pedestal by O-rings. The metal former was clamped to the base. The upper metal ring of the former was kept inside the top end of the rubber membrane and was held with the help of clamp before placing the funnel and the rubber bung in position.

The funnel and membrane were filled with oven-dried sand which was to be tested. Negative pressure of about 2KN/ m^2 was applied on membrane while filling the sand in membrane. The funnel was then removed and the sample was compacted if required. The surface of the sample was leveled and a porous stone was placed on its top. The loading cap was placed gently on the top porous stone and O-rings were fixed over the top of the rubber membrane [7].

Table 5. Index Properties of both types of sands

Sr. No.	Property	Unit	Values for Orsang river Sand	Values for Narmada river sand
1.	Specific Gravity	-	2.55	2.62
2.	Coefficient of Uniformity (Cu)	-	2.711	1.49
3.	Coefficient of Curvature (Cc)	-	0.94	1.108
4.	Type of sand as per IS code method (sieve analysis)	-	SP- poorly graded sand	SP- Uniformly graded sand
5.	Maximum Density	(gm/cm^3)	1.83	1.67
6.	Minimum Density	(gm/cm^3)	1.5	1.43
7.	Density of sand at 40% Relative Density	(gm/cm^3)	1.62	1.52
8.	Density of sand at 60% Relative Density	(gm/cm^3)	1.68	1.57
9.	Density of sand at 80% Relative Density	(gm/cm^3)	1.75	1.62
10.	Angle of internal friction at 40% I_D(from Direct shear box test)	degree	32	30
11.	Angle of internal friction at 60% I_D(from Direct shear box test)	degree	35	32
12.	Angle of internal friction at 80% I_D(from Direct shear box test)	degree	39	35

Model Plate Load Test Setup: Modulus of subgrade reaction was obtained by performing Model plate load test as per IS:9214-1979[5].

Model Plate Load Test was performed on both types of sand at 40%, 60% and 80% Relative Density. The size of plate used in Plate load test was 220 mm x 220mm x 25mm and size of tank was 1200 mm x1200mm x 1200mm. To achieve the desired density tank was filled layer by layer and each layer was vibrated for a particular duration by surface vibrator. Surface vibrator having surface area 32cm x 31cm , 16.9 kg weight and 1400 rpm frequency was used in this study. The detail of this procedure is given in Table 6 below.

Table 6. Details of parameters used to achieve the desired density of sand using surface vibration technique

Type of Sand	Density (gm/cm3)	Relative Density (%)	Thickness of Layer (mm)	Duration of Vibration (sec)
Orsang	1.62	40	150	45
	1.68	60	100	85
	1.75	80	50	60
Narmada	1.52	40	100	30
	1.57	60	100	60
	1.62	80	50	90

3 Result and Discussion

Average Modulus of Elasticity of Sand Es and Modulus of Subgrade Reaction k_s ($kN/m^2/m$) from Triaxial test and Model Plate load test (MPLT) were obtained as shown in Table-7. Modulus of Elasticity of Sand Es was taken an average of three slopes of the initial tangent of deviator stress-strain curve at different cell pressures (Fig.2 and Fig.3). The values of Poission's ratio were considered as 0.3, 0.25 and 0.2 for 40%, 60% and 80% relative density respectively in this study based on table 11.2 of K.R.Arora book [7], which was used to calculate theoretical value of Modulus of Subgrade Reaction k_s using correlations developed by various researchers and it was compared with the value obtained by performing Model Plate Load Test.

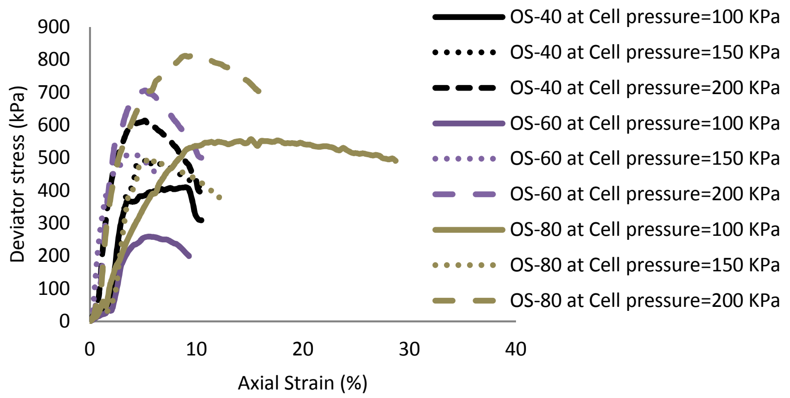

Fig.2. Deviator Stress-strain curve of Orsang sand at 40%, 60% and 80% relative density

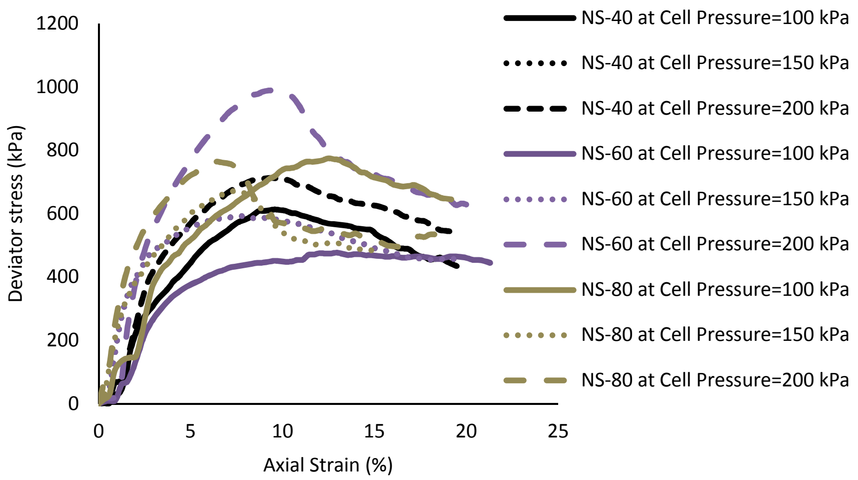

Fig.3. Deviator Stress-strain curve of Narmada sand at 40%, 60% and 80% relative density

3.1 Modulus of Elasticity of Sand Es

As the relative density (I_D) of sand was increased the Modulus of Elasticity of sand was also increased.

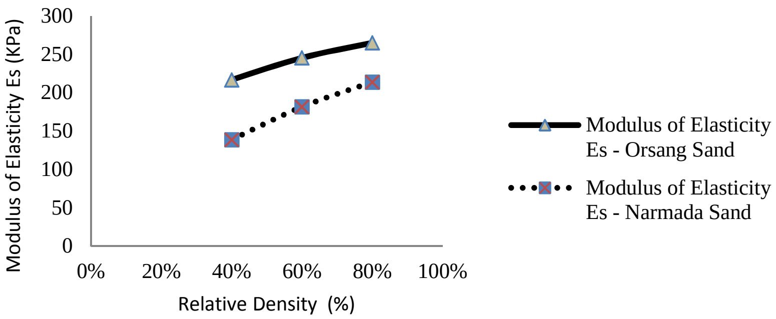

Fig.4. Variation in Modulus of Elasticity Es for Relative Density

3.2 Modulus of Subgrade Reaction (K_s)

Modulus of subgrade reaction was obtained from the Model plate load test as per IS:9214-1979.Load settlement curves for both types of sand at different relative densities were obtained as shown in Fig.5 and Fig.6.
Three types of corrections namely corrections for load deflection curve, correction for bending of the plate and correction for size of plate was applied for calculating corrected Modulus of Subgrade reaction according to IS: 2950 (Part I) -1981 [6].

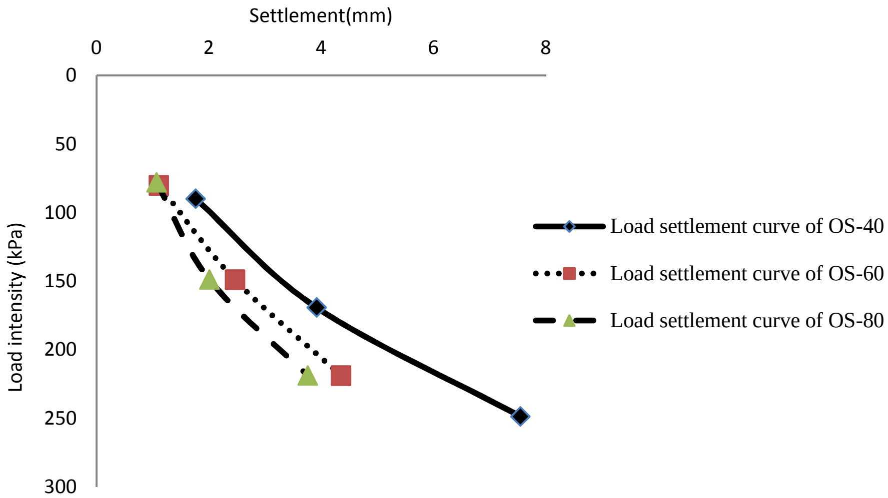

Fig.5. Load Settlement curve for Orsang Sand at 40%,60% and 80% relative density

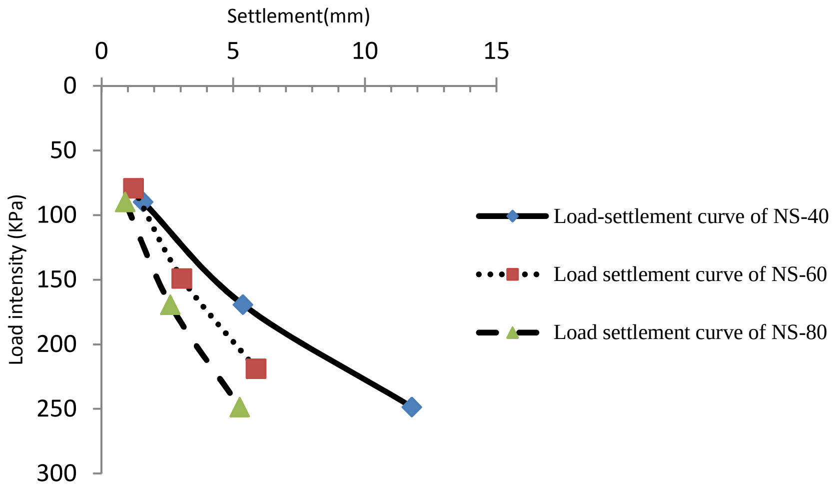

Fig.6. Load Settlement curve for Narmada Sand at 40%,60% and 80% relative density

Table 7. Elastic properties of sands at different relative densities and comparison of Modulus of Subgrade reaction

Type of sand	Modulus of Elasticity of Sand Es (KPa)	Modulus of Subgrade Reaction k_s ($KN/m^2/m$) from Present study and from theoretical relation developed by various investigators (Table 4)					
		Present Study	Vesic	Biot	Meyer-hof and Baike	Klopple and Glock	Selvadu-rai
OS-40	21667	49219	69640	102516	108225	151515	70346
OS-60	33889	87500	109744	162822	164310	246465	106801
OS-80	39167	112500	125366	186187	185448	296717	120541
NS-40	13879	26250	42985	62587	69327	97058	45063
NS-60	20952	56250	65186	95573	101587	152381	66032
NS-80	25229	65625	77847	114366	119454	191127	77645

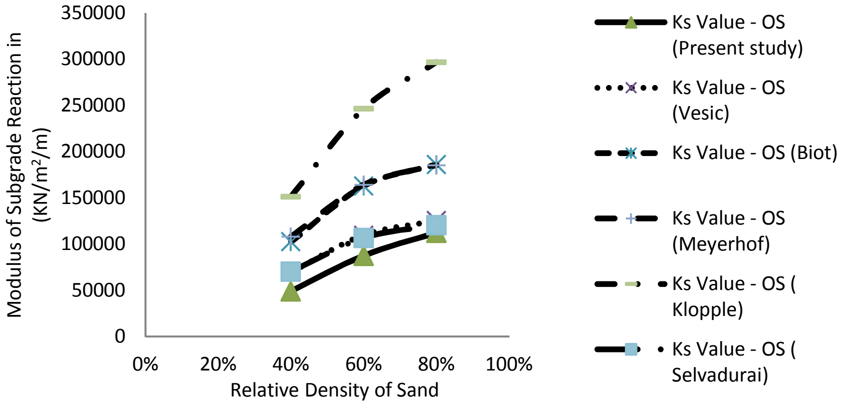

Fig.7. Comparison of Various Theoretical method and Present study for finding out Modulus of Subgrade Reaction of Orsang Sand at different relative densities

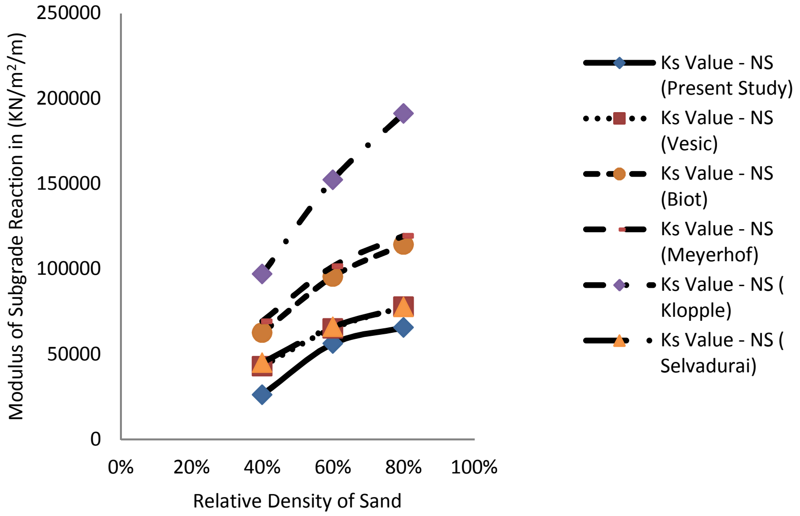

Fig.8. Comparison of Various Theoretical Methods and Present study for finding out Modulus of Subgrade Reaction of Narmada Sand at different relative densities

4 Conclusion

The Following Conclusions were obtained from this study:

1. As the relative density of sand increased the Modulus of Elasticity and Modulus of Subgrade reaction of both types of sand were increased.
2. The value of Subgrade reaction obtained by performing model plate load test(IS:9214,1979) was compared with the theoretical relation given by various researchers and found that the pattern of changing of Modulus of Subgrade reaction of Both sands was nearly same as of Vesic and Selvadurai. From this experimental study it can be concluded that in absence of Plate load test data of sand the theoretical relationship developed by Vesic and Selvaduraican can be used to determine Modulus of Subgrade Reaction by knowing Modulus of Elasticity of sand from Triaxial Test.
3. The value of Modulus of Subgrade reaction of Orsang sand at 60% and 80% relative density was increased by 78% and 128% respectively with respect to at 40% relative density and for Narmada sand, it was increased by 114% and 150% respectively**.**
4. As the relative density (I_D) of Orsang sand was increased to 60%(I_D) and 80%(I_D) from 40% (I_D) the Modulus of Elasicity of sand were found to be increased by 56% and 81% respectively as compared to Es at 40% (I_D) and-for Narmada sand it was 51% and 82% respectively.

References

1. Jamshid, Sadrekarimi.: Comparative study of methods of determination of coefficient of subgrade reaction. Electronic Journal of Geotechnical Engineering 14 (E), 1-14 (2009).
2. DaeSang Kim, SeongYong Park.: Relationship between the subgrade reaction modulus and the strain modulus obtained using a plate loading test. WORLD CONGRESS RAILWAY RESEARCH 2011, vol.9, pp. 1-11 (2011).
3. Ziaie_Moayed, R., Naeini, S.A.: Evaluation of modulus of subgrade reaction (Ks) in gravely soils based on standard penetration test (SPT). In: Ng, Zhang & Wang (eds.), INTERNATIONAL CONFERENCE ON PHYSICAL MODELLING IN GEOTECHNICS 2006, vol.6, pp. 801-805, Taylor & Francis Group, London (2006).
4. IS: 2720-Part-11-1993. Determinaton of the shear strength parameters of a specimen tested in unconsolidated undrained triaxial compression without the measurement of pore water pressure. India: Bureau of Indian Standards,Manak Bhavan, 9 Bahadur Shah Zafar Marg,New Delhi 110002,(1993).
5. IS: 9214 -1979. Method Of Determination Of Modulus Of Subgrade Reaction (K-Value) of Soils in Field. India: Bureau of Indian Standards,Manak Bhavan, 9 Bahadur Shah Zafar Marg,New Delhi 110002,(1997).
6. IS: 2950 (Part I) -1981,Indian Standard Code of Practice for Design and Construction of Raft Foundations. New Delhi: Bureau of Indian Standards Manak Bhavan, 9 Bahadur Shah Zafar Marg New Delhi 110002,(2004).
7. K. R. Arora: Soil Mechanics and Foundation Engineering. 3rd edn. Standard Publishers Distributors, Delhi (1992).
8. Bowles, J. E.: Foundation Analysis and Design. 6th edn., McGrow-Hill International press, New York (1998)