Probabilistic Evaluation of Liquefaction Potential Using Multivariate Adaptive Regression Splines

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Abstract. Liquefaction poses major technical challenges for key infrastructures such as nuclear power stations and large earth dams. In recent years, several efforts have been made to assess the liquefaction potential of a site using simplified methods, namely, the Seed and Idriss (1970) method, the Youd et al. (2001) method, the Idriss and Boulanger (2004) method, and the IS Code(1893) method. In general, the problem of liquefaction is highly nonlinear in nature and the parameters involved, namely, the engineering properties of soil and the earthquake characteristics, are all subjected to uncertainties. Although the role of soil plasticity in predicting the liquefaction potential in fine-grained soils is well recognized, none of the above-mentioned methods considered the effects of liquid limit and plasticity index in addition to the parameters such asN value, fines content, peak ground acceleration, and cyclic stress ratio. Keep- ing the above in view, the aim of this study is to (i) develop a comprehensive surrogate model considering all the above-mentioned six parameters using the multivariate adaptive regression splines (MARS), (ii) train and test the devel- oped model using the dataset available in literature, and (iii) perform probabilis-tic analysis using the first order reliability method (FORM) for predicting the liquefaction response of soils. Considering the computational efficiency, predic-tive accuracy, and the adaptivity associated with the developed model, the useof MARS in assessing liquefaction potential has been found to be promising.

Keywords: Liquefaction potential, Multivariate adaptive regression splines, Probabilistic analysis, First order reliability method.

1 Introduction

Ground shaking during earthquakes can cause loss of strength or stiffness in soils leading to settlement, landslides, earth dam failure, and other forms of damage in different structures. A phenomenon having the potential to result in such a loss of strength or stiffness is referred to as *liquefaction* in the literature.

Investigation and research carried out in the past have revealed that the mitigation of soil liquefaction is a difficult task and is sometimes uneconomical too. To handle this challenge, instead of liquefaction mitigation, main focus has always been on pre- diction of soil liquefaction. To achieve this, at first, several deterministic methods are applied. Values of Factor of Safety (FOS) are found out and the results are analyzed to judge whether liquefaction will occur or not. Within the framework of a determin- istic approach, a comparative study among the commonly recommended methods should be carried out and a site-specific guideline for the evaluation of liquefaction potential should be established for the benefit of practicing engineers. Moreover, a reliability-based evaluation of liquefaction would provide the risk associated with any possibility of soil liquefaction, apart from predicting the dominant parameters in- volved.

Juang et al. [10] used the approach of first-order reliability index for evaluation of liquefaction probability. This method necessitates an understanding of parameter and model uncertainty. Through "neural network learning" of case histories, an empirical model was used for estimating liquefaction resistance based on cone penetration test (CPT). Jha and Suzuki [8] used an upgraded FOSM, an advanced FOSM (Hasofer–Lind), point estimation method (PEM), and Monte Carlo simulation (MCS) method to evaluate the probability of liquefaction. Ku et al. [11] developed a rigorous maximum likelihood analysis of the adopted database based on the Robertson and Wride [16] method. For geotechnical practitioners, this model is more useful than the Factor of safety (FOS) approach. This model can easily be used in an excel spreadsheet for practical application. Bagheripur et al. [2] used reliability-based method to calculate the probability of failure and assessment of the liquefaction potential by using Genetic Algorithm (GA). Johari et al. [9] developed a probabilistic model based on the Jointly distributed random variable (JDRV) method, to predict the liquefaction potential of soil utilising the shear wave velocity test data. Umar et al. [18] also calculated relia- bility index and probability of seismic liquefaction potential. The chances of liquefac- tion of soil have been predicted using spreadsheet and MATLAB based on the pro- posed relation between P_L (Probability of Failure) and FOS (Factor of Safety). Ghani and Kumari [3] developed the regression model to predict the liquefaction using im- portant parameters such as liquid limit (LL), SPT blow count, fine content (FC), and moisture content with multi linear regression analysis. Zhang et al. [20] used the ELM(Extreme Learning Machine) to predict the model based on 266 CPT samples. In this study, it has been clearly indicated that the prediction model based on CPT perform better than that of SPT and can be accurately predicted for up to 100% of the liquefiedcase and overall accuracy of 87.5% which can be improved by adding more non- liquefied cases into the training set. Zhang et al. [21] used the GWO (Grey wolf Op- timization) algorithm to improve the prediction and effectiveness of the SVM model. Its optimization capability had been observed to be better than other methods. Zhang et al [22] used deep neural network and shear wave velocity (v_s) to predict model for soil liquefaction based on the SPT – v_s. Prediction parameters have been determined and analysed according to the mathematic prediction model. It was advocated that the existence of v_s can improve the accuracy of the prediction model and thus, it should be considered as an essential parameter for the prediction of soil liquefaction in fu- ture. Zhao et al. [23] developed a PSO-KELM model for soil liquefaction potential evaluation based on the cone penetration test (CPT) and shear wave velocity test (v_s). But it has one limitation that it cannot be used for other than Holocene, un-aged, and non-cohesive soil.

From the literature review presented herein it may be noted that various researchers have worked on several aspects of the topic under consideration. Many useful results have been obtained and conclusions drawn. But very few comparative studies have been reported involving Seed and Idriss [17], Youd et al. [19], Idriss and Boulanger [6] and IS 1893 (Part 1) [7]. It is also well accepted now that plasticity of soil has a dominant role in predicting liquefaction potential of soil, but very few studies have been reported in this direction. The parameters related to plasticity of soil, namely, theliquid limit (LL) and the plasticity index (PI) are found to have a very significant role in the evaluation of liquefaction potential of fine-grained soils.

Keeping the above in view, the objectives of the present study are to develop a comprehensive surrogate model considering all the above-mentioned six parameters (the N value, LL, PI, FC, PGA and CSR) using the multivariate adaptive regression splines (MARS)[1, 13, 14], to train and test the developed model making use of a dataset of liquefaction case study (Ghani and Kumari [3]), and finally, to perform probabilistic analysis using the First order reliability method (FORM) for predicting the liquefaction response of soils.

2 Comparison of Factor of Safety Determined Using Different Methods

Liquefaction potential of soil has been determined using the simplified procedure developed by Seed and Idriss [17], Youd et al. [19], Idriss and Boulanger [6] and the IS CODE [7] to compute Cyclic Stress Ratio (CSR), Cyclic Resistance Ratio (CRR), using the dataset of Chi-Chi earthquake liquefaction well documented by Hwang and Yang [5]. Separate computing codes have been developed for each of the methods in MATLAB environment. The validation of these codes has been done using the site data published in Jha and Suzuki [8] [12]. It is observed that the mean absolute error (MAE) associated with Idriss and Boulanger method is the minimum and the coeffi- cient of determination value (R^2) is the maximum amongst the four methods (Table1). Therefore, it can be concluded that Idriss and Boulanger method is the most accu- rate method.

Table 1. Summary of results of different statistical parameters

Sl. No.	Methods	Maximum error (%)	Minimum error (%)	Mean Ab-solute Error (MAE)	Coefficient of determination (R^2)
1	Seed and Idriss Method	2.79	0.025	0.005	0.996
2	Youd et al. Method	1.45	0.13	0.02	0.979
3	Idriss and Bou-langer Method	2.3	0.33	0.003	0.998
4	IS 1983 (Part 1) 2016 Method	8.51	0.85	0.004	0.996

In addition to the above studies using different deterministic methods to assess the liquefaction potential of a particular site, the factor of safety against liquefaction (F_L) have been determined and compared in Table 2 using the Chi-Chi earthquake dataset. It has been observed that factor of safety is less than one for all the depth which indicates that entire site is susceptible to liquefaction. The F_L values determined using the Idriss and Boulanger Method is more close to one than the other methods which is as per expectation as this is the most accurate method.

Table 2. Summary of F_L Based on Different Methods

Sl. No.	Depth (m)	γ (kN/m^3)	SPT, N	FC	Water table (m)	Values of F_L using different methods		
						Youd et al. Method	Idriss and Boulanger Method	IS 1983: 2016 Method
1	5	19.8	9	20	4	0.487	0.494	0.297
2	5.8	19.0	7	25	5	0.436	0.474	0.298
3	8.3	19.6	12	13	2.8	0.398	0.386	0.195
4	6.3	22.0	16	15	1.2	0.517	0.368	0.241
5	2.8	18.5	6	22	0.7	0.381	0.612	0.256
6	7.3	20.5	11	21	5.0	0.459	0.406	0.274
7	3.0	20.0	5	24	2.4	0.443	0.715	0.316
8	7.5	18.5	12	55	2.8	0.453	0.332	0.287
9	5.8	19	4	35	2.8	0.317	0.707	0.245
10	5.8	18.3	10	30	1.5	0.408	0.614	0.265

3 Development of MARS Model and Reliability Studies

As already mentioned in the *Introduction*, existing methods for the evaluation of liquefaction potential has one shortcoming that plasticity of the soil has not been considered as a parameter in the overall assessment. Thus, the one of the objectives of the present study is to develop a surrogate model considering the plasticity properties of soils (namely, LL and PI) in addition to other significant parameters [$(N_1)_{60}$, FC, PGA, CSR]. The multivariate adaptive regression splines (MARS) is one such regres- sion technique, the application of which is already well-established in other field of engineering, which is intended to be applied to predict the factor of safety against liquefaction in this study.

3.1 Study Area and Datasets Selected for liquefaction Analysis

The datasets selected for the training and testing of the MARS model have been taken from Ghani and Kumari [3]. In the same study, it is mentioned that the soil samplesfor those datasets have been collected from the five districts around Patna, Bihar and these places are in the zone IV of earthquake. As the Idriss and Boulanger method hasbeen observed to be the most accurate among other models considered in this study, this method has been made use of to determine the factor of safety (FOS) against liquefaction and taken as observed FOS. Initially 100 datasets have used for the training the MARS model and 13 datasets have then been employed for testing/ verifying the developed MARS model. Once the MARS based surrogate model has been developed, the statistical performance parameters, namely, the root mean square error (RMSE), and the coefficient of determination (R^2) have been determined to assess the predictive capacity of the model. It may be noted that for an ideal model, RMSE value should be close to 0.0 (zero) and R^2 value should be close to 1.0 (one). Table 3 presents the summary of the performance parameters, in which it has been observed that R^2 value during training is 0.968. Fig. 1 shows the plot between the values of factor of safety (FOS) predicted by MARS model and those calculated by the Idriss and Boulanger [6] method, which also indicates very well fitting and predictive capability of the MARS with only 100 samples. The R^2 value during testing is 0.893 which may be due to only 13 samples are available for testing purpose. Another interesting observa- tion noted from this study that R^2 value for the liquefied cases is 0.749 while that for the non-liquefied cases are 0.957. Thus, the performance of the developed MARS model in predicting non-liquefied cases is very good as compared to the liquefied cases. Again, this may be due to very less numbers of liquefied cases available for verification.

Table 3. Statistical performance detail for the developed MARS model.

Sl. No.	Item	RMSE	R^2
1	Training	0.088	0.968
2	Testing	0.133	0.893
3	Liquefied Cases	0.117	0.749
4	Non-liquefied Cases	0.092	0.957

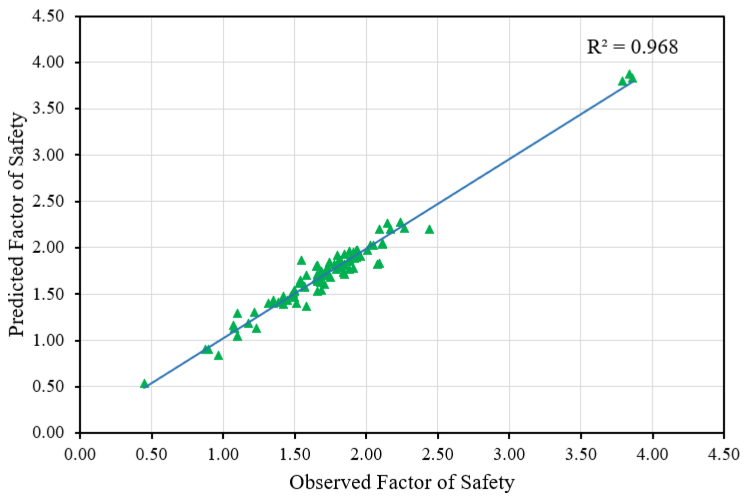

Fig. 1. Illustration of observed and predicted values of factor of safety

3.2 Effect of Soil Plasticity against Liquefaction

One of the objectives of this study is also to observe the effect of plasticity properties of soil to liquefaction assessment. From the datasets available for analysis [3] and the factor of safety against liquefaction (F_L) determined using MARS model, it has been observed that fine grained soil with LL > 25% (Fig. 2) and/ or PI < 8% (Fig. 3) has more chances of failure, this also corroborate the similar observation reported by Ghani and Kumari [3].

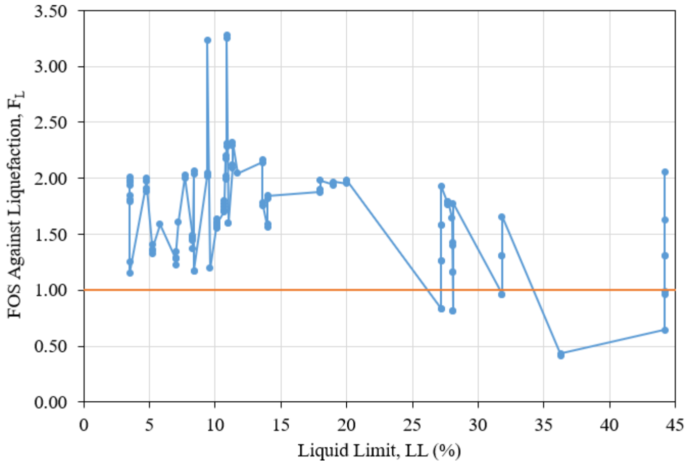

Fig. 2. Variation of FOS with liquid limit, LL

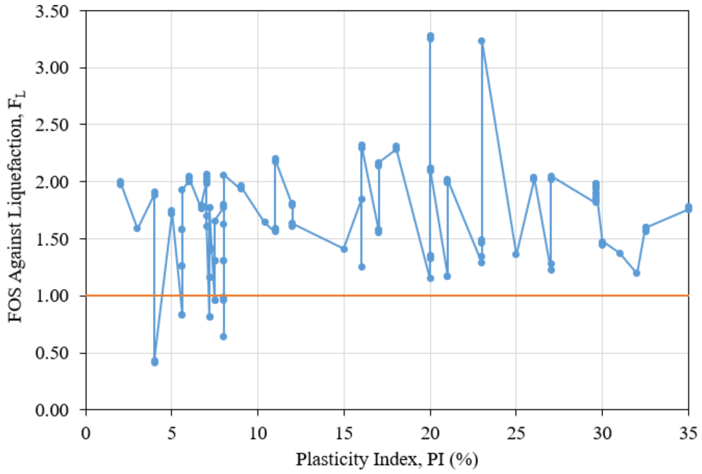

Fig. 3. Variation of FOS with plasticity index, PI

Table 4. The COV values for random parameters.

Sl. No.	Parameters	COV	Probability Distribution
1	$(N_1)_{60}$	0.4	lognormal
2	FC (%)	0.35	lognormal
3	PGA	0.1	lognormal
4	LL (%)	0.25	lognormal
5	PI (%)	0.2	lognormal
6	CSR	0.15	lognormal

3.3 Reliability Analysis Based on the Developed MARS Model

For the purpose of reliability analysis, all the six parameters considered in this study for liquefaction assessment have been treated as random. The observed/ reported values of those parameters are considered as mean values and the values of coefficient of variation (COV) used are taken from literature [3, 4, 15] and tabulated in Table 4. As some of the random variables cannot be negative, all the random variables are as- sumed to be lognormally distributed.

Table 5. Summary of reliability results using FORM coupled with MARS model.

Sl. No.	FOS Against Liquefaction, F_L	Reliability Index, β	Liquefaction Probability, P_L	P_L (%)
1	0.54	1.95	0.03	2.56
2	0.84	0.67	0.25	25.21
3	1.13	0.09	0.46	46.26
4	1.46	1.15	0.13	12.60
5	1.39	0.54	0.29	29.40
6	1.31	0.51	0.31	30.59
7	1.59	0.95	0.17	17.17
8	0.97	0.26	0.40	39.64
9	1.17	0.34	0.37	36.56
10	1.19	0.39	0.35	34.82
11	1.76	1.64	0.05	5.07
12	1.19	0.45	0.33	32.79
13	2.04	1.75	0.04	3.99

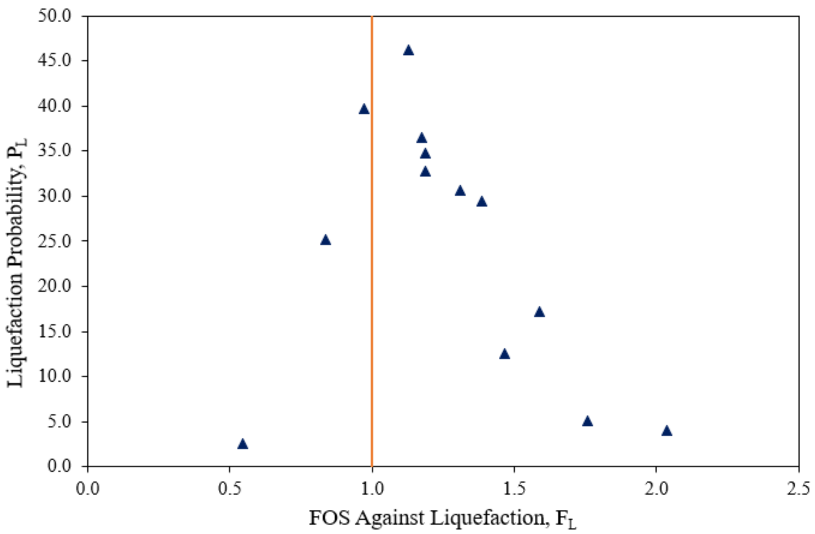

Fig. 4. Relationship between the FOS against liquefaction (F_L) and liquefaction probability, P_L

Table 5. presents the summary of reliability results (in terms of reliability index, β, and liquefaction probability, P_L) alongside the FOS against liquefaction (F_L). It is observed from Table 5 that the F_L is not the consistent measure risk particularly when it indicates liquefied case. This is also reflected from Fig. 4.

3.4 Probabilistic Sensitivity Analyses of The Random Variables

Based on the FORM method, a comparison study between the sensitivity indexes of the random parameters has also been done for all the 13 sites, of which reliability results are already presented in Fig. 5.

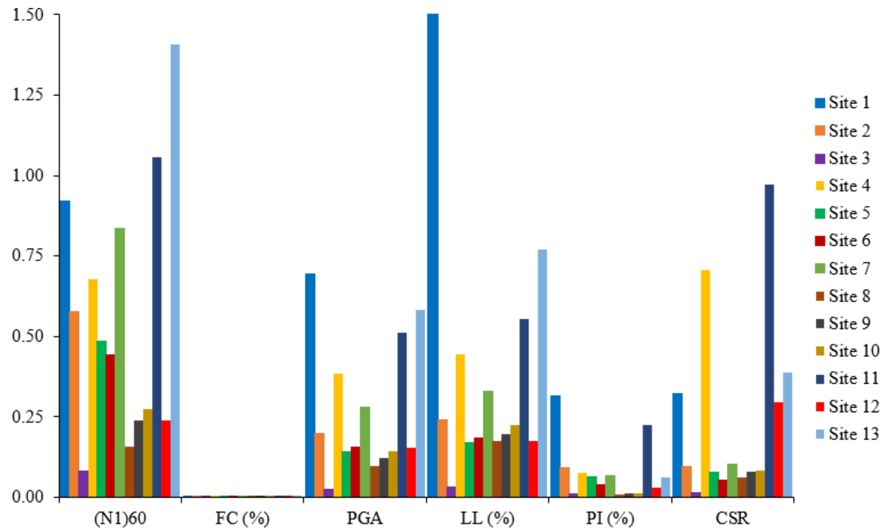

Fig. 5. Sensitivity analysis study based on the FORM method

From Fig. 5, it is observed that for most of the sites, LL (%) is found out to be second most significant parameters and in site 1, it is the most dominated parameters amongst the six parameters considered in this study. Therefore, it may be concluded that for fine grained soil, the plasticity properties of soil have a significant contribution in liquefaction assessment.

4 Conclusions

Based on the observations from the studies contained in this paper, the following concluding remarks can be made:

1. A performance study of the methods for assessment of liquefaction potential with respect to the Chi-Chi earthquake field data has revealed that the Idriss and Boulanger method (2004) has the minimum mean absolute error (MAE) and the maximum coefficient of determination. On this basis it can be stated that this method is the most accurate amongst the four such methods considered in this study.

2. It has also been observed that out of all the four values of factor of safety against liquefaction determined by using the four different methods, the one obtained from the Idriss and Boulanger Method is the closest to unity (1.0) for the site that is well-known to be susceptible to liquefaction. This is quite similar to the obser- vation made in Chi-Chi earthquake site data reported by Hwang and Yang [5].
3. Although it is well accepted that plasticity of soil has a dominant role in predicting liquefaction potential of soil, very few studies have been reported in this direction. In this study, a MARS (Multivariate Adaptive Regression Splines) based surrogate model has been developed incorporating plasticity properties of soils (namely, LL and PI) in addition to other significant parameters [$(N_1)_{60}$, FC, PGA, CSR] to predict the factor of safety against liquefaction.
4. From the studies on the effect of plasticity on liquefaction, it has been observed in the present study that fine-grained soil with LL > 25% (Fig. 2) and/ or PI < 8% (Fig. 3) has a higher probability of liquefaction.
5. From the probabilistic evaluation of liquefaction potential based on the developed MARS model, it is observed that the factor of safety against liquefaction is not a consistent measure of risk particularly when it indicates a case of liquefaction.
6. From the probabilistic sensitivity analysis carried out here it is observed that, for most sites, LL (%) is found out to be the second most (significant) dominant parameter while, for some sites, it is the most dominant parameter amongst the six parameters considered in this study. Therefore, it may be concluded that in the case of fine- grained soils, the plasticity properties of soil have a significant in- fluence on the assessment of liquefaction potential.

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