

Comparative Study of Analytical and Numerical

Modelling of Bearing Pressure of Shallow Foundation

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Abstract. Settlement is the predominant criterion which governs the design of footing on frictional and cohesive frictional soil. Hence bearing pressure plays a key role in design of footing. To evaluate the bearing pressure, Teng, Meyerhof and other researchers proposed empirical formula and charts in terms of N values obtained from Standard Penetration Test (SPT). In the present study, soil investigation data obtained from four sites have been used to estimate the bearing pressure through empirical formula and finite element analysis. Finite element model is validated through the case studies available in the literature. Bearing pressure values for strip footing of width varying from 1.5 to 5.5 m were estimated. Further, an attempt has been made to check the reliability of empirical formula by comparing with values obtained through numerical analysis. It is observed that if the SPT-N values are directly taken from the field tests, the results from Meyerhof expression were found to be in good agreement with the numerical model. Whereas, if only shear strength parameters are obtained from site, SPT-N value calculated indirectly using friction angle excludes the effect of cohesion. Hence, Meyerhof expression resulted in comparatively lower bearing pressure values.

Keywords: Shallow foundation, Bearing pressure, Finite element model, Empirical formula.

1 Introduction

For the structures including buildings, earth fills, bridges, and concrete dams, it is the earth which provides the overall support. Therefore, properties of the supporting ground will largely affect the stability of the structures. The ground which supports the structure is inevitably a soil that is weaker than any other construction materials such as masonry, wood, steel or concrete. Foundations are the substructure that transmits the structural load to the ground in such a manner that the supporting soil is not overstressed and does not undergo excessive settlement. Hence, it is necessary to understand and learn the response of the soil under loads and the parameters that influence it. Allowable bearing pressure of foundations is one of the primary concerns for geotechnical engineers. There are two main considerations to predict the allowable bearing pressures of shallow foundations: (a) there must be adequate factor of safety against ultimate shear failure and (b) the settlements should be within the acceptable limits. These are extensively studied, both experimentally and theoretically over the past decades (Jayamohan 2018).

For footing on granular soil, the allowable bearing pressure is generally governed by the settlement criterion unless the soil is loose or the footing is narrow. For footings of normal sizes the net safe bearing capacity is quite high for most natural sand deposits. Footings on granular soils are proportioned commonly by the use of Standard Penetration Test (SPT) 'N' values. Most of the methods propose empirical equations or charts to determine safe bearing pressure for a specified maximum total settlement in terms of N values. Peck, Hanson and Thornburn (1974) have modified the original Terzaghi and Peck (1948) recommendations and presented charts to obtain allowable bearing pressure. Teng's (1962) equation is based on Terzaghi and Peck (1948) charts with an additional factor introduced to account for the influence of depth of foundation. Meyerhof (1974) neglected the water table correction factor in his expression, assuming that N value obtained from the field below water table already considers its affect. Bowles (1982) suggested an increase in the bearing pressure obtained from Meyerhof's equation by 50%.

To evaluate the bearing capacity of shallow footing, laboratory model studies and full-scale field experiments have been conducted in the past (Consoli et al. (1998), Nabil (1996), Dash et al. (2001) and Murat et al. (2012)). However, with the advent of numerical methods, Finite Element Method has become the most important tool besides experiment and theory in understanding the engineering problems. Similar to other field of engineering, geotechnical engineering too benefited from numerical methods in predicting the soil behavior under the influence of various loading conditions. Many researchers (Halder et al. (2018), Arab et al. (2017), Acharyya et al. (2018) and Javdanian (2017)) carried out studies to analyse the behaviour footing using various numerical tools.

In the present study, an attempt has been made to check the reliability of bearing pressure values of shallow foundation obtained from various empirical formulas, by comparing with values obtained through numerical analysis using PLAXIS 2D (Reference Manual 2021). Numerical models are validated by using the field load tests reported by Consoli et al. (1998) and Nabil (1996). Soil investigation data obtained from four different sites have been used to estimate the bearing pressure through empirical formula and finite element analysis.

2 Validation of numerical model

Validation of numerical model is the process of establishing evidence which confirms that the model can be used for user's needs or specified requirements. For validation, field or lab studies can be used. In the current study, the field load tests on circular footing plates reported by Consoli et al. (1998) and Nabil (1996) are used to validate

PLAXIS 2D numerical model. The load settlement curves obtained from tests, reported in literature are compared with the results obtained from numerical models.

2.1 Consoli et al. (1998)

Load-settlement response obtained from field experiments carried out on circular steel plates of diameter varying between 0.30 and 0.60 m are used to validate the numerical models (PLAXIS 2D). Soil behavior is modeled by using Mohr Coulomb (MC) and Hardening Soil (HS) models. The values of parameters used to model soil are listed in Table 1. The obtained responses are compared with the load settlement behavior reported in literature. The load settlement behavior of surface circular footing obtained from field experiments and numerical modeling (PLAXIS 2D) is plotted in Figure 1.

Table 1. List of parameters used in Numerical models (Consoli et al., 1998)		
Parameters	Mohr Coulomb (MC) model	Hardening Soil (HS) model

	(MC) model	(HS) model
Young's modulus $(E_{50})^{ref}$ (kN/m 2)	10000	10000
Tangent stiffness (E_{oed}^{ref}) (kN/m ²)	-	10000
Unloading/reloading stiffness $(E_{ur}^{ref})(kN/m^2)$	-	30000
Cohesion (c') (kPa)	17	17
Friction angle (Φ ') (degree)	26	26
Poisson's ratio (v')	0.3	0.3



Fig. 1. Load settlement behavior of circular footing obtained from Consoli et al. (1998) and numerical models.

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From Figure 1, it can be observed that the numerical results are in good agreement with the field values. For a given settlement, MC model gives higher value of load than the field values whereas, HS model predicts the results which are lower than the field results.

2.2 Nabil (1996)

The load-settlement response of circular foundation on cemented very dense sand obtained by plate load test at a site situated in Kaifan, Kuwait is compared with results from numerical analysis using PLAXIS 2D. The effective strength parameters c' (31 kPa) and \emptyset' (36°) of soil were obtained from the literature. The tests are conducted at a depth of 0.4 m below ground level using solid plates of diameter 0.6 m, 0.45 m and 0.3 m. The values of parameters used to model soil are listed in Table 2. Comparison of load settlement curve from Nabil and Numerical model is shown in Figure 2.

Parameters	Mohr Coulomb (MC) model	Hardening Soil (HS) model
Young's modulus $(E_{50})^{ref}$ (kN/m ²)	55000	55000
Tangent stiffness (E_{oed}^{ref}) (kN/m ²)	-	55000
Unloading/reloading stiffness $(E_{ur}^{ref})(kN/m^2)$	-	165000
Cohesion (c') (kPa)	31	31
Friction angle (Φ ') (degree)	36	36
Poisson's ratio (v')	0.292	0.292
Unit weight (γ) (kN/m ³)	18.4	18.4

 Table 2. List of parameters used in Numerical models (Nabil, 1996)



Fig. 2. Load settlement behavior of circular footing obtained from Nabil. (1996) and numerical models.

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From the figure, it can be observed that, the results predicted by HS numerical model matches well with the results reported in the paper whereas, results from MC model are on higher side than the field results.

Based on the validation, it was found that Mohr Coulomb model gives 18.09 % higher value of load than the field values for a given settlement as it does not consider the effect of stress on stiffness. Whereas Hardening Soil model takes into account the stress dependency of stiffness and hence predicted load 8.89 % lower than the field values which was comparatively close. Thus in present study, HS model has been used for the further modeling of soil.

3 Comparative study

3.1 Analytical modelling of bearing pressure

Based on the works of Terzaghi and Peck, Teng (1962) gave a relation for allowable bearing capacity for a given value of permissible settlement as:

$$q_{np} = 1.4(N-3) \left(\frac{B+0.3^{2}}{2B}\right)^{2} C_{D}R_{w}'S_{a} KN/m^{2}$$

Meyerhof (1974) proposed an empirical formula different from that of Teng's as shown below.

$$q_{np} = 0.49NC_DS_a \ KN/m^2$$
 For B<1.2m
 $q_{np} = 0.32N \left(\frac{B+0.3}{B}\right)^2 C_DS_a \ KN/m^2$ For B>1.2m

Bowles (1982), assuming Meyerhof's expression to be on the conservative side proposed a recommendation to increase the bearing pressure values obtained from Meyerhof's equation by 50%.

In the current study, bearing pressure corresponding to 25 mm settlement (s_a) for strip of width (B) varying from 1.5 m to 5.5 m have been estimated using Teng, Meyerhof and Bowles recommendations.

3.2 Numerical modelling of bearing pressure

The strip footing is modelled in PLAXIS 2D as a plain strain model with 15 noded triangular elements. Hardening Soil (HS) model has been used to model soil behaviour. The input parameter Young's modulus (E_{50}^{ref}) has been back calculated from elastic settlement equation 1;

$$\Delta H = q_0 B \times \frac{1-\mu^2}{E} I_f -\dots -(1)$$

Where, ΔH is settlement, q_0 is pressure, *B* is width of footing, μ is Poisson's ratio, E is Young's modulus and I_f is influence factor. The values of Young's modulus thus

obtained are checked for the range suggested by Bowles (1995). PLAXIS in-built relation between E_{50}^{ref} , E_{oed}^{ref} and E_{ur}^{ref} ($E_{oed}^{ref} = E_{50}^{ref}$ and $E_{ur}^{ref} = 3 * E_{50}^{ref}$) have been used. Poisson's ratio of soil model is taken as 0.3. Water table was assumed to be present at the ground level. Footing is modelled using elastic plate of axial stiffness of 5*10⁶ kN/m and flexural rigidity of 8500 kNm²/m (Arab et al., 2017) kept at a depth of 1.5m below ground level. The strip footing plate is subjected to a uniform line load.

Meshing is done by selecting coarseness factor of medium type. Calculation type in initial phase is selected as K_0 procedure. Plastic calculation has been done in phase-1 by activating footing plate and line load. Deformed mesh obtained from PLAXIS 2D is shown in Figure 3. Bearing pressure corresponding to 25 mm settlement is obtained by plotting curve between load and settlement.



Fig. 3. Deformed mesh scaled up to 4 times (PLAXIS 2D)

3.3 Soil investigation data

In order to calculate bearing capacity and bearing pressure, it is necessary to determine basic soil parameters such as cohesion, angle of internal friction, SPT N value, etc. These parameters can be calculated by carrying out in-situ and laboratory tests. In this study, the soil investigation data obtained from four different sites have been used as input parameters for analytical and numerical modelling. From bore log data, SPT N values have been determined. By performing direct shear test (IS-2720-13, 1986), cohesion and angle of internal friction have been calculated.

Site-1:

Bore log data was collected from the site near Kottara, Mangalore, Karnataka. Overburden correction has been applied to the obtained SPT N value. The correlation between SPT N value and angle of internal friction (Φ) given by IS 6403:1981 has been used to find corresponding Φ value.

Obtained SPT N value is 18.83 and corresponding Φ value = 33°. This Φ value is used to model the soil behavior in PLAXIS 2D. The corrected SPT N value is used in empirical equations to find bearing pressure.

Site-2:

Laboratory tests are carried out on the soil sample obtained from a site near Hebri, Udupi. The soil is classified as silt of high compressibility (IS-1498, 1970). Shear strength parameters are used to find the bearing pressure through numerical model. From Φ value, SPT N value is calculated from chart provided in IS 6403:1981. Obtained SPT N value (for $\Phi = 34^{\circ}$) is 22. This SPT N value is used in empirical formula to find bearing pressure.

Site-3:

The laboratory tests are carried out on the soil samples obtained from the site near Dharmasthala, Karnataka. The soil is classified as silty sand (IS-1498, 1970). From known value of angle of internal friction, SPT N value is obtained by using chart provided in IS 6403:1981. For Φ =32°, SPT N value obtained is 10. Using these properties of soil, bearing pressure are determined and compared.

Site-4:

Standard Penetration test data from a site near Padubidri has been collected along with core cutter samples. Corrections for SPT N values obtained from the field are made. Direct shear test has been conducted on undisturbed soil sample to determine angle of internal friction and cohesion of soil. As both the shear strength parameters and SPT N values are directly obtained from tests, actual value of these parameters are directly used in analytical equations and empirical formula and also in the numerical analysis. No correlations or charts are used in calculating parameters.

Bore log datas collected from site 1 and site 4 are shown in Figure 4 and Figure 5 respectively. The laboratory test results on the soil samples collected from site 2, site 3 and site 4 are shown in Table-3.

In the case of sites where SPT N values are not available (site 2 and site 3), where SPT N values are determined on the basis of Φ value, numerical analysis considering cohesion as zero is also carried out to observe the effect of cohesion on determined SPT N value and corresponding bearing pressure value.

	Depth (m)	SPT 'N' Value
Clayey Sandy Gravelly Silt	1.5	_ N=16
Gravelly	3	N=14
Sandy Silty Clay	4.5	N=14
Weathered Rock	6	N=100
	7.5	N=100
	9	N=100
	10.5	N=100
	12	N=100
	13.5	N=100

Fig. 4. Bore log data collected from site-1



Fig. 5. Bore log data collected from site-4

	-	-		
Site		2	3	4
Grain size distri- bution (%)	Gravel	25	5	0
	Sand	26	58	100
	Silt and Clay	49	37	0
Unit weight (kN/m ³)	In-situ bulk unit weight	18.72	17.34	15.16
	In-situ dry unit weight	14.86	13.65	14.65
Specific gravity		2.61	2.61	2.61
Shear strength	Cohesion (kPa)	10	32	9
	Angle of internal friction (degree)	34	30	34

Table 3. Laboratory test results on the soil samples collected from site 2, 3 and 4

4 **Results and discussions**

The variation of bearing pressure with width of the strip footing for sites 1, 2, 3 and 4 are depicted in Figure 6 to Figure 9.

From the plots, it can be seen that as per Teng's correlation, bearing pressure decreases rapidly with increase in width of footing. It over predicts the bearing pressure values for width less than 2.5 m and under estimates for width of footing greater than 2.5 m. This is due to the reason that in Teng's expression, bearing pressure is inversely proportional to $4B^2$. Bowles recommendation to increase bearing pressure obtained by Meyerhof's expression by 1.5 times is on higher side of numerical model results.



Fig. 6. Variation of bearing pressure for site-1

Fig. 7. Variation of bearing pressure for site-2



Fig. 8. Variation of bearing pressure for site-3 Fig. 9. Variation of bearing pressure for site-4

Meyerhof's expression for finding bearing pressure value predicts considerably well with results from numerical model provided SPT N value used in the empirical formula is obtained directly from the field (Figure 6 and Figure 9).

If only shear strength parameters are collected from field and SPT data are not available (site-2 and site-3), usually relation between SPT N value and angle of internal friction (Φ) is used to obtain SPT N value. By doing this, the effect of cohesion is completely neglected. Hence Meyerhof's expressions are slightly on the lower side than the numerical results (Figure 7 and 8). In numerical analysis when the value of cohesion is neglected, the results obtained from numerical analysis compares well with the results calculated using Meyehof's expression. Figure 9 reflects all the above observations as both shear strength parameters and SPT N values are obtained from the site. It can also be observed that the effect of cohesion becomes insignificant with increase in width of the footing.

5 Conclusions

- In this study, comparison of bearing pressure of strip footing obtained from empirical formula and numerical analyses have been carried out. Field investigation data obtained from four different sites have been used as input parameters for analytical and numerical analysis.
- Numerical models were validated using field studies and it was found that Mohr Coulomb model gave 18.09 % higher value of load than the field values for a given settlement as it does not consider the effect of stress on stiffness. Whereas Hardening Soil model takes into account the stress dependency of stiffness and hence predicted load 8.89 % lower than the field values which was comparatively close.
- It was observed that for strip footing, Meyerhof's expression for finding bearing pressure value predicts considerably well with results from numerical model

provided SPT N value used in the empirical formula is obtained directly from the field SPT tests.

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