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Prediction of Ultimate Bearing Capacity of Eccentrically Inclined Loaded Strip Footing Resting over Dense and Medium Dense Sand using Generalized Regression Neural Network

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Abstract. Sometimes in remote areas geotechnical testing becomes more complex which makes conventional methods more complicated and cumbersome. So to conserve more time and money, computer science has developed "Neural network technique" which is a mimic of biological neural network like ANN, GRNN, PNN etc. to attain results in less time with more accuracy. In this study, ultimate bearing capacity of eccentrically inclined loaded strip footing is predicted resting over dense and medium dense sand with the help of RF (Reduction factor) value. A model test results were utilized for modeling GRNN (Generalized regression neural network) model using DTREG software to predict this RF value using Embedment ratio, inclination ratio, eccentricity ratio as input parameters and RF as output parameter using Gaussian type activation function. The results of experimentally calculated RF on the same study is compared with GRNN results and found more convenient and reasonable in terms of error minimization and accuracy. Also ANN MATLAB results were also analyzed with the GRNN results in which no such variations were spotted.

Keywords: Ultimate bearing capacity, Reduction factor, Strip footing, GRNN

1 Introduction

The evaluation of bearing capacity is the major criteria in the construction of any infrastructures like buildings, dams, bridges etc. There are several empirical, semi empirical formulas or methods to estimate the ultimate bearing capacity of footing like Terzhaghi, Meyerhoffs, plate load method etc. But these are conventional methods which are both time consuming and less accurate and even sometimes the field methods are less applicable in remote area as well. These laboratorial and field methods are also limited to simpler problems and could not easily manipulate complexities which are generated mostly during performing these conventional methods. There are several geotechnical calculations which requires complexities to attain a final result like settlement, slope failure, estimation of bearing capacity etc. The estimation of bearing capacity of eccentrically inclined loaded strip footing by field methods are itself a challenging task. Therefore, the neural network technique is used to eliminate these complications. With the help of this technique a well-trained and tested software model can be prepared.

The neural network works on the learning of the experimental or theoretical data. It relates the data with the output in the form of "activation functions". It provides the approximate result or output as compared to the desirable output by minimizing errors through iterations. The objective of the current study is to develop a GRNN-General regression neural network prediction model using experimental datasets from laboratory model tests performed by Patra [1] over dense sand and medium dense sand. Three input parameters (Df/B, e/B, α/ϕ) are used to predict a single output in the form of reduction factor (RF). The results found by GRNN are then compared with the empirical as well as ANN results [2]. The software used to apply GRNN in the present study is DT-REG, the results of which are further compared with the results of ANN prediction.

2 Literature Review

2.1 Laboratory model test

Laboratory model test was conducted by Patra [1] to determine ultimate bearing capacity of shallow strip footing subjected to eccentrically inclined load resting over dense and medium dense sand. Reduction factor (RF) value which is defined as a ratio of ultimate bearing capacity considering eccentrically inclined load to the bearing capacity centrally loaded with no inclination. The poorly graded dense sand having coefficient of curvature (Cc) is 1.15, coefficient of uniformity (Cu) and effective size of 1.15, 1.45 and 0.325 mm, respectively was used in the investigation. The embedment ratio (Df/B), eccentricity ratio (e/B) and inclination ratio were varied from 0 to 1, 0 to 0.15 and 0 to 20^{o}, respectively. Empirical equations were also used to calculate the value of reduction factor and treated as calculated RF which was further compared with the experimental values of RF. The variation of around 15% or less was seen and in some cases deviation was about 30% or less. Experimental value of RF is given by:

$$RF=[q_{u(Df/B,e/B,\alpha/\phi)}] / [q_{u(Df/B,e/B=0,\alpha/\phi=0)}] \quad \ldots\ldots\ldots\ldots\ldots\ldots..1$$

2.2 ANN modeling

The experimental datasets were utilized for training and testing ANN. A total of 120 datasets were used for model preparation out of which 70% was utilized for training data and 30% was utilized as testing or validation data. Embedment ratio, Eccentricity ratio and inclination ratio were used as predictor variables and a single output as reduction factor (RF) which further utilized to get the bearing capacity. MATLAB software was used for ANN modeling, the training function utilised in this model building was TRAINLM, adaptation learning function was LEARNGDM and the performance function was MSE. The number of hidden layers used in the model was one. Fig. 1 shows the connection strength of several inputs in neural network diagram.

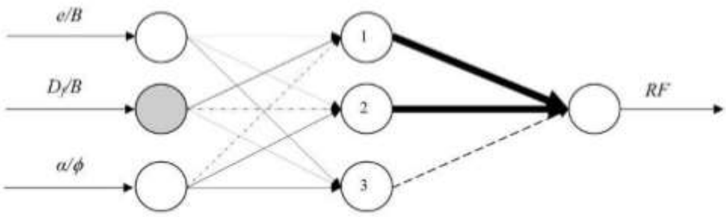

Fig.1 Neural network diagram showing connection strength of several inputs [2]

3 Methodology

Probability density function used in GRNN is normal distribution function with each training sample. In GRNN modelling network, the output is calculated on the basis of weight adjustment mechanism with the help of "Euclidean distance" which is approximately the square of the difference between the training data sample and the testing data sample. If the Euclidean distance of a certain variable is large then it means that the weight will be less and connection strength will be less for that variable. But if the Euclidean distance is small for certain variable it will have large amount of weight and connection strength. The equation used in GRNN is:

$$Y(x) = \frac{\Sigma Y_i \, e^{-(\frac{d_i^2}{2\sigma^2})}}{\Sigma \, e^{-(\frac{d_i^2}{2\sigma^2})}} \quad \text{……………………………..2}$$

Input sample is denoted as "X" and input sample in training as "Xi". Yi is the output sample regarding input sample of Xi. Euclidean distance is denoted as di^2 which is the distance between X and Xi. The activation function which actually denotes the weight of that input sample. is given by $e^{-(\frac{d_i^2}{2\sigma^2})}$.

The activation function utilised is the Gaussian type which comes under the type of radial basis function. The normal distribution are widely described by Gaussian function. It is the best kernel function whose equation is given by:

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \quad \text{……………………………………3}$$

3.1 Architecture of GRNN model

The network model is divided into four layers starting from input layer and ends at output layer.
INPUT LAYER: Each input layer is provided with one neuron in the input layer. The range of input neurons are standardized by the subtraction of median and division of

interquartile range. At the end of layer, the value of each neuron is then provided to the next hidden layer neurons for further processing.

HIDDEN LAYER: The hidden layer neurons are fed values by the input layer neurons. Then comes the hidden layer which is provided with a single neuron for each case in the training data set. The values of predictor and target variables for the similar case are stored by the neuron. The input values are presented over the X- axis, the distance from the neurons centre point known as Euclidean distance and is computed by the hidden layer after which using sigma values radial basis activation function is applied. This layer is mainly provided to compute the Euclidean distance which helps in adjusting the weightage of certain predictor variables and in the application of suitable activation function. The output of the hidden layer is then fed to the next layer known as Pattern layer.

PATTERN LAYER OR SUMMATION LAYER: This layer takes values from hidden layer as input. It contains only two neurons one, is Numerator neuron and the other is Denominator neuron. The value of denominator is computed by the summation of all values of activation function.The value for numerator neuron is computed by summation of multiplicative values of activation function and output data set values. The output values of both numerator and denominator are fed to the next layer known as decision layer.

OUTPUT OR DECISION LAYER: This layer contains only a single neuron. This layer ultimately predicts the target variable by simply computing its value from division of Numerator neuron and Denominator neuron values which are fed from the Pattern layer. Fig. 2 shows the GRNN architecture model used.

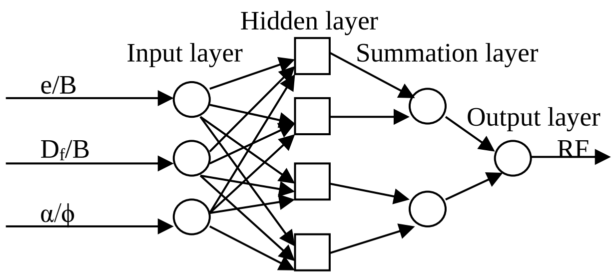

Fig.2 GRNN architecture model

3.2 Training principle

The primary work in training with generalised regression neural network technique is to select optimum value of sigma (σ) which helps to control the spread of radial basis function (RBF). The conjugate gradient algorithm is used by the DTREG software for computation of optimum sigma values. Separate sigma (σ) values for each predictor variable are used. The software uses leave one out method for evaluation of σ values during optimization. In this method measurement of error of model is done with the removal of each neuron, the neuron which shows the least error increase will then be removed and the process is repeated until the stopping criteria is achieved.

4 Database and Preprocessing

The laboratory model test conducted by Patra [1] over shallow strip footing subjected to eccentrically inclined load resting over dense and medium dense sand were used. Data sets are divided into two categories training and testing, around 70% data sets are used as training data whereas 30% data sets are utilised as testing data. Total 120 data sets are available, so first 90 sets are taken as training sets whereas last 30 sets are used as testing sets.

Table 1. Soil parameters and its characteristics [1]

Sand type	Unit weight of compaction (Kg/m^3)	Relative density of sand (%)	Friction angle (ϕ) degree	D_f/B	e/B	Load inclination (α) degree
Dense	14.36	69	40.8	0 0.5 1.0	0 0.05 0.1 0.15	0 5 10 15 20
Medium dense	13.97	51	37.5	0 0.5 1.0	0 0.05 0.1 0.15	0 5 10 15 20

Table 1 shows the soil charactertics and parameter used by Patra [1] in the investigation. Eccentricity ratio was varied from 0 to 0.15, inclination ratio was varied from 0 to 20 degree and embedment ratio was varied from 0 to 1 for dense and medium dense type sand. Experimental datasets used to model the training and testing process given in Table 2.

Table 2. Experimental model datasets [1]

Data Type	Expt. No.	e/B	Df/B	α/ϕ	Experimental q_u (kN/m^2)	Experimental RF	Calculated RF
Training	1	0.05	0	0	133.42	0.800	0.900
	2	0.1	0	0	109.87	0.659	0.800
	3	0.15	0	0	86.33	0.518	0.700
	4	0	0	0.123	128.51	0.771	0.770
	5	0.05	0	0.123	103.01	0.618	0.693
	6	0.1	0	0.123	86.33	0.518	0.616
	7	0	0	0.245	96.14	0.576	0.570
	8	0.05	0	0.245	76.52	0.459	0.513
	9	0.15	0	0.245	51.99	0.312	0.399
	10	0	0	0.368	66.71	0.400	0.400
	11	0.1	0	0.368	44.15	0.265	0.320
	12	0.15	0	0.368	35.12	0.211	0.280
	13	0.05	0	0.49	34.83	0.209	0.234
	14	0.1	0	0.49	29.43	0.176	0.208
	15	0.15	0	0.49	23.54	0.141	0.182
	16	0	0.5	0	264.87	1.000	1.000
	17	0.05	0.5	0	226.61	0.856	0.900
	18	0.1	0.5	0	195.22	0.737	0.800
	19	0	0.5	0.123	223.67	0.844	0.822
	20	0.05	0.5	0.123	193.26	0.730	0.740
	21	0.15	0.5	0.123	140.28	0.530	0.575
	22	0	0.5	0.245	186.39	0.704	0.656
	23	0.1	0.5	0.245	137.34	0.519	0.525
	24	0.15	0.5	0.245	116.74	0.441	0.459
	25	0.05	0.5	0.368	129.49	0.489	0.453
	26	0.1	0.5	0.368	111.83	0.422	0.402
	27	0.15	0.5	0.368	94.18	0.356	0.352
	28	0	0.5	0.49	115.76	0.437	0.364
	29	0.05	0.5	0.49	98.10	0.370	0.328
	30	0.15	0.5	0.49	72.59	0.274	0.255
	31	0	1	0	353.16	1.000	1.000
	32	0.1	1	0	278.60	0.789	0.800
	33	0.15	1	0	245.25	0.694	0.700
	34	0.05	1	0.123	277.62	0.786	0.790
	35	0.1	1	0.123	241.33	0.683	0.702
	36	0.15	1	0.123	215.82	0.611	0.614
	37	0	1	0.245	264.87	0.750	0.755
	38	0.05	1	0.245	239.36	0.678	0.679
	39	0.1	1	0.245	212.88	0.603	0.604
	40	0	1	0.368	225.63	0.639	0.632
	41	0.1	1	0.368	179.52	0.508	0.506
	42	0.15	1	0.368	155.98	0.442	0.443
	43	0.05	1	0.49	166.77	0.472	0.459
	44	0.1	1	0.49	143.23	0.406	0.408
	45	0.15	1	0.49	126.55	0.358	0.357
	46	0	0	0	101.04	1.000	1.000
	47	0.05	0	0	84.37	0.835	0.900

48	0.15	0	0	54.94	0.544	0.700
49	0	0	0.133	79.46	0.786	0.751
50	0.1	0	0.133	52.97	0.524	0.601
51	0.15	0	0.133	42.18	0.417	0.526
52	0.05	0	0.267	47.09	0.466	0.484
53	0.1	0	0.267	38.46	0.381	0.430
54	0.15	0	0.267	31.39	0.311	0.376
55	0	0	0.4	38.26	0.379	0.360
56	0.05	0	0.4	32.37	0.320	0.324
57	0.1	0	0.4	26.98	0.267	0.288
58	0	0	0.533	24.03	0.238	0.218
59	0.05	0	0.533	19.62	0.194	0.196
60	0.15	0	0.533	13.34	0.132	0.152
61	0	0.5	0	143.23	1.000	1.000
62	0.1	0.5	0	103.99	0.726	0.800
63	0.15	0.5	0	87.31	0.610	0.700
64	0.05	0.5	0.133	103.99	0.726	0.726
65	0.1	0.5	0.133	90.25	0.630	0.645
66	0.15	0.5	0.133	72.59	0.507	0.565
67	0	0.5	0.267	98.10	0.685	0.628
68	0.05	0.5	0.267	84.86	0.592	0.565
69	0.1	0.5	0.267	72.59	0.507	0.502
70	0	0.5	0.4	79.46	0.555	0.465
71	0.05	0.5	0.4	67.89	0.474	0.418
72	0.15	0.5	0.4	48.07	0.336	0.325
73	0	0.5	0.533	58.27	0.407	0.319
74	0.1	0.5	0.533	43.16	0.301	0.255
75	0.15	0.5	0.533	36.30	0.253	0.223
76	0.05	1	0	193.26	0.925	0.900
77	0.1	1	0	175.60	0.840	0.800
78	0.15	1	0	156.96	0.751	0.700
79	0	1	0.133	186.39	0.892	0.867
80	0.05	1	0.133	168.73	0.808	0.780
81	0.1	1	0.133	153.04	0.732	0.693
82	0	1	0.267	160.88	0.770	0.733
83	0.05	1	0.267	144.21	0.690	0.660
84	0.15	1	0.267	112.82	0.540	0.513
85	0	1	0.4	133.42	0.638	0.600
86	0.1	1	0.4	106.93	0.512	0.480
87	0.15	1	0.4	94.18	0.451	0.420
88	0.05	1	0.533	92.21	0.441	0.420
89	0.1	1	0.533	84.37	0.404	0.373
90	0.15	1	0.533	75.54	0.362	0.327

Testing	1	0	0	0	166.77	1.0	1.000
	2	0.15	0	0.123	65.73	0.394	0.539
	3	0.1	0	0.245	62.78	0.376	0.456
	4	0.05	0	0.368	53.96	0.324	0.360
	5	0	0	0.490	43.16	0.259	0.260
	6	0.15	0.5	0	164.81	0.622	0.700
	7	0.1	0.5	0.123	165.79	0.626	0.658
	8	0.05	0.5	0.245	160.88	0.607	0.590
	9	0	0.5	0.368	151.07	0.570	0.503
	10	0.1	0.5	0.490	85.35	0.322	0.291
	11	0.05	1	0	313.92	0.889	0.900
	12	0	1	0.123	313.92	0.889	0.877
	13	0.15	1	0.245	188.35	0.533	0.528
	14	0.05	1	0.368	206.01	0.583	0.569
	15	0	1	0.49	183.45	0.519	0.510
	16	0.1	0	0	68.67	0.680	0.800
	17	0.05	0	0.133	63.77	0.631	0.676
	18	0	0	0.267	55.92	0.553	0.538
	19	0.15	0	0.4	20.60	0.204	0.252
	20	0.1	0	0.533	16.68	0.165	0.174
	21	0.05	0.5	0	123.61	0.863	0.900
	22	0	0.5	0.133	120.66	0.842	0.807
	23	0.15	0.5	0.267	60.82	0.425	0.440
	24	0.1	0.5	0.4	56.90	0.397	0.372
	25	0.05	0.5	0.533	50.03	0.349	0.287
	26	0	1	0	208.95	1.000	1.000
	27	0.15	1	0.133	137.34	0.657	0.607
	28	0.1	1	0.267	129.49	0.620	0.587
	29	0.05	1	0.4	118.70	0.568	0.540
	30	0	1	0.533	98.10	0.469	0.467

In this study, trial version of DT-REG software was used which helps in providing results but unable to generate an equation for required output. After the selection of type model to build, values of sigma are selected or decided for the model. In this model preparation, sigma for each variable is used. The minimum and maximum sigma value is kept as 0.0001 and 10, respectively and 20 search steps are set for the model. Leave one out method is used in model testing and validation. The target variable is the reduction factor and three input variables are used for prediction. The validation method applied in the modelling is "Leave one out method" in which cross validation is performed by leaving one row out for each model built.

5 Results and Discussions

Best performance analysis is done by using correlation coefficient (Cr), determination coefficient (R^2), mean square error (MSE), root mean square error (RMSE) and mean absolute error (MAE).

Table 3. Experimental model datasets [1]

Parameters	Training	Testing
Mean target value for input data	0.652293	0.652293
Mean target value for predicted values	0.6530206	0.6544816
Variance in input data	0.0393364	0.0393364
Residual (unexplained) variance after model fit	0.000158	0.0005374
Proportion of variance explained by model (R^2)	99.598 %	98.634%
Coefficient of variation (C_v)	0.019267	0.035538
Normalised mean square error (NMSE)	0.004015	0.013661
Correlation between actual and predicted	0.998156	0.994194
Maximum error	0.492199	0.089428
Root mean square error (RMSE)	0.012568	0.0231815
Mean squared error (MSE)	0.000158	0.0005374
Mean absolute error (MAE)	0.0090726	0.0177511
Mean absolute percentage error (MAPE)	0.0179782	0.0323818

Table 3 reveals the training and testing parameters of experimental model test conducted by Patra [1]. 70% data sets are used as training data and a GRNN model is built. 30% datasets are used in the testing data. The coefficient of correlation for training and testing data was 0.998 and 0.994, respectively. It shows the linearity between value predicted and the actual output with greater precision. More close the value to 1 shows higher linearity. The values of statistical parameters for GNN for all continuous variables are shown in Table 4. As RF is the target variable and the other three are predictor variables. Maximum value for RF was 1 whereas its minimum value was 0.132 and mean value was 0.65229. The value of standard deviation was 0. 19833.The statistical parameters from ANN are shown in Table 5. In the statistical analysis, maximum value of output was 1 and its minimum value was 0.132. Whereas as mean or average value was set as 0.555. The standard deviation of the output variable was 0.217.

Table 4. Statistical parameters from GRNN

Variable	Rows	Minimum	Maximum	Mean	Standard deviation
Df/B	120	0	1	0.66956	0.37450
A	120	0	20	7.98676	6.80215
e/B	120	0	0.15	0.06603	0.05531
RF	120	0.132	1	0.65229	0.19833

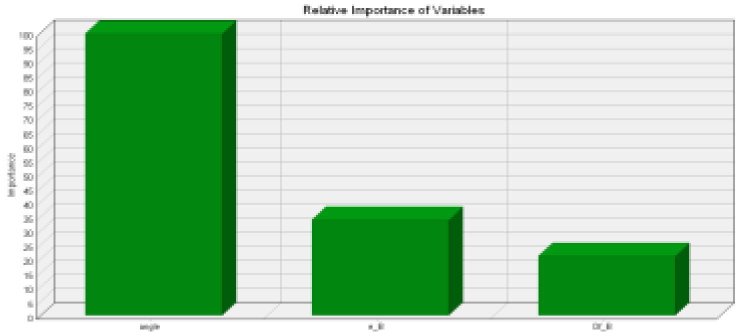

Fig. 2. Relative importance of input varibles on output variable RF

Fig. 2 shows the relative importance of variable in the prediction model. It can be easily seen in Fig. 2 that the inclination variable shows 100% importance or impact over the target variable while eccentricity ratio variable shows 33.946% impact and embedment ratio make least impact or least important as compared to other two variables with around 22.112% importance over the target variable. Fig. 3 and Fig. 4 show the variation of experimental RF (Actual) vs predicted RF value from GRNN and experimental RF vs empirically calculated RF(CRF), respectively. Higher variation can be seen, but still the graph proceeds in the linear direction but less linearity is shown as compared to the GRNN prediction model graph which shown in Fig. 3.

Table 5. Statistical parameters from ANN [2]

Parameter	Maximum	Minimum	Average	Standard deviation
e/B	0.15	0	0.075	0.056
Df/b	1	0	0.5	0.408
α/ϕ	0.533	0	0.256	0.181
RF	1	0.132	0.555	0.217

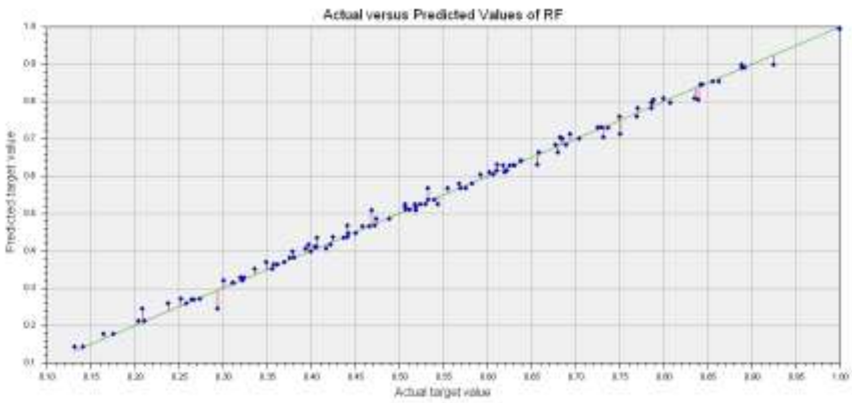

Fig. 3. Plot Between Experimental RF (Actual) and predicted RF value from GRNN

Fig. 5 shows variation of actual target variable and AN N predicted output variable for training and testing data. It can be seen from the Fig. 5 that the model built was providing good results, which was analyzed with the help of coefficient of correlation. The value of Cr for training was 0.997 which is much closer to the value 1 whereas Cr value for testing was 0.996 which represents good prediction according to this model.

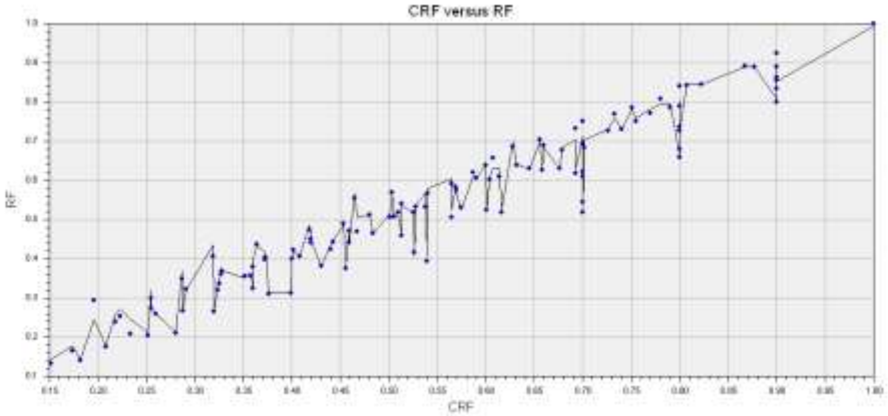

Fig. 4. Plot between Experimental RF and Empirically calculated RF(CRF)

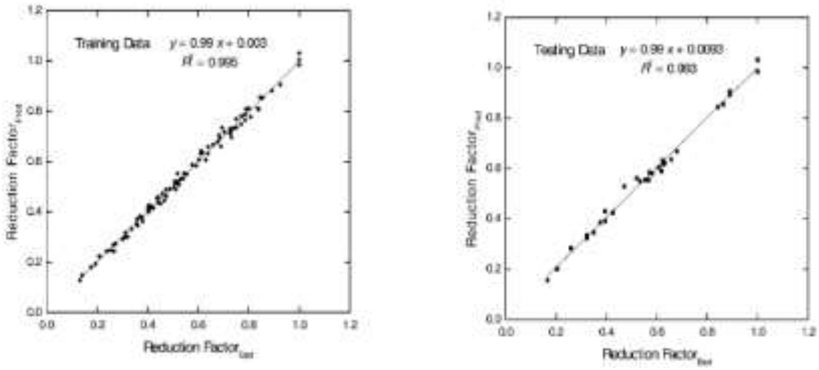

Fig. 5. Plot of experimental RF and predicted RF from ANN [2]

Table 6. Comparison between ANN and GRNN with Mathematical indices

TYPE OF MODEL	GRNN (present study)		ANN (results from [2])	
	TESTING	TRAINING	TESTING	TRAINING
MSE	0.0005	0.00015	0.0019	0.001
RMSE	0.0231	0.0125	0.043	0.032

R^2	0.986	0.996	0.992	0.994
Cr	0.994	0.998	0.996	0.997

The comparison between ANN and GNN with different indices is shown in Table 6. No such variation in results was seen in between GRNN network model and ANN model. The mean square error was less in GRNN prediction work. There was slight difference between the correlation coefficient like for training work was 0.998 for GRNN model whereas 0.997 for ANN model and fro testing it was 0.994 for GRNN and 0.996 for ANN. Results of both GRNN and ANN shows higher accuracy than empirically calculated results also shown in Figs.3-5.

6 Conclusions

Highlights of the present study are shown below:

1. No such variations in results between ANN and GRNN was spotted, both models were equally accurate.
2. The data available was never enough for back propagation neural network., this GRNN neural network technique founds to be advantageous because of the ability of this technique in utilizing fewer data samples efficiently to converge the function.
3. The standard deviation found in output reduction factor (RF) was lesser in GRNN model as compared to ANN model.
4. In the GRNN model inclination ratio was provided higher importance as compared to other two input variables like embedment ratio and eccentricity ratio (α/ϕ as 100%, e/B as 33.946% and Df/B as 21.112%) whereas in the ANN network model as per Garson's algorithm inclination ratio (α/ϕ) was given more importance as compared to other two followed by embedment ratio (Df/B) and then eccentricity ratio (e/B).

References

1. C. Patra, R. Behara, N. Sivakugan & B. Das (2012) Ultimate bearing capacity of shallow strip foundation under eccentrically inclined load, Part I, International Journal of Geotechnical Engineering, 6:3, 343-352, DOI: 10.3328/IJGE.2012.06.03.343-352
2. R. N. Behera1, C. R. Patra1, N. Sivakugan and B. M. Das (2012). Prediction of ultimate bearing capacity of eccentrically inclined strip footing resting over dense sand, part-1. International Journal of Geotechnical Engineering. 7(1), 36-34.
3. Terzaghi, (1943). Theoretical Soil Mechanics, Wiley, New York.
4. GG Meyerhof (1953). The bearing capacity of foundations under eccentric and inclined loads. Proceedings. III International Conference on Soil Mechanics and Foundation Engineering. Vol. 1, 440-445.