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Chronological and Mathematical Development of True Triaxial Failure Criteria of Brittle Rocks

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Abstract. Earlier researchers such as von Kármán, Mohr, Coulomb and Griffith did not con- template the strengthening of materials due to intermediate principal stress σ2. Böker and Handin *et al.* showed the strengthening effect of σ2 in marble, and in limestone, dolomite and glass, respectively. Murrell extended the Griffith's criteria to its three-dimensional form residing in octahedral plane, and even included the uniaxial tensile strength of material. Afterwards, Nadai, Drucker and Prager, Bresler and Pister, and Wiebols and Cook and more included all the principal stresses in their respective criteria. In 1971, Mogi put the dubiety to rest by testing dolomite and trachyte in a high capacity true triaxial testing machine. Then, many researchers such as Lade and Duncan, Michelis, Takahashi and Koide, Smart, Haimson and Chang, Alexeev *et al.*, Tiwari and Rao, He *et al.* and several others developed their own true triaxial testing machines and conducted their own experiments. This paper discusses this chronological and mathematical de- velopment of brittle failure criteria which describes the true stress state existing at the depths. Simultaneously comparative assessment is made to segregate the conditional nature of the failure criteria. These developments are pertinent to know how the underground brittle failure such as strainburst is perceived today.

Keywords: brittle; fracture; true triaxial; principal stress; testing machine; failure.

1 Introduction

Studies pertaining to stresses around underground spaces and failure behaviour of rock mass is crucial to allow deformations in excavation stage and restrict the same after the construction of the structure is completed and commissioned. Hence there is a quintessential need of a functioning failure criterion. Geomaterials behave either in a brittle or ductile fashion when stressed under an external load. Based on this behaviour failure can be classified as either fracture (brittle material) or yielding (ductile material) behaviour. These failure criteria are in terms of stress or strain experienced by a de- forming body. In stress space, generally it is denoted that the major principal stress σ_1 is an implicit function of other two principal stresses, intermediate and minor principal stresses, σ_2 and σ_3 respectively, i.e. $\sigma_1 = f_1(\sigma_2, \sigma_3)$ or $f_2(\sigma_1, \sigma_2, \sigma_3) = 0$, which are determined either theoretically, empirically, numerically or using tools such as fuzzy logic or machine learning. In general, rocks are tested for triaxial stress state, which is a special case wherein $\sigma_2 = \sigma_3$. However, it has been established that it is not always the case in any underground excavation. σ_2 presents a strengthening effect on the overall stress state in the underground space [1–6].

In geomechanics, positive convention is used for compression, hence negative convention is reserved for tensile forces, however in solid mechanics the notion is opposite. Hence the material under discussion has to be noted to avoid conflict of understanding. Usually, the failure is observed in rock mechanics when the specimen under compression fails either in tension or shear. This failure can be either microscopic, which deals with crack mechanics, and other, macroscopic failure, which deals with load or energy carrying capacity of the structure in a whole. The microscopic failure criteria are developed from the micromechanical failure models, which considers the initiation of cracks and its coalescence to form major cracks, or from continuum damage mechanics models. The macroscopic failure criteria are based on the stress-strain response of the material, energy exchange (via, S-criterion, etc.), or empirical models and damage characteristics models.

The failure of brittle materials (in solid mechanics) is established as per either the phenomenological failure criteria, linear elastic fracture, elasto-plastic fracture, energy-based criteria, or cohesive zone models to model the non-linear expansion at the crack tip. Amongst the phenomenological failure criteria, some are applicable to brittle materials and some to ductile materials. Within them the criteria applicable to brittle fracture are maximum stress or strain criteria, Mohr-Coulomb criteria, Bresler-Pister failure criteria, William-Warnke failure criteria, Johnson-Holmquist damage models (for high strain rate) and Hoek-Brown criteria.

Hoek and Bieniawski (1965) [6] proved that the crack length during the triaxial test on brittle rocks depends on the confinement with extension approaching less than 10% of the crack length observed for confinement exceeding 10% of σ_1. The authors showed that Griffith's criterion was applicable for tensile and very low confinement conditions (with $\sigma_3 < 10\%$ or $\sigma_c/10$). However, the authors also stated that for σ_3 exceeding $\sigma_c/10$, the modified Griffith criterion with frictional coefficients between 0.75 and 1.5 was observed to be shear failure. Similarly, Diederichs (2003) [7] that extensile crack propagation was observed for low confinement, which ultimately led to brittle and spalling failure, or a combination of spalling and local shear failure. At high confinement such easy coalescence of cracks and shear or kink-band did not occur, which led to shear failure. Although Hoek – Brown criterion considers the effect of confinement, it does not account for change in the physical process. Also, the Coulomb criterion which considers the mobilization of cohesion and friction at failure, but for cohesive soils, and not for brittle rocks wherein it observed that cohesion is lost completely before the frictional strength at failure can be mobilized, as observed by Martin (1997) [8].

2 Review of existing stress-based criteria

2.1 Coulomb Criterion

As early as in 1773 Charles-Augustin de Coulomb while researching on the effect of lateral earth pressure acting on the retaining walls to improve the shear strength of the soil, devised the earliest form of failure theory. Lamé proposed that the material undergoes brittle failure when the maximum principal stress σ_1 reaches the material strength σ_c (maximum principal stress theory), which is depicted as

$$\sigma_1 \leq \sigma_c \tag{1}$$

Coulomb observed that the shear strength of soil (in soil mechanics) was dependent on the parameters such as internal friction angle (φ) and cohesion (c) with the former dependent on stress imposed on the soil. Then Coulomb plotted shear stresses on y-axis against normal stresses applied on the soil mold on x-axis and found a linear correlation as follows with the failure observed when the shear stress exceeds the shear strength of the soil mass.

$$r = c + \sigma_n \tan \tag{2}$$

2.2 Mohr-Coulomb Criterion

As early as 1900, Otto Mohr theorized that the shear strength of a material on plane stress is a function of the normal stress applied on it as $\tau = f(\sigma_n)$. Mohr devised the Mohr failure envelope as a line drawn tangent to the plot of circles at different combinations of major and minor principal stresses (σ_1 and σ_3) on a plot of τ (on y-axis) vs σ_n (on x-axis). Therefore, the Mohr failure envelope suggests that the shear strength at failure is a dependent on the normal stress developed across the failure plane. Additionally, it can be observed that the Mohr-Coulomb criteria is a convolution of both Mohr's theory and Coulomb's criteria. However, no tension cut-off was proposed. Mohr's circle can also be used to find the principal stresses for a stress element as expressed below.

$$\sigma_1 = \sigma_c + \emptyset\sigma_3 \tag{3}$$

or,

$$N\sigma_1 - (N+1)\sigma_3 = V_0 \tag{4}$$

Where $N = (1\text{-}\sin\varphi)/(2\sin\varphi)$, V_0 is the theoretical isotropic tensile strength, which can be computed from the equation as $V_0 \sin\varphi = S_0 \cos\varphi$ and $\sigma_c = 2S_0 \cos\varphi/(1\text{-}\sin\varphi)$ is the uniaxial compressive strength of the material. The Mohr's criterion considers torsion (displays helicoidal failure) along with uniaxial compressive strength and uniaxial tensile strength, which results into a non-linear envelope drawn on Mohr's circles. Since the Rankine's theory overestimated the ultimate strength, Mohr-Coulomb theory was more utilized. The linearized envelope (Mohr-Coulomb theory) considers just the uniaxial compression and uniaxial tension. For brittle materials, the critical principal stress points were computed as follows, where n is the design safety factor.

$$\sigma_1 = \frac{\sigma_{ut}}{n}; \frac{\sigma_1}{\sigma_{ut}} - \frac{\sigma_3}{\sigma_{uc}} = \frac{1}{n}; \sigma_3 = -\frac{\sigma_{uc}}{n} \tag{5}$$

But, since the Mohr-Coulomb theory is conservative in the fourth quadrant, Modified Mohr criteria was deemed to be more suitable to study the brittle materials as it accounts for the nonlinear brittle failure in the fourth quadrant. The Modified Mohr criteria is computed as

$$\sigma_1 = \frac{\sigma_{ut}}{n}; \frac{(\sigma_{uc} - \sigma_{ut})\sigma_1}{\sigma_{uc}\sigma_{ut}} - \frac{\sigma_3}{\sigma_{uc}} = \frac{1}{n}; \sigma_3 = -\frac{\sigma_{uc}}{n} \tag{6}$$

2.3 Griffith's Criterion

The linear elastic fracture mechanics assesses the unstable crack growth as the stresses induces mode I opening up the crack in the plane of stress (Griffith (1921; 1924) [9, 10]. Griffith's theory as per the law of thermodynamics evaluates the critical stress σ or strain energy consumed to propagate these microcracks during the uniaxial tensile strength (in solid mechanics) as

$$\sigma = \sqrt{\frac{2E\gamma}{\pi a}} \tag{7}$$

Where E is the Young's modulus of the material, γ is the surficial energy consumed per unit area of the crack, and $2a$ is the crack length for 2D cracks. Griffith hypothesized that in expense of some energy called the dissipation energy a new crack is developed. The mode I fracture toughness in plane strain condition is

$$K_{Ic} = Y\sigma_c\sqrt{\pi a} \tag{8}$$

Where σ_c is the far-fields critical stress and Y is the dimensionless parameter dependent on geometry, material properties and loading conditions. If stress intensity factor determined experimentally, is greater than the fracture toughness of the material, crack propagation and localized coalescence is observed. However, this method presents a challenge to compute the fracture stress in a complex geometry or loading condition. Alternatively, in a plane strain condition, the strain energy release rate criterion is more pertinent to brittle failure. It is defined for a mode I crack through the thickness of the plate as

$$G_I = \frac{P}{2t}\frac{du}{da} \tag{9}$$

Where P is the force applied, through the t thickness of the plate which is deformed by a displacement u at the point of application of the force leading to crack growth, and a is the crack length at the edge or $2a$ is the plane crack length. It is assumed that when the strain energy release rate exceeds the critical value of G_I, namely G_{Ic}, i.e., the critical strain energy release rate, the crack propagation takes place. For a plane stress condition, the fracture toughness K_{Ic} and critical strain energy release rate G_{Ic} are associated as

$$G_{Ic} = \frac{1}{E}K_{Ic}^2 \tag{10}$$

Hence, Griffith's criterion is applicable in tensile and tensile-shear failure. Griffith's criterion suggests that the crack extension in the plane of compression is influenced by the principal stresses and tensile strength of the material undergoing cracking as follows.

$$\begin{cases} (\sigma_1 - \sigma_3)^2 = 8T_0(\sigma_1 + \sigma_3); if\ \sigma_1 + 3\sigma_3 > 0 \\ \sigma_3 = -T_0;\ if\ \sigma_1 + 3\sigma_3 \leq 0 \end{cases} \tag{11}$$

Where T_0 is the uniaxial tensile strength of the uncracked section of the thin plate. In the above stated equation, it is assumed that the compressive stress is positive. Further to refine the above criterion, Griffith (1924) developed a theoretical model of a randomly oriented elliptical crack in a plane of stress in an elastic homogenous material. Griffith observed high concentration of tensile stresses at the edge of the elliptical cracks (wing crack model) under the application of the compressive stresses. It was observed that the fracture initiates from the edge defect when the localized tensile strength of the uncrack section of the material is exceeded. Hence, the theory of crack initiation, propagation and coalescence of the microscopic fractures created due to high stress concentration at the sharp tip of the crack was recommended. Further, when the critically oriented fractures are propagated through the material and the ultimate stress is reached, failure of the material as whole occurs.

Walsh and Brace (1964) [11] adjoined the shear mechanism at the tip of the crack and friction between the faces of the crack, and hence modified Griffith criterion (1962)

[12] was born. According to this criterion, the authors considered that crack closure may appear as the compressive forces are applied on the material, which can cause the shear failure to occur before the tensile stress reaches its ultimate value at the tip of the crack. To account for the shear fracture, the authors also proved that the aforementioned friction influences the shear fracture. Hence, the modified Griffith criterion can be applied to anisotropic rocks. The modified Griffith criterion is expressed as below.

$$\sigma_1\left[\sqrt{\mu^2+1}-\mu\right]-\sigma_3\left[\sqrt{\mu^2+1}+\mu\right]=4\sigma_t \qquad (12)$$

$$\begin{cases} r^2+4\sigma_t\sigma_n-4\sigma_t^2=0; & if\ \sigma_n<0 \\ r=2\sigma_t+\mu\sigma_n; & if\ \sigma_n>0 \end{cases} \qquad (13)$$

Where μ is the coefficient of internal friction, σ_t is the tensile strength of the material, σ_n is the normal stress and τ is the shear stress developed at the crack closure faces.

Murrel (1964) [13] extended the Griffith criterion further by considering all the three principal stresses, σ_1, σ_2 and σ_3. The extended Griffith criterion in the π-plane has a circular surface and denoted as follows.

$$(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2=24T_0(\sigma_1+\sigma_2+\sigma_3) \qquad (14)$$

2.4 Nadai Criterion

Nadai (1950) [14] suggested a general 3D failure criterion that suggests that the material failure occurs when octahedral shear stress τ_{oct} has reached its critical value. However, brittle fracture does not follow it accurately since the strength varies incredibly with varying confinement. This octahedral shear stress and octahedral effective normal stress is calculated as.

$$r_{oct}=\frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2} \qquad (15)$$

$$\sigma_{oct}=\frac{(\sigma_1+\sigma_2+\sigma_3)}{3} \qquad (16)$$

2.5 Wiebols and Cook Criterion

Experimental investigations of the polyaxial test data did not agree with the circular surface of the failure criteria in the π-plane. Hence, researchers such as Wiebols and Cook (1968) [15] augmented stress invariant J_3 into the ever evolving failure criteria. Since $|\tau|-\mu\sigma_n>0$ when shear sliding occurs at the surfaces of the crack as mentioned by modified Griffith's criterion, this quantity is called the effective shear stress, and the strain energy per unit volume of material stored at the cracking surface is called effective shear strain energy. The aforementioned energy is dependent on the magnitude of the principal stresses and the orientation of the crack axis with the principal stress direction. The normal and shear stresses on such cracks can be computed as.

$$\begin{cases} \sigma_n=l^2\sigma_1+m^2\sigma_2+n^2\sigma_3 \\ r^2=l^2\sigma_1^2+m^2\sigma_2^2+n^2\sigma_3^2-\sigma_n^2 \end{cases} \qquad (17)$$

For the uniaxial stress condition, the normal and shear stresses reduce to

$$\sigma_n=\frac{\sigma_1}{2}(1+cos2\theta) \qquad (18)$$

$$|r|=\frac{\sigma_1}{2}sin2\theta \qquad (19)$$

Thus, for a uniaxial stress condition, the effective strain energy can be computed as

$$W_{eff}=\frac{N\pi\sigma_1^2}{2}\left[\frac{7\mu^{6\mu}+15\mu^4+9\mu^2+1}{30(\mu^2+1)^{\frac{5}{2}}}+\frac{5\mu^2+3}{6(\mu^2+1)^{\frac{1}{2}}}-\frac{16\mu}{15}\right] \qquad (20)$$

Whereas in biaxial stress condition ($\sigma_1 = \sigma_2$, $\sigma_3 = 0$), the normal and shear stress reduce to

$$\sigma_n = (l^2 + m^2)\sigma_1 \quad (21)$$

$$r^2 = (l^2 + m^2)\sigma_1^2 - (l^2 + m^2)^2\sigma_1^2 \quad (22)$$

Thus for a biaxial stress condition, the effective shear strain energy can be computed as

$$W_{eff} = \frac{N\pi\sigma_1^2}{2}\left[\frac{\mu^7 + 9\mu^5 + 15\mu^3 + 7\mu}{30(\mu^2+1)^{\frac{5}{2}}} - \frac{2\mu^5 + 6\mu^3 + 4\mu}{3(\mu^2+1)^{\frac{3}{2}}} - \frac{3\mu^2 + \mu}{2(\mu^2+1)^{\frac{1}{2}}} + \frac{32\mu^2 + 8}{15}\right] \quad (23)$$

Moreover, in polyaxial stress state, the effective shear stress energy can be computed as

$$W_{eff} = \sum \frac{2N\pi}{K}(|r| - \mu\sigma_n)^2; \quad where\ |r| > \mu\sigma_n \quad (24)$$

And the directions cosines for each relevant point in a unit octant are

$$l = cos\psi;\ m = sin\psi\ cos\lambda;\ n = sin\psi\ sin\lambda \quad (25)$$

Such that the longitude angle λ and polar angle ψ of each point are

$$\psi = \sqrt{\frac{\pi}{2K}}\left(\frac{1}{2} + a\right);\ a = 0, 1, 2, etc;\ \text{such that}\ a = \left(0, \sqrt{\frac{K\pi}{2}} - \frac{1}{2}\right] \quad (26)$$

$$\lambda = \frac{1}{\sin\psi}\sqrt{\frac{\pi}{2K}}\left(\frac{1}{2} + b\right);\ b = 0, 1, 2, etc;\ \text{such that}\ b = \left(0, \sqrt{\frac{K\pi}{2}}\ sin\psi - \frac{1}{2}\right] \quad (27)$$

A limitation of this criterion is the unavailability of any recognized laboratory method of determination of the coefficient of sliding friction on microcracks.

Colmenares and Zoback (2002) [16] proposed the Modified Wiebols and Cook criterion and defined the second invariant of deviatoric stress J_2 as.

$$J_2^{\frac{1}{2}} = A + BJ_1 + CJ_1^2 \quad (28)$$

Where, A, B, C are the material constants, that depend on the particular rock type and minimum principal stress. These constants can be determined from the conventional triaxial compression tests.

2.6 Mogi Criterion

Hobbs (1970) [17] proposed a simple power law for fracture and yield stress with varying B and b parameters for each rock types. In addition, a simple power law equation for shear stress as given below was suggested.

$$\sigma_1 = B\sigma_3^b + \sigma_3 \quad (29)$$

$$r = k\sigma^a;\ \text{where k and a are cconstants varying for each rock types} \quad (30)$$

Mogi (1971) [18] proposed a criterion which considers all the three principal stresses and based it on the octahedral shear stress and a mean stress, thus the Mogi criterion has a form $\tau_{oct} = f(\sigma_m, 2)$, where octahedral shear stress and mean normal stress have their usual meaning. The linearized Mogi criterion is called the Mogi-Coulomb criterion. Al-Ajmi and Zimmerman (2005) [19] based on the work of Mogi devised a linear relation between τ_{oct} and σ_m by curve fitting polyaxial data from eight different rocks in the corresponding space, which is written as.

$$r_{oct} = a + b\sigma_m \quad (31)$$

Where a parameter is the intersection of the fitting line on the τ_{oct} axis and the b parameter is the angle of inclination with the σ_m axis. This criterion was found to not only complement the acquired polyaxial data brilliantly but also the triaxial test data. Later of which reduces the linear Mogi-Coulomb criterion to Mohr-Coulomb criterion. As reported by Chang and Haimson (2012) [20], this criterion requires laboratory true triaxial test data and the parameters differs slightly for all rocks with varying σ_2 and σ_3.

Mogi (1971) also observed through tests on marble, dolomite that ductility (ε_n) increases with increasing σ_3 but decreases with increasing σ_2, which is expressed as.

$$\varepsilon_n = f_3(\sigma_3 - \alpha\sigma_2); \text{where } f_3 \text{ is a montonically increasing function} \quad (32)$$

From the above equation Mogi (1971) also understood that at deeper levels the sudden release of elastic strain energy is higher than calculated from conventional triaxial compression test since higher value of σ_2 can lead to decrease in ductility. Also, Mogi observed the striking of fractures in the 2D plane parallel to the intermediate principal stress direction. Hence Mogi updated the Nadai criterion by accounting for the average of major and minor principal stresses instead of octahedral normal stress. The Mogi power criterion can be rewritten as an empirical criterion as follows.

$$r_{oct} = A\sigma_{m,2}^{n} \quad (33)$$

where A and n are material constants.

2.7 Hoek-Brown Criterion

Hoek and Brown (1980) [21] on the basis of tests on intact specimens proposed a criterion as follows.

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \sqrt{m_b \frac{\sigma_3}{\sigma_c} + s} \quad (34)$$

Where s = 1.0 for intact rocks and m_b >> 1.0. However, in-situ conditions will defer widely from laboratory ideal conditions. Thus, Hoek and Brown (2002) [22] recommended a failure criterion for brittle rock mass as expressed here.

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^{a} \quad (35)$$

Pan and Hudson (1988) [23] expressed the 3D stress state of the Hoek and Brown criterion as.

$$\frac{9}{2\sigma_c} r_{oct}^2 + \frac{3}{2\sqrt{2}} m r_{oct} - m\sigma'_{m,3} = s\sigma_c \quad (36)$$

Whereas, Zhang and Zhu (2007) [24] proposed the 3D Hoek-Brown criteria as,

$$\frac{9}{2\sigma_c} r_{oct}^2 + \frac{3}{2\sqrt{2}} m r_{oct} - m\sigma'_{m,2} = s\sigma_c \quad (37)$$

2.8 Rao Criterion

Rao (1984) [25] and Rao et al. (1986) [26] proposed a nonlinear failure criterion for intact rocks that originated from Mohr-Coulomb criterion. It is expressed as follows.

$$\frac{\sigma_1 - \sigma_3}{\sigma_3} = B\left(\frac{\sigma_c}{\sigma_3}\right)^{\alpha} \quad (38)$$

Where, α is the slope of the plot between $\left(\frac{\sigma_1-\sigma_3}{\sigma_3}\right)$ and $\left(\frac{\sigma_c}{\sigma_3}\right)$ on the log-log scale, and B is a material constant which is sensitive to geological nature of rock, computed as $\left(\frac{\sigma_1-\sigma_3}{\sigma_3}\right)$ [27]. This equation works well for brittle rocks undergoing shear and shear-tension fracture and under low confinement. The theory was later on extended for strength prediction of anisotropic and weathered rockmass. However, the theory underestimates the potential strength under high confinement and different nature of fracture under high confinement and hard brittle rocks.

2.9 Bresler-Pister Criteria

Bresler-Pister (1985) [28] yield criterion is a failure criterion that was devised to predict the strength of concrete under multiaxial stress states. It is the extension of the Drucker-Prager [29] yield criterion that is expressed in stress invariants. Although the DP criterion is simple and smooth, it tends to overestimate rock strength for general stress states and is not accurate when one of the principle stresses is tensile. Whereas, the Bresler-Pister [28] yield criterion is expressed in terms of the stress invariants as.

$$\sqrt{J_2} = A + BI_1 + CI_1^2 \tag{39}$$

Where A, B and C are material constants. These parameters can be expressed in terms of yield stress in uniaxial compression, yield stress in uniaxial tension and yield stress in biaxial compression.

2.10 Johnson-Holmquist Model

In 1992 and 1994, the Johnson-Holmquist damage models were proposed, which are used to model mechanical behaviour of damaged brittle materials, such as rocks, under a great range of strain rate [30, 31]. These materials exhibit high compressive strength and low tensile strength. The first version of the model, called the 1992 Johnson-Holmquist 1 (JH-1) model, was developed to simulate high strain, but ignored the progressive damage with increasing deformation. The second version called the Johnson-Holmquist 2 (JH-2) model or Johnson-Holmquist damage material model accurately simulated the damage of brittle materials under ballistic impact, as well as the effect of the hydrostatic pressure and damage on the strength of such brittle materials.

2.11 Ottosen Criterion

The Ottosen (1977) yield criterion [32] which is a four parameter failure criterion applicable for a short duration loading of concrete. It is generally expressed as.

$$F_y = \frac{a}{\sigma_c} J_2 + \lambda(\theta)\sqrt{J_2} + bI_1 - \sigma_c \tag{40}$$

Where, a and b are dimensionless parameters and the dimensionless function $\lambda(\theta)$ depends on the Lode angle θ. The Ottosen yield criterion is the same as DP criterion when $a = 0$ and λ is independent of the Lode angle.

2.12 Feng et al. Criterion

Feng et al. (2020) [33] proposed a 3D failure criterion for hard brittle rocks wherein the authors accounted for cohesion and friction angle at unsymmetrical horizontal stress loading, along with the difference in strength of the rock at TC and TE states and influence of σ2. The authors proposed a linear and nonlinear (hyperbolic) failure criteria as follows, respectively.

$$(\sqrt{1 - b + sb^2} + t\,(1 - \sqrt{1 - b + b^2})\,sin_0)\,(\sigma_1 - \sigma_3) = (\sigma_1 + \sigma_3)sin_0 + 2c_0cos_0 \tag{41}$$

Where, b is the intermediate principal stress coefficient calculated as the ratio of difference of principal stresses, i.e., $\frac{\sigma_2-\sigma_3}{\sigma_1-\sigma_3}$, and c_0 indicates the cohesion at $b = 0$ and φ_0 indicates friction angle at $b = 0$.

$$\left(\frac{\sigma_1 - \sigma_3}{sin_b}\right)^2 = (\sigma_1 + \sigma_3 + 2c_0 cot_0)^2 \quad + a \tag{42}$$

Where a is the material constant. It is determined from either unconfined compression test or uniaxial tensile test as shown in Feng et al. (2020) [33].

3 Review of existing strain-based criteria

3.1 Stacey extension strain criterion

Stacey (1981) [34, 35] proposed empirical strain based criterion which is applicable in low confinement underground excavation faces. Stacey suggested that the fracture in a brittle rock will initiate when the total extension strain exceeds the characteristic critical strain of that rock, i.e., $\varepsilon_3 \geq \varepsilon c$.

The brittle fracture will form in the plane normal to the direction of extension strain, i.e., minor principal stress direction. For a linearly elastic material, the least principal strain is depicted as.

$$\varepsilon_3 = \frac{1}{E}(\sigma_3 - u(\sigma_1 - \sigma_2)) \tag{43}$$

According to this, the extension fracture can form when all the principal stresses are compressive and the net effect even at failure is compressive.

3.2 Sakurai critical strain criterion

Sakurai (1981) [36] proposed a direct strain evaluation technique to estimate the allowable maximum principal strain ε_1. It was suggested that for brittle fracture that as $\varepsilon 1$ approaches the critical strain ε_0 which is determined from axial stress-strain curve, as shown below, the fracture possibility increases.

$$\varepsilon_0 = \frac{\sigma_c}{E_i} \tag{44}$$

Later in 1995, Sakurai et al. (1995) [37] proposed that since the shear failure occurs in the minimum principal strain, maximum shear strain γ_{max} approaching critical shear strain γ_0 can serve as a warning of tunnel instability, and it can be computed from uniaxial compression test as follows.

$$\gamma_{max} = \gamma_0 = \frac{r_{max,f}}{G_{50}} = \frac{(\sigma_1 - \sigma_3)_f}{2G_{50}} = \frac{(\sigma_1 - \sigma_3)_f}{2} \cdot \frac{2(1+u)}{E_{50}} = \varepsilon_0(1+u) \tag{45}$$

Where G_{50} and E_{50} are the secant modulus of shear and Young's modulus at 50 % of the ultimate strength.

3.3 Fujii critical tensile strain criterion

Fujii et al. (1993a, b, 1994a, b) [38–40] stipulated on the basis of unconfined, triaxial compression tests and Brazilian tensile tests that the brittle fractures in rocks are commanded by ε_3 which when reaches critical tensile strain ε_{TC}, fracture appears. The authors also observed that the effect of confinement on ε_{TC} is negligible.

3.4 Kwasniewski and Takahashi critical strain criterion

Kwasniewski and Takahashi (2010) [40] found that shear octahedral shear strain γ_{oct} increases linearly with extensile strain ε_3, however, a better correlation exists between γ_{oct} and ε_1, as fractures are aligned parallel to σ1 direction. Hence, the criterion can be written as follows.

$$\gamma_{oct} = a + b\varepsilon_1 \tag{46}$$

In summary, since the deformation can be easily and readily measured in the field, the strain based criteria is used frequently in practice. Under the influence of low and medium confinement, Stacey criterion, Fujii criterion and Kwasniewski and Takahashi criterion will prove to be useful, however, Sakurai criterion will be applicable in non confined/low confinement loading environment or in hard brittle rocks found in deep excavations which undergo shear failure.

4 Comparative studies of stress response of rocks under polyaxial stress conditions

For this study well known and widely accepted failure criteria for brittle materials are selected, namely extended Griffith's criterion (Murrell criterion), Modified Weibols and Cook criterion (Colmenares and Zoback's modified WC criterion), Mogi nonlinear true triaxial criteria, Modified 3D Hoek-Brown criterion (Zhang and Zhu's 3D HB criterion), Bresler-Pister criterion, and Feng et al. criterion are discussed.

With preliminary calculations performed for extended Griffith's criterion, it can be seen that this criterion underestimates the strength of Westerly granite [16, 41] in low confinement (σ_1 = 177.7 MPa at σ_2 = 18 MPa, σ_3 = 2 MPa; tested σ_1 = 300 MPa), and massively underestimates the strength in high confinement (σ_1 = 549.8 MPa at σ_2 = 310 MPa, σ_3 = 77 MPa; tested σ_1 = 1005 MPa). Comparing the trendline from modified WC criterion with the plots of σ_1 vs σ_2 for varying σ_3 (Figure 1 (a)), a mean misfit of approximately 27 MPa and overestimating at higher σ_2 i.e., near TE state can be observed. Similarly, from Figure 1 (b) comparing the trendline from Mogi (1971) criterion with the polyaxial test data for Dunham dolomite shows that Mogi (1971) has a mean misfit of approximately 28 MPa, overestimates strength at very high σ_3, i.e., near TE state and the criterion in discussion is not a good fit for very high σ_2 and σ_3.

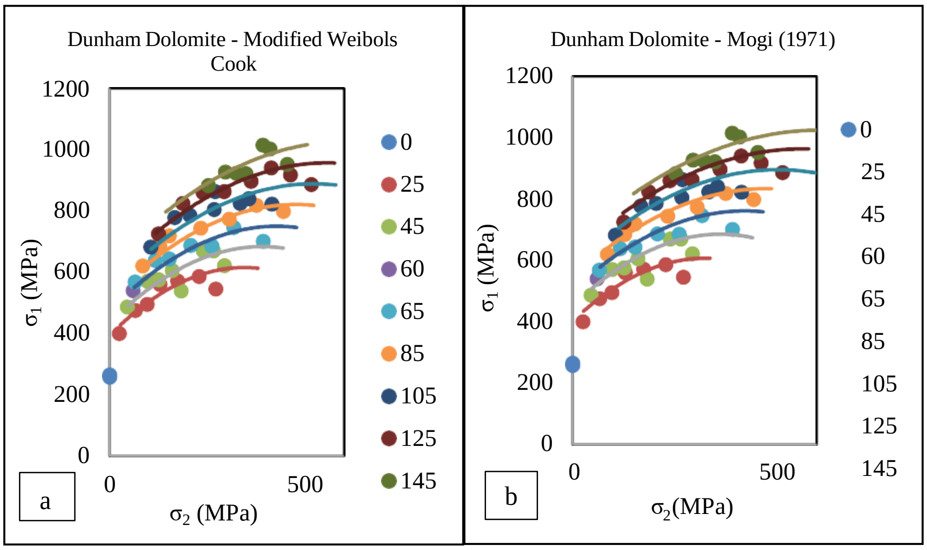

Figure 1. Best fitting solutions for Dunham dolomite using (a) Modifeid Weibols - Cook (WC) criterion, (b) Mogi (1971) criterion in principal stress domain.

From Figure 2, parameters m and s can be calculated from the trendline for the best fitting of Zhang and Zhu's modified Hoek-Brown criterion (Table 1). It can be noted that the criterion underestimates and overestimates values for Dunham dolomite and Westerly granite, respectively. Also, it can be observed that the mean misfit for Dunham dolomite is 3.31MPa spread along the length with more misfit towards higher confinement, and for Westerly granite it is 0.64 MPa which is spread all along the length of the confinement. Also, the Hoek-Brown parameters (m and s) calculated are incorrect as the actual m and s parameters are 9±3 and 32±3 for Dunham dolomite and Westerly granite, respectively.

Table 1. m and s parameters from Zhang and Zhu's Modified Hoek-Brown criterion.

Rock	m	s
Dunham dolomite	6.682	0.55
Westerly granite	47.58	1.582

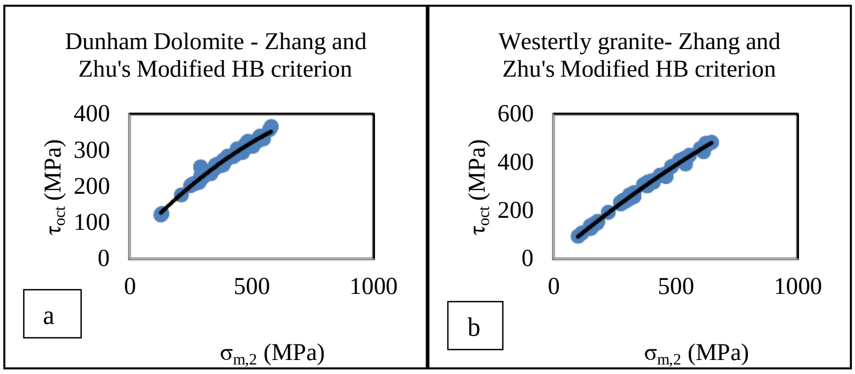

Figure 2. Best fitting solutions using Zhang and Zhu's Modified Hoek-Brown criterion for (a) Dunham dolomite, (b) Westerly granite in octahedral plane.

Using σ_c of Westerly granite as 201 MPa, σ_b as 460.08 MPa and σ_t as 12.5 MPa, parameters A, B and C for Bresler-Pister criterion are 14.4, 0.57 and -3.23E-4. A comparison has been made between the test data and Bresler-Pister criterion in Figure 3. It can be observed that it is applicable to low strength brittle materials with fair accuracy, but as confinement increases, the criterion deviates away from the test data.

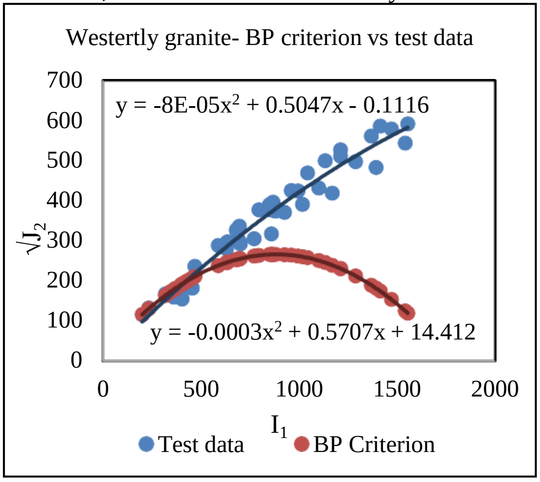

Figure 3. Comparison of computed Bresler-Pister criterion with test data for Westerly granite.

From the test data, c_0 and ϕ_0 for Feng et al. (2020) criterion can be calculated as 41.27 MPa and 51.08˚. Also, parameter a is computed as -122241.23. Thereafter, parameter $\sin(\phi_b)$ is computed based on the deviator stress (σ_1-σ_3). For a non-zero b, at low confinement of σ_2 = 18 MPa, σ_3 of 2 MPa, the strength according to nonlinear criterion is 284.63 MPa (tested σ_1 is 300 MPa), and at σ_2 = 310 MPa, σ_3 = 77 MPa, the strength is 959.63 MPa (tested σ_1 is 1005 MPa). Although Feng et al. criterion observes strengthening with increasing confinement and lower misfit at higher σ_3. It also observes no significant influence of σ_2, quotes lower strength for varying σ_3 and has an average misfit of 44.9 MPa. Hence, this criterion needs to be improved upon.

5 Discussion and Conclusion

Although new true-triaxial failure criteria have been developed, Mogi (1971) criterion (3D) and Hoek-Brown criterion remain a standard for further research, especially in brittle fracture mechanics of rocks, due to their empirical nature and adaptability in varying stress states. If static and quasi-static loading condition is in focus, then the classical works of Mogi, Haimson and Chang, Feng et al., C.D. Martin et al., Colmenares and Zoback, Tiwari and Rao, Zimmerman, and Weibols and Cook play a huge role in the study of brittle fracture of rocks. However, if dynamic loading or impact loading is taken into consideration then the works of Johnson-Holmquist and Ottosen play a crucial role in the study of brittle rock fracture of rocks under ballistic impact.

Based upon the review done here, Mogi (1971), Modified 3D Hoek-Brown, and Modified Weibols and Cook criteria are still the best failure criteria for true triaxial stress conditions. It is to be noted that minimal overestimation by Modified WC criterion and Mogi (1971) criterion are suitable, since the support system provided according to these failure criteria will withstand higher load as the rock approaches its ultimate strength in underground openings.

Since, deformations can be monitored in underground openings using total station easily. The observations made by Stacey, Sakurai and Kwasniewski and Takahashi are important. If during the fracture of rocks there is insignificant plastic deformation and the calculated minimum principal strain reaches its critical strain, there is high possibility of bursting or spalling in underground openings. Hence, such brittle geology demands suitable application of shotcrete, wire mesh along with yielding rockbolts or energy absorbing rockbolts and yielding steel arch supports in tunnels and caverns. Temporary underground mine areas may require lattice girders and steel ribs along with rockbolts in such cases or destress blasting or caving in extreme stress conditions.

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