Visakhapatnam Chapter

*Proceedings of Indian Geotechnical Conference 2020*
*December 17-19, 2020, Andhra University, Visakhapatnam*

# Probabilistic Investigation on Seismic Bearing Capacity of Shallow Foundation on Unsaturated Fly Ash Deposit

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**Abstract.** Fly ash is a by-product of coal based thermal power plants. Utilization of fly ash in geotechnical projects may provide a sustainable solution for its efficient disposal. In this regard, bearing capacity of fly ash deposits is of paramount importance for satisfactory performance of foundations built on these deposits. Fly ash deposits used for filling up of low-lying areas are often under unsaturated or partially saturated state and therefore, incorporation of suction stress induced in fly ash due to the presence of matric suction becomes essential while estimating its bearing capacity. In this regard, unsaturated shear strength may be quantified using *Water Retention Characteristic Curve* (WRCC) fitting parameters. However, determination of WRCC parameters often involves various uncertainties arising mostly due to the limited number of test data, inherent limitation of measurement range of suction measuring instruments etc. Furthermore, very limited studies are available in literature to address the problem of seismic bearing capacity of fly ash deposits under unsaturated framework. In the present study, probabilistic based approach has been adopted to obtain the seismic bearing capacity of footing placed on fly ash deposit, incorporating the uncertainty of the random input parameters, by adopting a factorial design approach. Sensitivity of random variables, such as infiltration rate ratio ($q/k_s$), WRCC fitting parameters of the fly ash and horizontal seismic acceleration coefficient ($k_h$) on the seismic bearing capacity are presented. A prediction model has also been developed considering variation of random input parameters.

**Keywords:** Fly ash, Seismic bearing capacity, Probabilistic investigation.

## 1 Introduction

Fly ash is a fine-grained, coal combustion residue by-product generated at coal based thermal power plants, which if not disposed of properly, may become a hazardous waste [Nadaf and Mandal 2017]. The increase in urban population and consequently increased demand of power has put a lot of burden on the coal based thermal power plants. In this pursuit, tremendous amount of fly ash is generated globally while the utilization of fly ash is limited. In this regard, one of the most promising technique of bulk utilization of fly ash may be in its use as a geomaterial for reclamation of low lying areas, land fill, construction of embankment and mine fill [DiGioia and Nuzzo 1972, Ghosh and Subbarao 2006, Martin *et al*. 1990, Kim *et al*. 2005, Kaniraj and Havangi 2001, Leonards and Bailey 1982]. Bearing capacity and settlement are two key design parameters while evaluating the suitability of fly ash as a fill material. Various researchers [Gill *et al*. 2013, Nadaf and Mandal 2017, Cui *et al*. 2018] carried out model tests to study the load settlement characteristics of fly ash deposit/slopes.

Conventionally, shallow foundations are constructed in vadose zone which is partially saturated due to various environmental factors such as evapotranspiration and capillary action of soil. Classical bearing capacity theories neglect the suction stress while estimating the bearing capacity and therefore, yields a very conservative solution, which may lead to a uneconomical design of a footing. Recently, various researchers [Anand and Sarkar 2020, Vahedifard and Robinson 2016, Oh and Vanapalli 2013, Vanapalli and Mohamed 2007] investigated the problem of bearing capacity under unsaturated framework and based on the results, it has been well established that the matric suction significantly affects the shear strength and hence the bearing capacity of a footing. The matric suction distribution profile depends on several factors, such as depth of water table, surface flux boundary conditions etc. Incorporation of influence of these parameters are vital while designing a foundation.

Often, foundations are located in seismically active zone where the bearing capacity of the foundation is affected significantly due to the seismic forces acting on the foundation. Various researchers have attempted to address the problem of seismic bearing capacity of a shallow footing by adopting a pseudo-static approach (Budhu and Al-Karni 1993, Sarma and Iossifelis 1990, Zhu 2000). However, problem of seismic bearing capacity under unsaturated framework has not been given due consideration in the literature.

Therefore, in the present study an attempt has been made to provide a solution to the seismic bearing capacity of a shallow strip footing under unsaturated framework, based on a finite element analysis. Variation of seismic bearing capacity with seismic horizontal co-efficient ($k_h$) has been studied under a deterministic framework for different steady state surface flux boundary condition. Effects of uncertainty in the random input parameters have been presented through a detailed probabilistic analysis.

## 2 Problem Definition and Material Properties

### 2.1 Schematics of the problem

Fig. 1 illustrates the graphical representation of the problem considered in the present study. A surface, strip footing of width ($B$=1m) has been considered resting on the surface of a fly ash deposit. Ground water table (GWT) has been assumed to be at a depth of 10$B$ from the ground surface. Above the GWT, under a steady state surface flux boundary condition, fly ash would not be uniformly saturated, and the effective degree of saturation would vary with depth. Due to the variable saturation, effective unit weight ($\gamma_e$) and capillary cohesion ($c_{app}$) of fly ash deposit would also vary with depth as shown in Fig. 1. Partially saturated region above the GWT may be divided into $N$ number of layers and capillary cohesion and effective unit weight may be assigned to each layer. In Fig. 1, $c_i$ and $\gamma_i$ represents the capillary cohesion and effective unit weight respectively at the $i^{th}$ layer above the ground water table. The optimization of number of layers ($N$) and values of $c_i$ and $\gamma_i$ have been discussed in the subsequent sections.

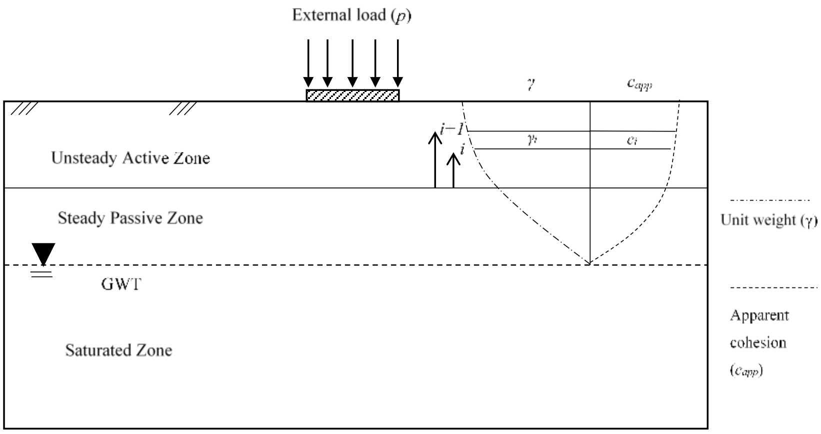


**Fig. 1.** Schematics of the problem geometry considered in Present study

### 2.2 Fly ash properties considered in the study

Based on an extensive literature survey on the geotechnical properties of several different Indian fly ash samples, Pandian (2004) reported the range of parameters for Indian fly ash samples. For the present study, properties of fly ash samples are chosen based on the typical range of values reported in Pandian (2004).

Water retention characteristic curve (WRCC) fitting parameters of the fly ash has been obtained from Prakash *et al*. (2019).

**Table 1.** Properties of fly ash considered in the study

| Property | Values |
|---|---|
| Unit weight, $\gamma$ | 16.6 kN/m$^3$ |
| Friction angle, $\phi$ | 34° |
| True cohesion, $c$ | 0 kPa |
| $\alpha$ | 0.032 kPa$^{-1}$ |
| $n$ | 2.161 |
| $G$ | 2.0 |

## 3 Quantification of Suction Stress Based Engineering Parameters

To incorporate matric suction into the original shear strength Mohr-Coulomb's (M-C) failure criterion, Fredlund *et al*. (1978) extended the M-C criterion and proposed the following closed-form expression for the modified failure envelope

$$\tau_f = c' + (\sigma - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b \tag{1}$$

Where, $c'$ = effective cohesion, $u_a$ = pore-air pressure, $u_w$ = pore-water pressure, $u_a$-$u_w$ = matric suction, $\phi^b$ = parameter representing internal friction angle due to presence of matric suction, $\phi'$ = Angle of internal friction, $\tau_f$ = shear stress at failure, $\sigma$ = normal stress.

It has, further been well attested by various researchers, that the behaviour of unsaturated geomaterials can be modelled with sufficient accuracy using the Soil water characteristic curve (SWCC) fitting parameters [Lu and Likos 2004, Vahedifard and Robinson 2016]. In this regard, the effective stress state of a partially saturated soil can be expressed as [Lu and Likos 2004]

$$\sigma' = \sigma - u_a - \sigma_s \tag{2}$$

Where, $\sigma'$ = effective stress, $\sigma$ = total stress and $\sigma_s$ = suction stress. The suction stress ($\sigma_s$) may be expressed as [Lu *et al*. 2010]

$$\sigma_s = -(u_a - u_w) \text{ for } u_a - u_w \le 0 \tag{3a}$$

$$\sigma_s = -S_e(u_a - u_w) \text{ for } u_a - u_w \ge 0 \tag{3b}$$

Where, $S_e$ = Effective degree of saturation, may be expressed as (Lu *et al*. 2010)

$$S_e = \frac{1}{\left[1 + \{\alpha(u_a - u_w)\}^n\right]^{\frac{n-1}{n}}} \tag{4}$$

Where, $\alpha$ = SWCC fitting parameter, closely related to the inverse of the air-entry value, $n$ = SWCC parameter representing the pore-size distribution.

Substituting the value of effective degree of saturation ($S_e$) into equation (3) yields the expression suction stress ($\sigma_s$). The contribution of matric suction on shear strength can be represented by considering additional resistance amongst the soil particles against shearing (apparent cohesion), arising predominantly due to the capillary effects, and may be expressed as

$$c_{app} = -\sigma_s \tan\phi' \tag{5}$$

Under a steady state surface flux boundary condition (infiltration or evaporation), the degree of saturation and therefore, matric suction does not remain constant with depth. Under a steady state infiltration condition, the expression for matric suction may be expressed as (Wang *et al*. 2019)

$$\psi = (u_a - u_w) = -\frac{1}{\alpha}\ln\left[\left(1 + \frac{q}{k_s}\right)e^{-\alpha\gamma_w(D_w - z)} - \frac{q}{k_s}\right] \tag{6}$$

Where, $q$ = steady state infiltration rate, $k_s$ = saturated hydraulic conductivity, $D_w$ = depth of water table from the ground surface, $z$ = distance of the point considered from the ground surface, $\gamma_w$ = unit weight of water.

### 3.1 Variation of capillary cohesion ($c_{app}$) with depth

Variation of capillary cohesion with depth may be obtained by substituting the expression of matric suction ($\psi$) (eqn. 6) and effective degree of saturation ($S_e$) (eqn. 4) in eqn. (3) and (5). After the substitution and simplification, the expression of capillary cohesion may be expressed as

$$c_{app} = \frac{1}{\alpha}\left[\frac{\ln\left\{\left(1+\frac{q}{k_s}\right)e^{-\alpha\gamma_w(D_w-z)}-\frac{q}{k_s}\right\}}{\left\{1+\left[-\ln\left\{\left(1+\frac{q}{k_s}\right)e^{-\alpha\gamma_w(D_w-z)}-\frac{q}{k_s}\right\}\right]^n\right\}^{1-1/n}}\right]\tan\phi' \tag{7}$$

### 3.2 Variation of effective unit weight ($\gamma_e$) with depth

As the effective saturation varies with depth, it is evident that the effective unit weight of the fly ash deposit would also vary with depth. Wang *et al*. (2019) proposed the following closed-form formulation for effective unit weight due to variable saturation of geomaterials under a steady state infiltration condition

$$\gamma_e = \frac{(\gamma_{sat}-\gamma_w)G_s + \{S_e-(1-S_e)S_r\}(G_s\gamma_w-\gamma_{sat})}{G_s-1} \tag{8}$$

Where, $S_r$ = Residual saturation of fly ash, $G_s$ = Specific gravity of fly ash particles, $\gamma_{sat}$ = saturated unit weight of fly ash. In the above formulation, substituting the expression for $S_e$ (eqn. 4) yield the variation of effective unit weight with depth

## 4 Numerical Modelling

For numerical investigation on the seismic bearing capacity, commercially available finite element software package "*PLAXIS*$^{2D}$" has been considered in the present study.

### 4.1 Problem Definition and Boundary Conditions

Figure 2 depicts the developed finite element model and boundary conditions. As it has already been discussed that under steady state surface flux boundary condition, the saturation of the soil does not remain constant, and therefore, fly ash deposit above ground water table (GWT) could not be modelled as a single homogeneous material. To overcome this, fly ash deposit above GWT has been divided into (N) number of layers and each layer has been assigned different unit weight and capillary cohesion obtained from eqns. (7) and (8). In FE model, both horizontal and vertical fixities were applied to the bottom boundaries, while horizontal fixities were provided to the side boundaries. The domain size was chosen sufficiently large based on a sensitivity study to minimize the boundary effects. The number of discretized layers of unsaturated zones ($N$) were chosen to be 10 based on a sensitivity analysis. For the present study, a '*fine*' mesh refinement has been adopted based on the results obtained from a mesh optimization study.

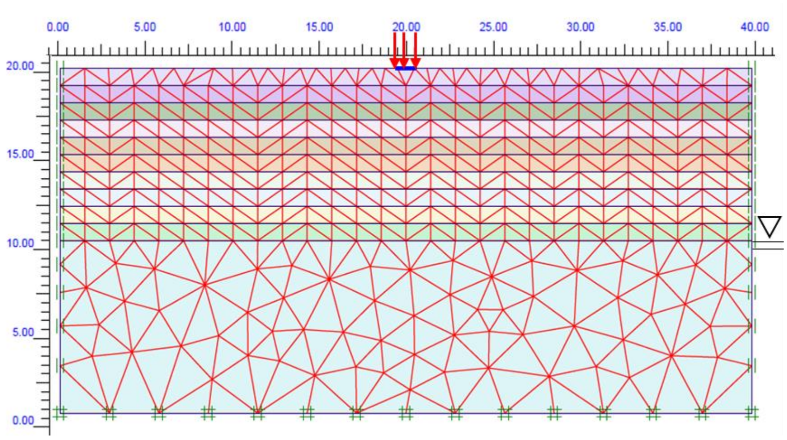


**Fig. 2**. Typical layers considered in the finite element analysis and boundary conditions for the system considered.

## 4.2 Validation of the Developed Model

In order to establish the efficacy of the developed model in predicting the seismic bearing capacity of strip footing, a set of pseudo-static analyses were carried out and the results are compared against the pioneering works reported in the literature. For the validation purpose, the friction angle of the fly ash was assumed to be 40°, which is within the range of friction angle reported for fly ash samples (Pandian 2004). Results are compared in terms of Terzaghi's (1943) bearing capacity factor ($N_\gamma$) for different values of horizontal seismic acceleration co-efficient. Fig. 3 depicts the comparison between the $N_\gamma$ obtained from the present study and the values reported in the literature. Very close agreement establishes the efficacy of the present developed model in predicting the seismic bearing capacity under pseudo-static method

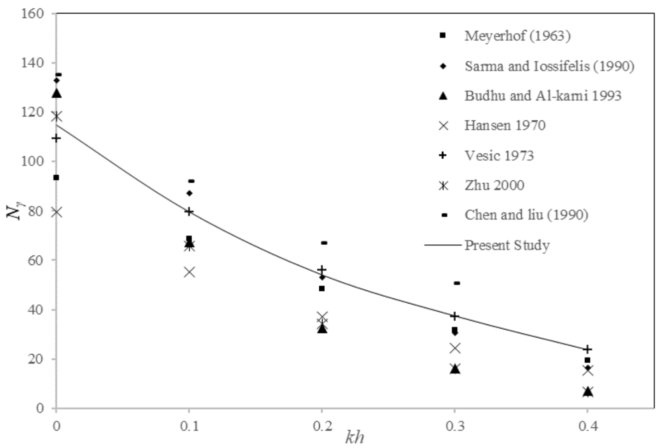


**Fig. 3.** Comparison of the seismic bearing capacity factor ($N_\gamma$) obtained from the present investigation with results reported in literature

# 5 Results and Discussions

## 5.1 Deterministic Seismic Bearing Capacity

In order to study the influence of horizontal pseudo-static acceleration co-efficient on the seismic bearing capacity, a set of numerical investigations were carried out considering different pseudo-static acceleration co-efficient values ($k_h$ = 0.0, 0.1, 0.2, 0.3, 0.4 and 0.5). Results are presented in dimensionless forms as Seismic Bearing Capacity Ratio (SBCR) which may be defined as

$$SBCR = \frac{p}{\gamma B} \tag{9}$$

Where $p$ = ultimate collapse load, $\gamma$ = unit weight of fly ash and $B$ = Footing width.

Variation of SBCR with infiltration rate ratio ($q/k_s$)

Variation of SBCR for different infiltration rate ratio ($q/k_s$ = 0.0, -0.2, -0.4, -0.6, -0.8 and -1.0) has been shown in Fig. 4.

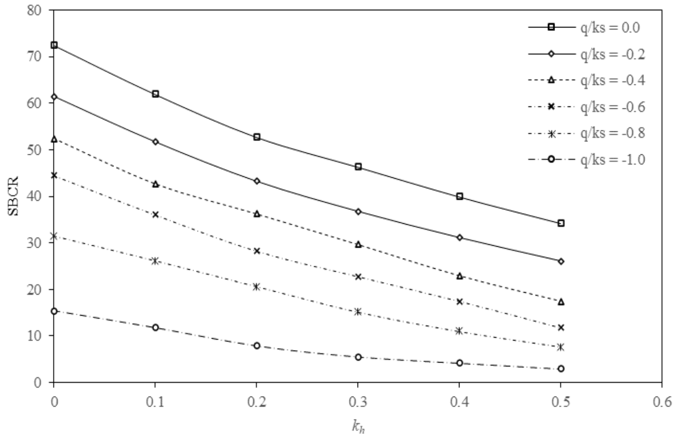


**Fig. 4.** Variation of SBCR with $k_h$ for different infiltration rate ratio ($q/k_s$)

From Fig. 4, it is quite evident that the variation of steady state infiltration rate ratio imparts substantial variation in the seismic bearing capacity of the footing on fly ash deposit. Furthermore, with increase in the magnitude of horizontal seismic acceleration co-efficient ($k_h$) an almost uniform reduction in SBCR was observed for all the values of $q/k_s$ considered in the present study.

## 5.2 Probabilistic Investigation

SWCC fitting parameters are usually obtained by fitting a prescribed parametric model to a limited number of test data. This induces a substantial uncertainty to the measured WRCC parameter. In addition, due to the inherent limitations of the measuring instrument to measure up to a limited range of matric suction, also induced some uncertainty to the measurement of the parameters [Prakash et al. 2019]. Occurrence of earthquake and rate of infiltration induced in soils due to environmental factors are also highly uncertain in nature. Therefore, a seismic bearing capacity analysis, based on deterministic set of parameters, may not yield reliable solution. To overcome above limitations, in the present study, the seismic bearing capacity analysis has been carried out under probabilistic framework. Uncertainty arising due to uncertainty in the input random variables have been quantified by development of prediction models based on analyses carried out using factorial design. The prediction models were constructed based on multiple regression analysis [Myers *et al*. 2016].

## Random parameters selection and their variability

In the present study, four different significant random variables ($q/k_s$, $\alpha$, $n$ and $k_h$) have been considered for the probabilistic investigation. A factorial design approach has been adopted in which the variation of parameter is described by using a high point estimate and low point estimate [Rosenblueth 1975]. The low point and high point estimate and their distribution has been presented in Table 2.

**Table 2.** Random parameters adopted in the present study

| Parameters | Distribution | High Point Estimate | Low Point Estimate |
|---|---|---|---|
| WRCC parameter ($\alpha$) | Log-normal | 0.017 | 0.040 |
| WRCC parameter ($n$) | Log-normal | 1.59 | 3.57 |
| Infiltration rate ratio ($q/k_s$) | Normal | -1.0 | 0.0 |
| Horizontal seismic acceleration co-efficient ($k_h$) | Normal | 0.0 | 0.5 |

For the low and high point estimates of WRCC fitting parameters, values and its distribution are chosen from the results presented in Prakash *et al*. 2019. For infiltration rate ratio, values are their distributions are adopted from Anand and Sarkar (2020).

For horizontal seismic acceleration co-efficient ($k_h$) a low point and high point estimates of 0 and 0.5 have been adopted which is conventionally adopted in pseudo-static investigation.

## Effect of random parameters on SBCR

Influence of random parameters on SBCR presented in Table 3. The negative sign and positive sign represents the corresponding low point estimate and high point estimate of the random variable respectively. ANOVA (Analysis of Variance) results are presented in Table 4.

**Table 3.** Model runs and corresponding SBCR

| Run | $q/k_s$ | α | n | $k_h$ | SBCR |
|---|---|---|---|---|---|
| 1 | + | + | + | + | 4.76 |
| 2 | - | + | + | + | 2.34 |
| 3 | + | - | + | - | 51.92 |
| 4 | + | - | - | + | 48.76 |
| 5 | + | + | + | - | 21.38 |
| 6 | + | - | + | + | 17.53 |
| 7 | - | + | - | - | 16.98 |
| 8 | - | - | + | - | 16.98 |
| 9 | - | + | + | - | 16.98 |
| 10 | + | - | - | - | 16.98 |
| 11 | + | + | - | + | 17.10 |
| 12 | - | + | - | - | 33.37 |
| 13 | + | - | + | + | 2.34 |
| 14 | - | - | - | - | 45.18 |
| 15 | - | - | - | + | 2.34 |
| 16 | - | + | - | + | 2.34 |

**Table 4.** ANOVA results for SBCR

| Terms | Factors | Percentage Contribution | F-Value | p-Value |
|---|---|---|---|---|
| Main | $q/k_s$ | 39.49 % | 30.76 | 0.003 |
| | $\alpha$ | 11.23 % | 8.75 | 0.032 |
| | $n$ | 3.55 % | 2.77 | 0.157 |
| | $k_h$ | 22.29 % | 17.37 | 0.009 |
| | $\alpha * q/k_s$ | 11.23 % | 8.75 | 0.032 |
| Interaction | $n * q/k_s$ | 3.55 % | 2.77 | 0.157 |
| | $k_h * q/k_s$ | 0.04 % | 0.03 | 0.868 |
| | $n * \alpha$ | 0.00 % | 0.00 | 0.996 |
| | $k_h * \alpha$ | 0.01 % | 0.01 | 0.946 |
| | $n * k_h$ | 2.19 % | 1.71 | 0.248 |

From the ANOVA results presented in Table 4, it is quite evident that the infiltration rate ratio ($q/k_s$) and horizontal seismic acceleration co-efficient ($k_h$) are the most significant parameters affecting the seismic bearing capacity of the footing with their percentage contribution being 39.49% and 22.29% respectively.

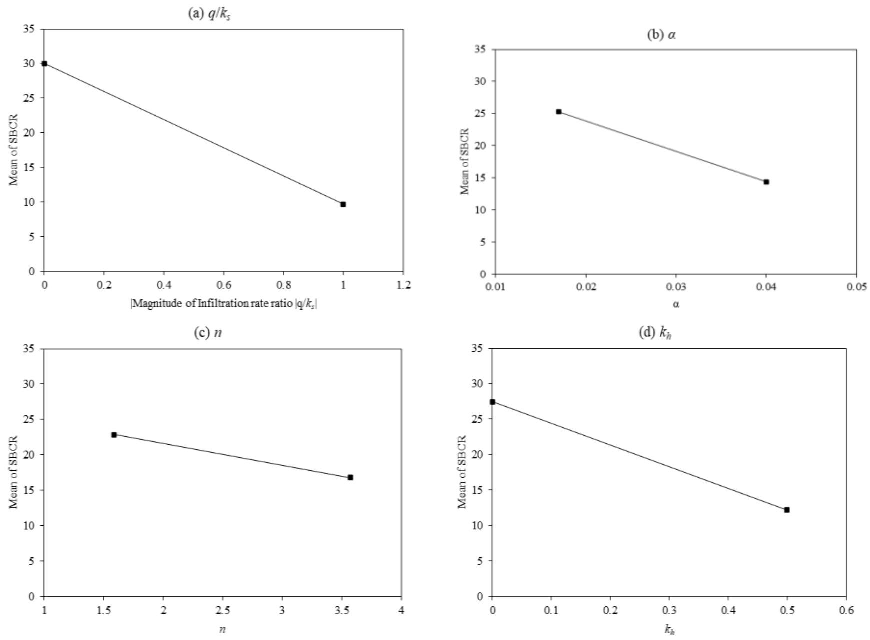


**Fig. 5.** Effect of random variable (a) $q/k_s$ (b) $\alpha$ (c) $n$ and (d) $k_h$ on SBCR of unsaturated fly ash deposit

Effect of random variables on SBCR has been presented in Fig. 5. A thorough quantitative comparison of Fig. 5. suggests that inflitration rate ratio substantially affects the SBCR of a surface strip footing placed on an unsaturated fly ash deposit. Subsequently, the magnitude of seismic acceleration co-efficient is the second most significant parameter. Amongst the SWCC fitting parameters, parameter *n* has the least significance.

Development of prediction model

Prediction model has been developed for SBCR based on the results obtained from the factorial design analysis. Prediction model for SBCR can be expressed as

$$SBCR = 74.2 + 63.8\frac{q}{k_s} - 934\alpha - 3.77n - 5.6k_h - 943\frac{q}{k_s}\alpha - 6.16\frac{q}{k_s}n - 2.6\frac{q}{k_s}k_h + \alpha n - 45\alpha k_h - 9.68nk_h \quad (10)$$

Prediction model shown in eqn. (10) can be used by practicing engineers while solving the problem of seismic bearing capacity of shallow strip footing placed on a variably saturated fly ash deposit.

## 6 Summary and Conclusions

Present study presents the seismic bearing capacity of a strip footing placed on an unsaturated fly ash deposit under a probabilistic framework. Influence of uncertainty in four different significant input variables on the overall seismic bearing capacity has been reported. Influence of steady state infiltration rate ratio ($q/k_s$) and horizontal seismic acceleration co-efficient ($k_h$) has been presented under a deterministic framework. Based on the results, the following major conclusions may be made

1. Seismic bearing capacity is substantially affected by variation of steady state infiltration conditions, as well as magnitude of seismic acceleration co-efficient ($k_h$).
2. Under deterministic framework, the reduction in SBCR with $k_h$ is uniform for all the infiltration rate ratio considered.
3. Prediction model developed in the present study may be useful for practicing geotechnical engineers while designing a shallow foundation on fly ash deposit in seismically active regions.
4. With increase in infiltration rate ratio, bearing capacity reduces substantially and therefore, results of in-situ field tests may be significantly affected due to the presence of matric suction or effective saturation of the geomaterials.

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