Visakhapatnam Chapter

Proceedings of Indian Geotechnical Conference 2020
December 17-19, 2020, Andhra University, Visakhapatnam

Consolidation: Critical Appraisal of Settlement versus Rate of Settlement (SRS) Approach with Fuzzy Logic

Sudhir Kumar Tewatia[1], Malaya Chetia[2], Taslima Nasrin[3] and Kanishck Tewatia[4]

[1] Jimma University, Ethiopia, Africa
[2,3] Assam Engineering College, Guwahati 781013, India
[4]Mindsarovar Technologies Private Limited, Bangalore, India
tewatia1961@gmail.com

Abstract. Analysis of time dependent behavior of clayey soils in vertical consolidation is carried out by plotting the experimental settlement, δ, with its differential (with respect to experimental time, t) $d\delta/dt$. This is called the settlement versus rate of settlement (*SRS*) approach. A fastest rapid loading method is suggested that gives the coefficient of consolidation, c_v values very close to true c_v and also shows how near the calculated and true c_v values are by fuzzy logic. The merits and demerits are compared with popular methods. The *SRS* methods work even when the settlement-time-pressure data at the beginning of load increment is not known. The simple procedure for quantitative isolation of secondary consolidation and creep (from primary consolidation) is suggested. Six phases of consolidation are quantitatively identified instead of three (initial, primary and secondary). Terzaghi's one dimensional consolidation equation is resolved in 3 parts; (1) parabolic 0-40%U, (2) transition 40-60%U and (3) exponential 60-100%U, where, U is the degree of consolidation. It is shown that the *SRS* plot is a very powerful and useful tool for consolidation analysis.

Keywords: Clay, Consolidation, Rate of Settlement, Coefficient of Consolidation, Fuzzy Logic, Creep.

1 Introduction

Some simple and very short concepts are sometimes path breaking in the development of research in some subjects. One such concept was given by Terzaghi [1] as:

$$\sigma = \sigma' + u \tag{1}$$

where, σ is total pressure, σ' is effective pressure and u is pore-water pressure. This was to pave the way for the development of modern soil mechanics, particularly in clays. Similarly, since the days of Newton, differential equations have been used for solving problems mostly as below

$$y = f(x); \quad y' = dy/dx = f_1(x); \quad y'' = d^2y/dx^2 = f_2(x) \tag{2}$$

That means differential equation was used as a function of *x*. little practice was there to use it as a function of *y*. This particularly, created the problems when *x* is not known. Similar happened in consolidation where all researchers were using δ-t or δ-σ plots. The solution failed when t or σ is not known at the time of load increment and that was the usual case in the field. Terzaghi [2] gave the average time of loading concept in case of continuous uniform loading that, however, was rough/untrue assumption (Fig. 1).

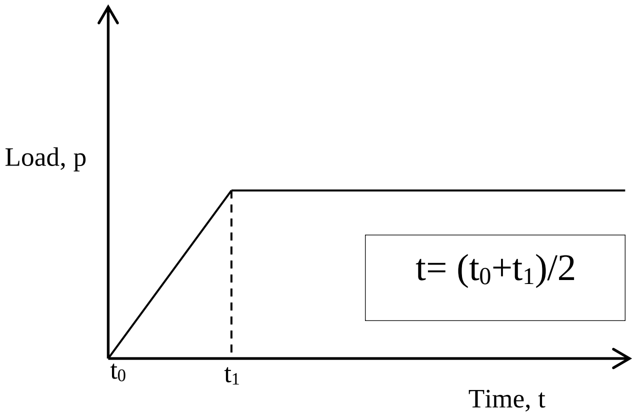

Fig. 1. Uniform ramp loading [2]

Tewatia [3-19] suggested the y-$f(y')$ and y-$f(y'')$ plots. In consolidation, it is called the settlement versus rate of settlement, *SRS* approach. They use characteristics of degree of consolidation, *U* versus theoretical velocity, *dU/dT* and *U* versus *dT/dU* plots in linear and semi-log formats. Where, *U* is the degree of consolidation and *T* is the time factor in Terzaghi's one dimensional consolidation equation

$$U = 1 - \frac{8}{\pi^2} \sum_{N=0}^{N=\infty} \frac{1}{(2N+1)^2} Exp\left(- \frac{(2N+1)^2 \pi^2}{4} T \right) \quad (3)$$

Earlier, Eq. 3 was resolved in two parts; (1) parabolic 0-60% *U* by Fox [20] as

$$T = \frac{\pi}{4} U^2 \quad (4)$$

and (2) exponential 60-100%*U* as

$$U = 1 - \frac{8}{\pi^2} Exp(-\frac{\pi^2}{4} T) \quad (5)$$

Beginning of the secondary consolidation in the range of the primary consolidation was a hypothesis but the *SRS* approach separated it quantitatively since its beginning. Creep and secondary consolidation (considered to be same) were defined separately due to high resolution power of the *SRS* approach [14, 15]. Terzaghi's equation was resolved in two parts parabolic Eq. 4 and exponential Eq. 5. Nothing was known of the part where it changes its nature from parabolic to exponential. Only three types of consolidation settlements were known; initial, primary and secondary but there may exist 6 phases. The c_v was calculated using a substantial data and portion of the δ-t plot and it was unnecessarily time consuming in the laboratory and field. All the methods failed when time, settlement and pressure at the instant of load increment were not known. All the methods provided c_v in the laboratory that was affected by secondary consolidation. To compare the calculated c_v with true c_v (i.e. c_v that is not affected by secondary consolidation) one had to compare the calculated values of hydraulic conductivity with the measured values of hydraulic conductivity. All such problems are solved by the *SRS* approach very easily and quickly.

In spite of all such outstanding clear merits and its publications in various most reputed journals for about 24 years, the approach could not get momentum. Instead, the reputed professors of reputed universities started stealing this approach from ASTM Geotechnical Testing Journal [3], Springer Journal of Geotechnical and Geological Engineering [15] and ASCE International Journal of Geomechanics [17] and published in most reputed geotechnical journals like ASTM Geotechnical Testing Journal [21], Applied Clay Science [22] and Géotechnique [23] etc. There was a parliament question also in India in 1997 on research espionage of this technique. This paper is an attempt to find the merits and demerits of the *SRS* approach in consolidation with further possible improvements like application of fuzzy logic and parabolic fitting [24] for finding the $d\delta/dt$ etc. The fuzzy logic is an approach to computing, based on "degree of truth" in which the truth values of variables may be any real number between 0 and 1 both inclusive. It is used to represent the concept of partial truth, where the truth value may range between completely true and completely false.

1.1 Settlement versus Rate of Settlement (*SRS*) approach

By definition

$$U = \frac{\delta - \delta_0}{\delta_{100} - \delta_0} \tag{6}$$

$$T = \frac{c_v t}{H^2} \tag{7}$$

In Eq. 3, U is differentiated with respect to T. The plots of U-T and U-dU/dT in various formats are given in the Figs 2(a) and 2(b) [3]. In Fig 2(b), the symmetrical S curve is divided into 3 parts: 1. Parabolic 0-40%U, 2. Exponential 60-100%U and 3. Transition zone U = 40-60%. From the Fig. 3(a)

$$\delta_0 = \delta - \frac{s}{2.3026} \tag{8}$$

and in Fig. 4(b)

$$\delta_{100} = \delta + \frac{s}{2.3026} \tag{9}$$

where, s is the slope of experimental δ-$log_{10}(t)$ S curve. The slope at the point of inflection,

$$s_{50} = 1.009\ (\delta_{100} - \delta_0) \approx (\delta_{100} - \delta_0) = \text{amount of primary consolidation} \tag{10}$$

The Fig. 3(a) shows the theoretical U versus dU/dT plot for finding δ_0. The Fig. 3(b) shows the experimental δ versus $d\delta/dt$ plot for finding δ_{100}. The Fig. 4 shows the quantitative isolation of secondary consolidation from experimental plot of SB dam soil [3, 4, 15].

$$Y_{a1} = Y_{50} + (Y_{50} - Y_a) \quad \text{and} \quad logX_{a1} = logX_{50} + (logX_{50} - logX_a)$$

$$\text{Hence, } Y_{a1} = 2Y_{50} - Y_a, \quad \text{and} \quad X_{a1} = X_{50}^2/X_a \tag{11}$$

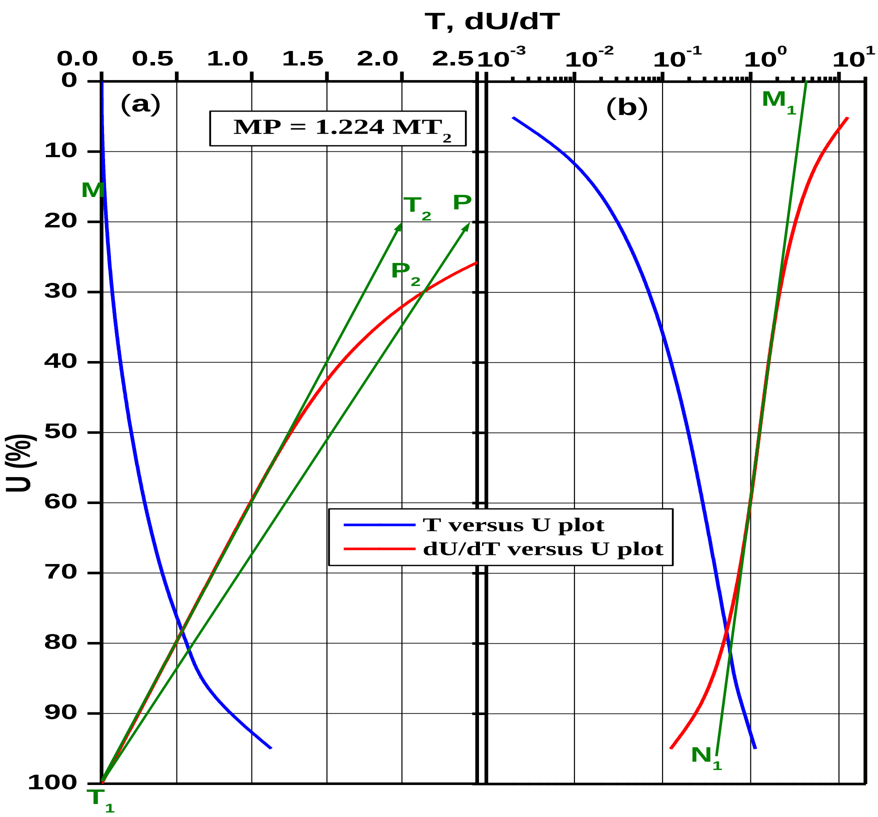

Fig. 2. Theoretical U versus (T, dU/dT) plots in **(a)** linear and **(b)** semi-log formats

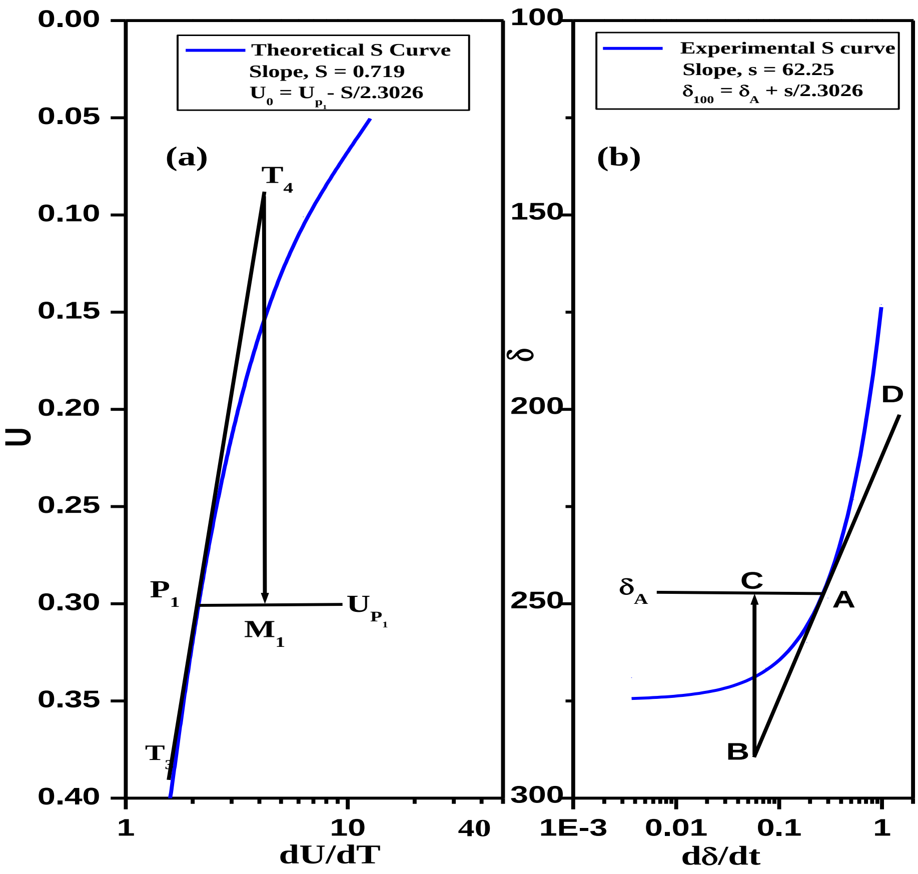

Fig. 3. *S* curve **(a)** Theoretical plot for finding δ_0 and **(b)** experimental plot for finding δ_{100}.

2 Determination of c_v, δ_0 and δ_{100} using δ-$(d\delta/dt)$ Plot

Tewatia [3] derived

$$\delta = -\frac{4H^2}{\pi^2 c_v}\left(\frac{d\delta}{dt}\right) + \delta_{100} \tag{12}$$

Where, *H* is the drainage path. The Eq. 12 is the equation of straight line in the form: $y = m\,x + c$, where m is slope and c is intercept on *y-axis*. The δ-$(d\delta/dt)$ plot is a straight line having a slope, $m = (4H^2)/(\pi^2 c_v)$ and intercept on δ axis, $c = \delta_{100}$. Thus, c_v can be evaluated as:

$$c_v = \frac{4\,H^2}{\pi^2\,m} \tag{13}$$

As per Terzaghi's assumptions if c_v/H^2 is constant, then δ-$(d\delta/dt)$ curve should be a straight line. As secondary consolidation starts and runs superposed over primary consolidation, therefore c_v/H^2 is not constant. Eq. 12, therefore, gives a curve, considerable portion of which is straight line (Fig. 5). When this line is extrapolated to cut δ axis it gives δ_{100} (Fig. 5) and δ_0 can be determined as from the Fig. 2(a) [4]

$$\delta_0 = \delta_{100} - \left(\frac{\delta_{100} - \delta_{30}}{0.70}\right) \tag{14}$$

2.1 Determining true c_v and fuzzy logic for c_v

The first step is to plot δ-$(d\delta/dt)$ on semi-log scale as shown in the Fig. 4. Draw tangent in the middle straight-line portion and find its slope s_{50} over one log cycle as in the Fig. 2(b) and Fig 6 [3, 4, 15]. The δ_0 is found using Eq 8 and δ_{100} is determined using Eq 10 as

$$\delta_{50} = 0.5(\delta_0 + \delta_{100}) = \delta_0 + 0.5(\delta_{100} - \delta_0) = \delta_0 + s_{50} \tag{15}$$

$$c_v = \frac{0.8033\left(\frac{d\delta}{dt}\right)_{50}}{s_{50}} H^2 \tag{16}$$

$$\mu = \frac{t_{50}\left(\frac{d\delta}{dt}\right)_{50}}{(0.245)(s_{50})} \tag{17}$$

where, μ is a fuzzy logic, that measures the trueness of c_v [3, 4, 15]. Its value varies between 0 and 1. Closer is μ to 1, closer is c_v to the true c_v. The true c_v is the one that is devoid of the effect of secondary consolidation. The μ and c_v values determined by various methods for various soils are given in the Table 1.

Table 1. μ and c_v values determined by various methods for various soils.

Method	c_v (x 10^{-5} cm^2/sec)			
	SB dam soil	BC soil	Bentonite-sand mix	Bentonite
	w_L = 61% w_P = 29%	w_L = 69% w_P = 33%	w_L = 100% w_P = 30%	w_L = 495% w_P = 49%
Casagrande [25]	2.58	12.3	3.92	-
Taylor [26]	3.31	16.3	4.07	1.44
SRS [3, 15]	4.82	24.5	5.57	2.81
Fuzzy value, μ	1	0.97	0.94	0.92

2.2 Isolation of secondary consolidation

On semi-log plot in Fig. 2(b), unlike *U-T S*-curve, *U-(dU/dT) S*-curve is symmetrical about the mid-point, U_{50}. This property is used to isolate the secondary consolidation from the primary consolidation in Fig. 4. Up to 50%*U*, there is no (or insignificant) secondary consolidation in most of the inorganic soils. It is primary consolidation only. Usually after 50%*U* and definitely after 60%*U*, the secondary consolidation essentially starts and runs superposed over primary consolidation [3,10,15]. Therefore, the experimental curve deviates from theory after mid-point [usually 50-60%*U*]. So, the upper portion is retraced after 50%*U* (i.e. δ_{50}) that gives the theoretical experimental curve up to 100%*U*. The vertical difference between these two curves is the secondary consolidation in Figs 4 and 6. The secondary consolidation is defined as any other experimental compression that is not initial and primary compression [3, 4, 15, 27].

2.3 Six phases of consolidation settlement

Literature shows 3 phases of consolidation only. They are (i) initial compression, (ii) primary compression and (iii) secondary compression. The other phases could not be visualized because of the limitations of all available methods before the *SRS* approach. The *SRS* approach shows 6 phases of consolidation due to its high-resolution power. They are initial compression, first primary compression, transition from first primary compression to second primary compression, second primary compression, and transition from second primary compression to creep and lastly creep (Figs 4 to 7).

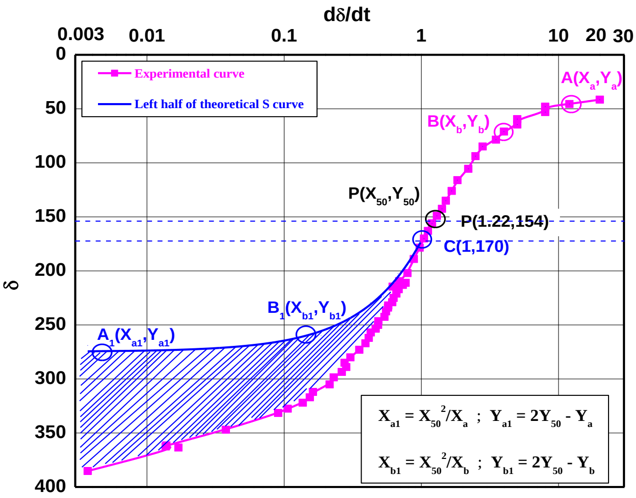

Fig. 4. δ versus *dδ/dt* semi-log plot for Sawan Bhado (SB) dam soil for isolation of secondary consolidation [3, 4, 15].

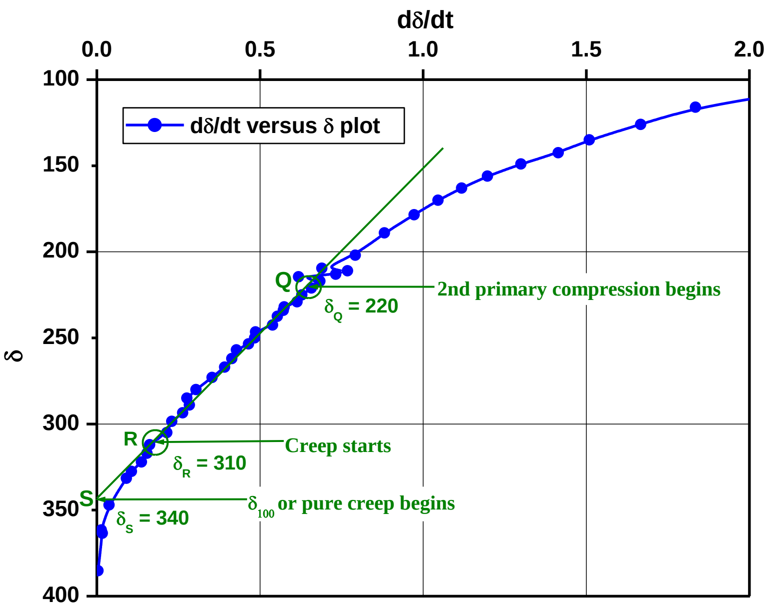

Fig. 5. δ versus *dδ/dt* linear plot for SB dam soil for finding c_v and δ_{100}

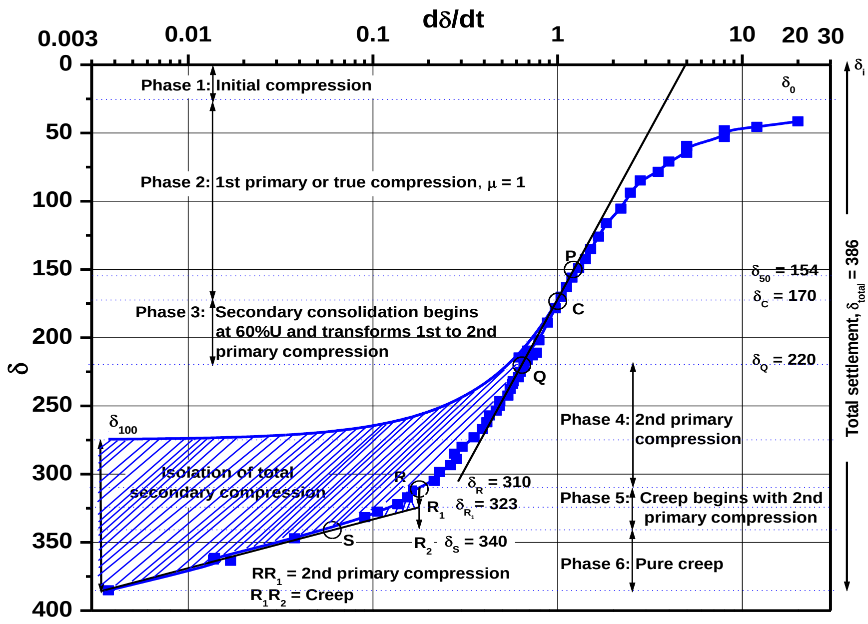

Fig. 6. Six phases of consolidation for SB dam soil.

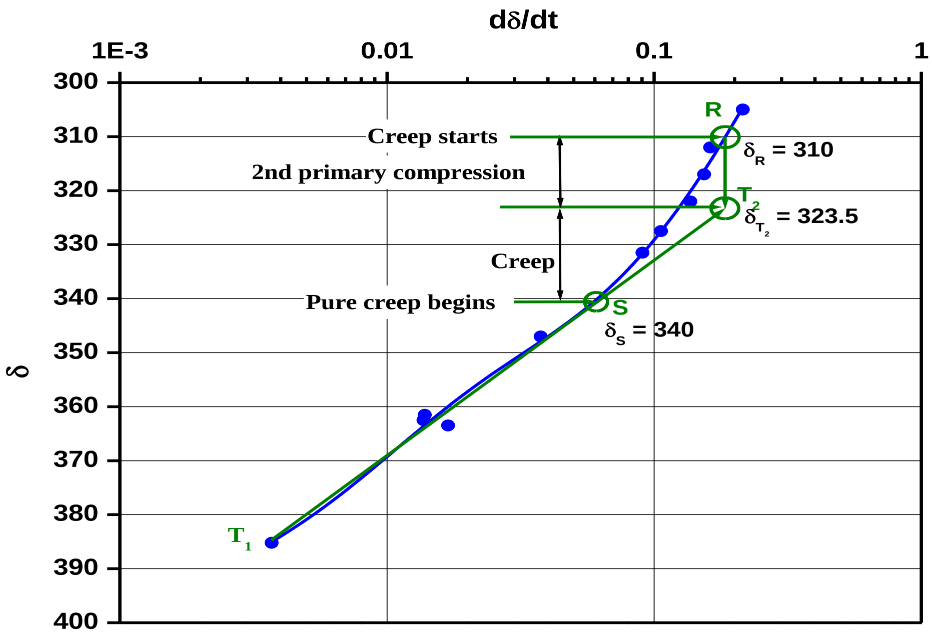

Fig. 7. δ versus $d\delta/dt$ semi-log plot for SB dam soil.

2.4 Isolation of creep

To isolate creep (Fig. 7), we draw a vertical line from *R* where creep starts. Extrapolate the straight line of later portion of the *S* curve. The point where vertical line cuts the extrapolated line is the point below which we get creep from the combination of 2nd primary compression and creep [15].

3 Limitations of *SRS* Approach

The plot of *SRS* has a weakness that it requires δ-t data to be recorded very precisely [28]. Unlike IS, ASTM or BS codes the settlement is recorded first at different intervals of time and later time in seconds is recorded when dial gauge needle coincides with the exact mark on the dial gauge [3, 4, 15]. The SB dam soil data were recorded like this, while other soils data were recorded as usual. Though this method works well in usual data recording system, but still the parabolic fitting of slope ($d\delta/dt$) can be used for better accuracy [28]. In this method, only three consecutive δ-t data points are fitted by a parabolic curve and the slope at 2nd point is taken as the $d\delta/dt$. However, now very precise instruments for measuring even less than a micron settlement are available, so the precise data recording is no longer a problem.

4 Discussion and Conclusions

The *δ-(dδ/dt)* method can determine c_v, δ_0, δ_{100} and (hence, using Eqs 3, 6 and 7) entire unknown *δ-t* data, when the time and settlement at the instant of load increment are not known, by observing data just for a few minutes (theoretically zero time or a point) as *δ-(dδ/dt)* plot (Fig. 5). It is not capable of giving some quantitative estimate to show how far the c_v is from the true c_v but a fuzzy logic. The method is capable of isolating not only creep but the whole secondary consolidation that runs superposed over the primary consolidation. The *δ-log(dδ/dt)* plot can give true c_v as well as all the six phases of consolidation. No other method is available in literature that determines c_v at less than 70%*U*. The *δ-log(dδ/dt) or S* curve plot requires data upto 50%*U* to determine the value of c_v. Thus, the *S* curve plot is the fastest rapid loading method for vertical consolidation that takes 1/4th time of Taylor method as $T_{50} \approx (T_{90})/4$.

References

1. Terzaghi, K.:Mathematisch-naturewissenschaftliche Klasse, Akademie der Wissenschaften in Wien, Sitzungsberichte, Part Iia132(3/4), 125-138 (1923).
2. Terzaghi, K.:Theoretical Soil Mechanics. John Wiley and Sons, Inc., New York, London (1943).
3. Tewatia, S.K.: Evaluation of true c_v and instantaneous c_v, and isolation of secondary consolidation. Geotechnical Testing Journal 21(2), 102-108 (1998).
4. Tewatia, S., Tewatia, K.: Consolidation of Soils: Rate of Settlement Approach. 1st edn. Lambert Academic Publishing, LAP, Riga Latvia (2019). https://www.morebooks.de/store/gb/book/consolidation-of-soils:-rate-of-settlement-approach/isbn/978-620-0-31301-0 https://www.researchgate.net/publication/336739313_CONSOLIDATION_OF_SOILS_RATE_OF_SETTLEMENT_APPROACH_RSA_A_Scholarly_book_on_the_State-of-the-Art
5. Tewatia, K., Tewatia, S.K., Rath, S.: Discussion on 'Evaluation of consolidation results by the settlement rate approach' by B.S. Olek, A. Borecka and H. Wozniak (2016). Electronic Journal of Geotechnical Engineering 21(07), 1-9 (2019).
6. Tewatia, K., Tewatia, A., Rath, S.: Discussion of 'Fastest rapid loading methods of vertical and radial consolidations' by S.K. Tewatia, Pratima Rani Bose, and Asuri Sridharan (2013). International Journal of Geomechanics 15(1), 1 (2015).
7. Tewatia, S.K.: Time Dependent Behavior of Clayey Soils. Ph.D. Thesis, Department of Civil Engineering, Delhi College of Engineering, Delhi University, India (2010).
8. Tewatia, S.K.: Trend of settlement in primary and secondary consolidations. Geomechanics and Geo-Engineering 8(2), 125-134 (2012).
9. Tewatia, S.K.: Equation of 3D consolidation in Cartesian co-ordinates. International Journal of Geotechnical Engineering, 7(1), 105-108 (2013).
10. Tewatia, S.K.: Principle of super position of rate of settlement in 2D and 3D consolidations. Geotechnical and Geological Engineering 33(5), 1369-1378 (2015).

11. Tewatia, S.K., Venkatachalam, K.: Closure for discussion on 'Improved √t method to evaluate consolidation test results'. Geotechnical Testing Journal 21(2), 155-156 (1998).
12. Tewatia, S.K., Venkatachalam, K., Sridharan, A.: T-Chart to evaluate consolidation test results. Geotechnical Testing Journal 21(3), 270-274 (1998).
13. Tewatia, S.K., Bhargav, D.N., Chouhan, G., Sridharan, A.: Velocity method for creep of clayey soils. Electronic Journal of Geotechnical Engineering 7(Bundle B), (2002).
14. Tewatia, S.K., Bose, P.R.: Discussion on 'A study on the beginning of secondary compression of soils' by R.G. Robinson. Journal of Testing and Evaluation 34(5), 1-6 (2006).
15. Tewatia, S.K., Bose, P.R., Sridharan, A., Rath, S.: Stress induced time dependent behavior of clayey soils. Geotechnical and Geological Engineering 25(2), 239-255 (2007).
16. Tewatia, S.K., Sridharan, A., Singh, M., Rath, S.: Theoretical equations of vertical and radial consolidations by equating degrees of consolidation by settlement analysis and dissipation of pore pressure. Geotechnical and Geological Engineering 30(4), 1037-1043 (2012).
17. Tewatia, S.K., Bose, P.R., Sridharan, A.: Fastest rapid loading methods of vertical and radial consolidations. International Journal of Geomechanics 13(4), 332-339 (2013). http://ascelibrary.org/doi/abs/10.1061/(ASCE)GM.1943-5622.0000213
18. Tewatia, S.K., Huat, B., Rath S., Tewatia, K.: U-chart for quick evaluation of radial and 3D consolidation test results. Ground Improvement 166(4), 188-195 (2013).
19. Tewatia, S.K., Bose, P.R., Sridharan, A.: Closure to 'Fastest rapid loading methods of vertical and radial consolidations' by S.K. Tewatia, P.R. Bose, and Asuri Sridharan. International Journal of Geomechanics 15(1), 1-3 (2015).
20. Fox, E.N.: In: Proceedings of 2nd International Conference on Soil Mechanics and Foundation Engineering, 1, pp. 41-42 (1948).
21. Al-Zoubi, M.S.: Settlement rate-settlement method for radial consolidation (SRSM-RC). Geotechnical Testing Journal 37(5), 907-916 (2014).
22. Al-Zoubi, M.S.: Consolidation analysis using the settlement rate-settlement (SRS) method. Applied Clay Science 50, 34-40 (2010).
23. McKinley, J.D., Sivakumar, V.: Coefficient of consolidation by plotting velocity against displacement. Géotechnique 59(6). 553-557 (2009).
24. Singh, S.K.: Confined aquifer parameters from temporal derivative of drawdowns. Journal of Hydraulic Engineering 127(6), 466-470 (2001).
25. Casagrande, A.: Notes on soil testing for engineering purposes. Soil Mechanics Series No. 8, Publication No. 268, Howard University, Cambridge, Mass (1939).
26. Taylor, D.W.: Fundamentals of soil mechanics. John Wiley and Sons Inc., New York (1948).
27. Olson, R.E.: State of the art: consolidation testing. In Young RN, Townsend FC (eds) Consolidation of Soils: Testing and Evaluation, ASTM STP 892, West Conshohocken, PA, 7-70 (1985).
28. Tewatia, S.K.: Discussion and Closure on 'Comparison of the hyperbolic and Asaoka observational method of monitoring consolidation with vertical drains' by Tan and Chew, 1996'. Soils and Foundations, Japanese Geotechnical Society 38(2), 224-227 (1998).