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Limit Equilibrium Analysis and Design of Geosynthetic-Reinforced Fill Walls under Special Conditions

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Abstract. Geosynthetics have been successfully used to reinforce fill for walls for different applications. Under certain circumstances, these walls may be designed and constructed under special conditions, for example, tiered walls, back-to-back walls, limited space fill walls, and fill walls with secondary reinforcement. Most design methods available in design standards or manuals based on lateral earth pressure theories cannot or must be modified to approximately handle such special conditions. Limit equilibrium methods have been commonly used to analyze and design slopes including reinforced slopes and can be used to analyze and design geosynthetic-reinforced fill walls under these special conditions. This paper describes the progressive failure of fill walls to slopes, presents the evidences of fill walls and slopes having similar failure modes, and justifies the use of the limit equilibrium approach to analyze and design geosynthetic-reinforced fill walls under special conditions. This paper also presents a top-down limit equilibrium design procedure and demonstrates the advantages of using this procedure for designing geosynthetic-reinforced fill walls under special conditions.

Keywords: Geosynthetic, limit equilibrium, slope, stability, wall.

1 Introduction

Geosynthetics have been successfully and increasingly used to reinforce fill for walls for different applications because of their easy construction, cost-effective, and sustainable advantages [9]. The term “fill” is used herein instead of “earth” because all geosynthetic-reinforced walls are fill walls instead of cut walls (e.g., soil nailed walls and anchored walls). The term “earth” is more appropriate when both fill and cut walls are discussed. In the practice, the terms “analysis” (or “analyse”) and “design” are sometimes used interchangeably, but they are different when we deal with geosynthetic-reinforced fill walls. Analysis is referred to the calculation of a performance index (mostly the factor of safety, FoS) performed based on assumed or known layout and tensile strength of geosynthetic reinforcement in a wall while design is referred to the calculation performed to determine the layout and/or tensile strength of geosynthetic reinforcement. Both analysis and design of geosynthetic-reinforced fill walls will be discussed in this paper.

In the literature, walls and slopes are defined based on an inclination angle of 70° (also called a slope angle) from the horizontal line. It is referred to as a wall when the inclination angle is equal or larger than 70°; otherwise, it is considered as a slope. Unfortunately, most design standards and manuals specify different design methods for geosynthetic-reinforced fill walls and slopes. In other words, reinforced fill walls are often designed based on the lateral earth pressure theory, e.g., Rankine or Coulomb's lateral earth pressure theory while reinforced fill slopes are often designed based on the limit equilibrium theory, e.g., Bishop or Spencer's slip surface approach [5, 25]. Actually, both Rankine and Coulomb's lateral earth pressure theories were also developed based on limit equilibrium. However, the linearly increasing lateral earth pressure distribution is an assumption. In this paper, the method based on Rankine or Coulomb's lateral earth pressure theory is referred to as the lateral earth pressure method while the method based on Bishop or Spencer's slip surface method is referred to as the limit equilibrium method. Since geosynthetic-reinforced fill walls are considered relatively flexible, active earth pressures and their corresponding failure plane are often assumed in the lateral earth pressure method. As illustrated by Han [8], the current design method for a geosynthetic-reinforced fill wall assumes a linearly increasing lateral earth pressure based on the Rankine or Coulomb theory (Figure 1a). Due to this linearly increasing lateral earth pressure distribution, the required tensile strength of the geosynthetic reinforcement also linearly increases with the depth (i.e., $T_1 < T_2 < T_3$) if reinforcement spacing is equal. Based on the limit equilibrium method, however, the required tensile strength of geosynthetic reinforcement in a reinforced slope does not change with depth (i.e., $T_1 = T_2 = T_3$) if reinforcement pullout does not control. This comparison demonstrates that these two design concepts result in different magnitudes and distributions of tensile forces for geosynthetics at different elevations for reinforced fill walls and slopes thus requiring different reinforcement strengths and/or lengths. These differences cannot be justified especially when a reinforced fill retaining structure (wall or slope) has a slope angle of 70° and is analysed by both approaches. As demonstrated later in this paper, failure of a fill retaining structure is progressive and a fill wall can become a fill slope if deformation and failure are allowed; therefore, fill wall and fill slope can be considered as the same fill structure at different stages.

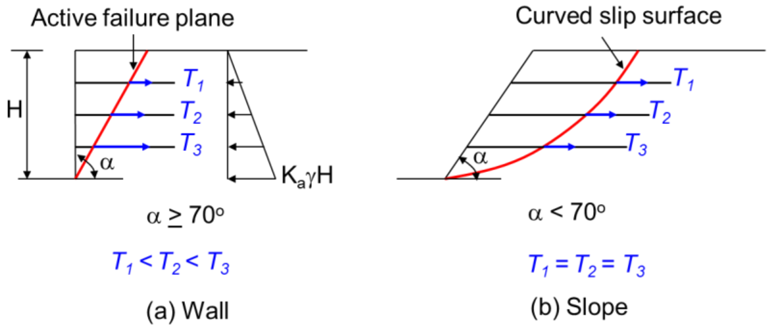


Fig. 1. Reinforced fill wall versus slope (modified from Han [8])

Leshchinsky et al. [21] identified this issue and proposed a unified design approach to geosynthetic-reinforced slopes and segmental walls. In their approach, they analysed a series of log spiral slip surfaces through the toes of the walls and slopes to determine the distribution of mobilized friction angles required to reach the limit equilibrium state for each slip surface. Han and Leshchinsky [11] developed a general analytical framework for design of flexible reinforced fill structures, in which planar failure surfaces were used to determine the distribution of required reinforcement tensile strengths along each reinforcement length using a top-down approach. Instead of planar failure surfaces used in Han and Leshchinsky [11], Leshchinsky et al. [19] and Leshchinsky et al. [20] improved the framework by using log spiral failure surfaces and circular failure surfaces, respectively. This design framework will be presented later.

The lateral earth pressure approach based on Rankine or Coulomb's theory included in many design standards and manuals can be used to analyse and design single nearly vertical fill walls with sufficiently wide fill to allow full development of the active earth pressure failure plane. However, geosynthetic-reinforced fill walls may be constructed under special conditions: (1) tiered, (2) back-to-back, (3) limited space, and (4) with secondary reinforcement as shown in Figure 2, which do not satisfy the condition required for the lateral earth pressure approach. Some manuals proposed modifications of the typical design methods to consider these special conditions; however, these methods are not well verified. This paper presents the past studies done by the author and others to examine these methods using the limit equilibrium method.

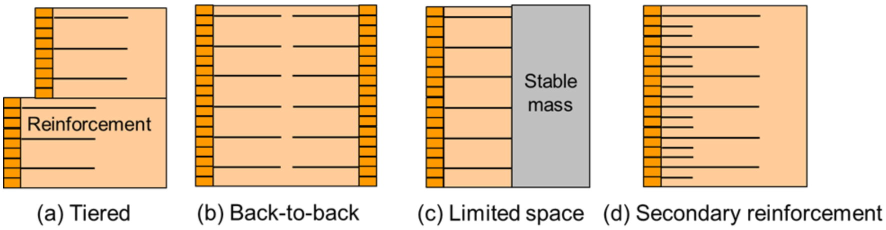


Fig. 2. Geosynthetic-reinforced fill walls under special conditions.

Limit equilibrium of fill walls can be analysed by an analytical method assuming failure surfaces and ensuring force equilibrium (e.g., the Bishop or Spencer method) or a continuum mechanics-based numerical method. A numerical method based on continuum mechanics uses a strength reduction approach to obtain a limit equilibrium state. Both methods can calculate a minimum factor of safety (FoS) and identify a critical failure surface for a fill wall. Han and Leshchinsky [10] found that the analytical limit equilibrium method and the continuum mechanics-based numerical method resulted in similar factors of safety and critical slip surfaces for typical geosynthetic-reinforced fill walls and fill slopes.

This paper describes the progressive failure of fill walls to slopes, presents the evidences of fill walls and slopes having similar failure modes, and justifies the use of the limit equilibrium approach to analyse and design geosynthetic-reinforced fill walls, especially under special conditions. This paper also presents a top-down limit equilibrium design procedure and demonstrates the use of this procedure for designing geosynthetic-reinforced fill walls under special conditions.

2 Progressive Failure and Critical Surfaces

Figure 3 shows the progressive movement and failure of a vertical fill wall to a flat slope with critical surfaces, which depend on lateral support and vertical load. When the lateral load, P, applied on the wall facing is equal to the overburden weight of the fill multiplied by its coefficient of lateral earth pressure at rest (i.e., K_o), the fill wall does not have any movement and no critical surface is formed. This state is also called the K_o state and this load is referred to as P_0 in this paper. When the lateral load, P, gradually decreases from P_o to P_1, the fill wall facing starts to move laterally and potential slip surfaces develop. At a limited movement, a critical surface (i.e., O-1) approximated by a bi-linear line with a top width of 0.3H (H is the height of the fill wall) may be formed. This critical surface is commonly used to design reinforced fill walls with metallic reinforcement (i.e., inextensible reinforcement) because of its high tensile stiffness [4]. When the lateral load, P, decreases to the load, P_2, which is equivalent to the active lateral earth pressure P_a, the fill wall facing has more movement and an active failure plane (i.e., O-2) with an angle of ψ (equal to $45^o+\phi/2$ based on the Rankine theory, ϕ is the frictional angle of the fill) is formed as shown in Figure 3. This failure plane is commonly used to design geosynthetic-reinforced fill walls using the lateral earth pressure method because geosynthetics are considered as extensible reinforcement. This state is also called the active state that is at limit equilibrium. Considering the friction angle of a typical fill material ranging from 30^o to 50^o, the slope angle of the Rankine failure plane ranges from 60^o to 70^o. Therefore, a fill structure with a slope angle less than 70^o may not have this failure plane so that the lateral earth pressure method cannot be used. This may be the reason why 70^o is commonly used to divide walls and slopes. However, if the lateral load, P, continues decreasing to P_2 (i.e., lower than P_a), the wedge would lose its stability and slide down along the critical surface, O-2. Immediately after the wedge slides down, the remaining fill becomes a steep slope and a lateral load, P_3, lower than P_a may be enough to maintain the stability of this steep slope. However, when P_3 continues decreasing to zero, this steep slope is not stable because the slope angle, ψ is still greater the fill friction angle, ϕ; therefore, another slip plane (O-3) at an angle of ϕ is formed and the soil wedge above this slip plane slides down. The steep slope eventually becomes a flat slope at another limit equilibrium. The slope angle corresponding to this limit equilibrium is often called the repose angle. Figure 3 also shows that the top distances from the wall facing to three typical critical surfaces change from 0.3H, $H/\tan\psi$, to $H/\tan\phi$. If a typical fill friction angle of 34^o is assumed as suggested by Berg et al. [4] as a default value, the above top distances become 0.3H, 0.5H, and 1.5H, respectively. These distances will be used to interpret some numerical or experimental results in later sections.

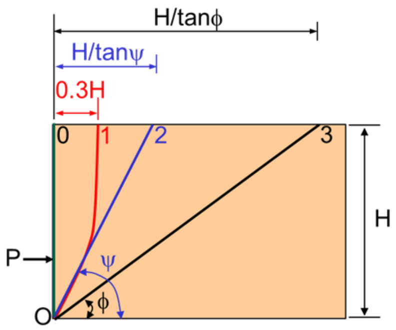


Fig. 3. Progressive failure of fill walls to slopes

The above progressive failure shows the gradual transition from a fill wall to a fill slope if the lateral load, P, is not sufficiently maintained. The required lateral load to maintain the stability of the fill wall to the fill slope (e.g., the use of geosynthetic reinforcement) decreases. The influence distance on the top of the wall can range from 0 to 1.5H if the fill friction angle is 34°.

3 Limit Equilibrium Analysis

3.1 Typical fill walls and slopes

Both analytical and numerical methods can be used to analyse typical fill walls and slopes and identify critical failure surfaces within the fill. The analytical method (e.g., Bishop's method and Spencer's method) determines its lowest factor of safety (FoS) among all the factors of safety corresponding to a large number of assumed slip surfaces analysed and identify the critical failure surface corresponding to the lowest factor of safety. The numerical method based on the continuum mechanics determines the factors of safety of fill walls and slopes using a strength reduction method, in which a factored fill strength is assumed by the numerical software and the fill strength is reduced gradually by a factor until a limit equilibrium is reached [10]. The factor corresponding to the limit equilibrium state is the factor of safety of the fill wall or fill slope. The "potential" critical slip surface is often identified based on the zone of high shear strain rates.

Figure 4 shows the comparison of the critical slip surfaces identified by the Bishop and numerical (finite difference) methods for the geosynthetic-reinforced fill walls (vertical) and slopes (70° slope angle) at the limit equilibrium state (i.e., FoS = 1.0). In both analyses, the fill friction angle of 34° was used. Figure 4(a) also includes the Rankine active failure plane within the fill wall for the comparison. Figure 4(a) shows that the critical slip surface within the wall at the limit equilibrium state identified by the numerical method is approximately linear and close to the Rankine active failure plane; however, the critical slip surface identified by the Bishop method is circular due to its assumption and only matches the slip surface by the numerical method within the lower portion. This comparison indicates that the limit equilibrium method based on the Bishop method can accurately predict the FoS but approximately capture the critical surface for the fill wall due to the limitation of the circular slip surface assumption. Mohamed et al. [22] compared the critical surfaces within the geosynthetic-reinforced vertical fill walls identified by the Bishop method, the Spencer method, the lateral earth pressure method (Rankine theory), and the centrifuge test and concluded that non-circular surfaces based on the Spencer method well captured the failure surfaces identified by the centrifuge test.

Figure 4(b) shows the comparison of the critical slip surface within the fill slope identified by the Bishop method and the critical zone of the high shear strain rates generated by the numerical method at the limit equilibrium state (i.e., FoS = 1.0). Figure 4(b) clearly shows the critical zone generated by the numerical method was curved and the Bishop method well captured the curved critical zone by the critical slip surface. This comparison demonstrates that the Bishop method can not only accurately predict the FoS but also well capture the critical surface for the fill slope.

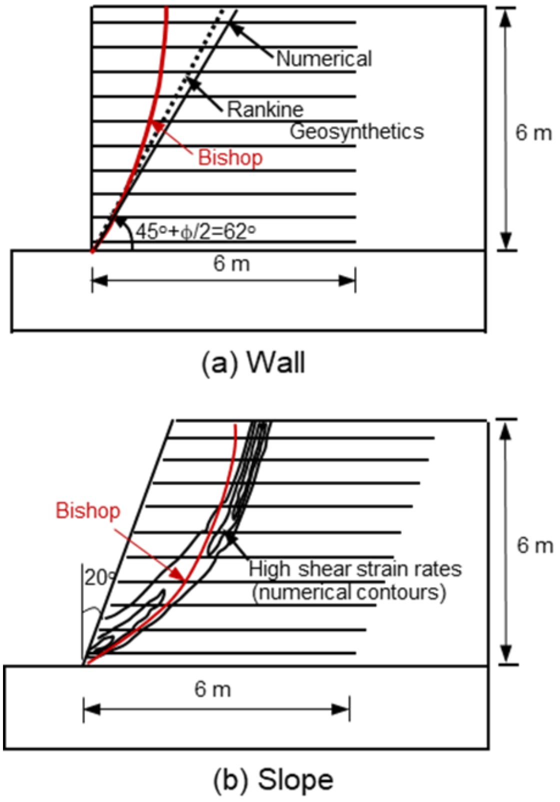


Fig. 4. Critical slip surfaces in wall and slope at the limit equilibrium state (modified from Han and Leshchinsky [10])

Yang et al. [27] evaluated different methods available in the literature for calculating the maximum of maximum tensile loads for the stability of the 3.6-m tall wrapped-face geosynthetic-reinforced fill wall with a slope angle of 82° tested by Bathurst et al. [2] at various surcharge levels. Figure 5 shows that the limit equilibrium method predicted the loads close to those by the finite element method and closer to the measured ones than the lateral earth pressure approaches (Rankine and Coulomb) and the K-stiffness method proposed by Bathurst et al. [1].

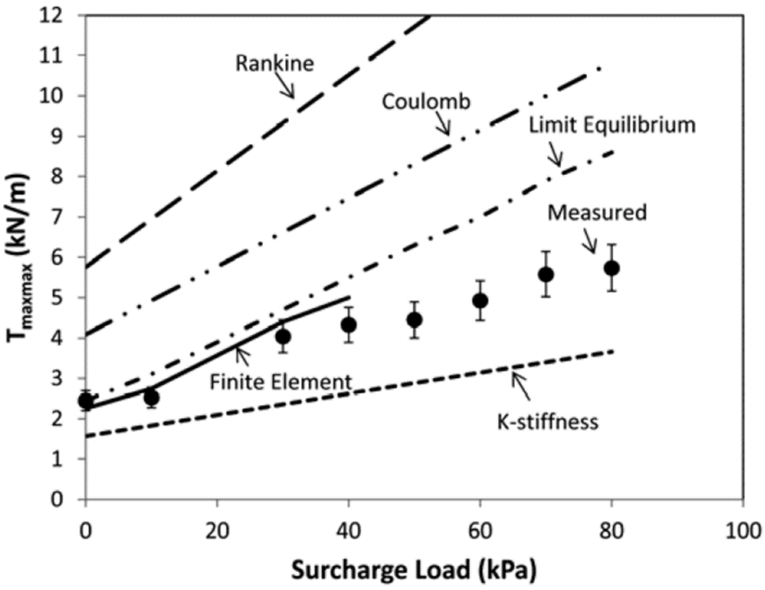


Fig. 5. Calculated maximum of maximum reinforcement load among all reinforcement layers at various surcharge levels [27]

The above discussions show that the limit equilibrium method can be used to analyse both geosynthetic-reinforced fill walls and slopes, calculate similar factors of safety or required similar tensile strengths of reinforcement, and identify similar slip surfaces as the numerical method based on the continuum mechanics or the experimental tests. This finding demonstrates that the limit equilibrium method can be used as a unified method for analysing fill walls and slopes without the separation between walls and slopes by the slope angle of 70°.

3.2 Tiered walls

Geosynthetic-reinforced tiered walls as shown in Figure 6 are often used in the practice when the wall height is relatively high and there is sufficient space to create offsets. The benefit of tiered walls is to reduce the required tensile strength for geosynthetic reinforcement. Tiered walls are often an alternative to steep slopes. New Civil Engineer [24] reported the 74-m tall fill structure constructed with the combination of geosynthetic-reinforced tiered walls and steep slopes for the Sikkim airport. The primary reinforcement had vertical spacing up to 2 m and high-strength geogrid with its tensile strength up to 800 kN/m. Tiered walls are considered as fill structures between walls and slopes and a special condition for fill walls. Berg et al. [4] modified the method for geosynthetic-reinforced fill walls by considering the offset distance, D, between upper and lower tiers for three cases as shown in Figure 5: (1) when $D \leq H_2\tan(45^{o}-\phi/2)$ (same as $H_2/\tan\psi$ as discussed earlier) (referred to as the lower bound), treat the upper tier as a uniform surcharge on the lower tier, (2) when $D > H_2\tan(90^{0}-\phi)$ (same as $H_2/\tan\phi$) (referred to as the upper bound), treat two tiers independently, and (3) when $H_2\tan(45^{0}-\phi/2) < D \leq H_2\tan(90^{0}-\phi)$, treat it between the previous two cases by distributing the upper tier load onto each reinforcement in the lower tier. Clearly, the lower and upper bounds were suggested based on the two critical surfaces discussed earlier.

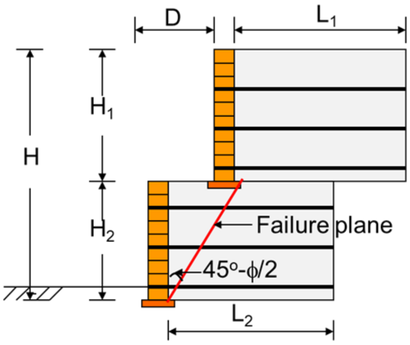


Fig. 6. Geometry and definitions of geosynthetic-reinforced tiered fill wall

Leshchinsky and Han [17] conducted a comprehensive study on the stability of tiered walls by investigating eight influence factors using the numerical (finite difference) and limit equilibrium (Bishop's) methods. The investigated influence factors are: (1) fill quality, (2) reinforcement length, (3) reinforcement stiffness, (4) reinforcement type, (5) foundation soil, (6) water, (7) surcharge, and (8) number of tiers. Leshchinsky and Han [19] set up the condition for all the cases with FoS = 1.0 (i.e., limit equilibrium) by selecting an appropriate tensile strength of reinforcement using the Bishop method and then analysed them using the numerical method. Except for one case controlled by the bearing capacity of the foundation soil in the numerical analysis, all other cases had the same FoS calculated by the numerical and limit equilibrium methods, demonstrating that the limit equilibrium method can analyse the stability of tiered walls except for the condition controlled by foundation bearing capacity. Figure 7 shows that the critical slip surfaces within three-tiered walls at the limit equilibrium identified by the numerical method and the Bishop method match well.

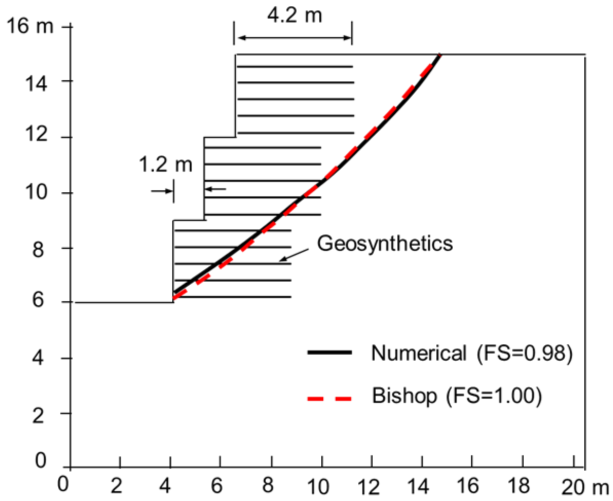


Fig. 7. Critical slip surfaces in tiered walls (modified from Leshchinsky and Han [17])

Leshchinsky and Han [17] investigated the effect of the offset distance between tiers on the required tensile strength of the reinforcement to maintain the limit equilibrium state and examined this effect as compared with the recommendations by Berg et al. [4] as shown in Figure 8. Figure 8 shows that the increase of the offset distance reduced the required tensile strength of the reinforcement. An increase from one tier to three tiers increased the required tensile strength of the reinforcement. Figure 8 shows that the tiered walls become independently at a certain offset distance. Using three tiers as an example, the required tensile strengths calculated based on the recommendations by Berg et al. [4] are higher than those by the limit equilibrium method. The lower and upper bounds suggested by Berg et al. [4] are much higher than those determined by the limit equilibrium method. The offset distance determined by the limit equilibrium method was $0.8H_2$, which is between the lower and upper bounds and close to $H_2/\tan(\psi/2+\phi/2)$. The centrifuge test results from Mohamed et al. [22] show the similar comparison with those based on the recommendations by Berg et al. [4].

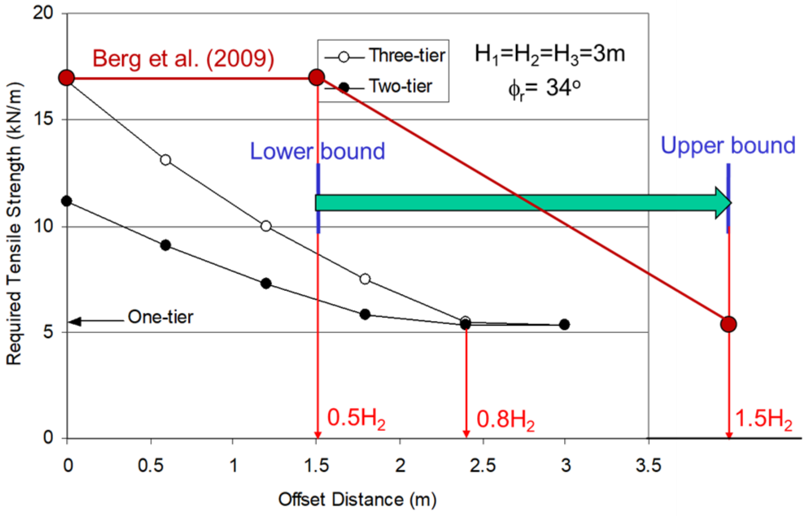


Fig. 8. Effect of offset distance on required tensile strength of reinforcement (modified from Leshchinsky and Han [17])

3.3 Back-to-Back fill walls

Geosynthetic-reinforced back-to-back fill walls as shown in Figure 9 are often used as bridge abutments for approaching embankments to bridges. Depending on the width of the wall, the geosynthetic reinforcement may have a gap, meet, or overlap at the middle of the wall. Similar to the tiered walls, the offset distance between two walls, D_b, is an important design parameter. Berg et al. [4] suggested the modification of the typical design by considering the following three cases: (1) when $D_b > H\tan(45^o – \phi/2)$, treated as two independent walls with full active thrust mobilized, (2) $D_b = 0$ and reinforcement overlap length at the middle $L_R > 0.3H_2$, no active thrust, and (3) $0 < D_b < H\tan(45^o - \phi/2)$, consider reduction of active thrust (linearly interpolated from the full active case to zero).

Han and Leshchinsky [12] conducted numerical and limit equilibrium analyses of geosynthetic-reinforced back-to-back fill walls under different wall width and fill quality. For each numerical or limit equilibrium analysis, the tensile strength of the reinforcement was appropriately selected to reach the limit equilibrium state (i.e., FoS = 1.0) and the critical surface was identified. In this study, the Spencer method was used as well for the comparison purpose.

Figure 10 shows the critical surfaces in the back-to-back fill walls with a wall width to height ratio of 3.0 obtained by Han and Leshchinsky [12] as an example. Since the limit equilibrium method can only analyse slope stability in one side, the critical slip surfaces from the Bishop and Spencer methods are only presented on the left side of the wall. As a result, the limit equilibrium method cannot analyse the interaction between two sides of the wall. This is one of the limitations of the limit equilibrium method for analysing geosynthetic-reinforced fill walls. Similar to what was discussed earlier, the circular slip surface assumption of the Bishop method could not predict accurately the critical surface for a vertical wall as compared with the Spencer and numerical methods. Figure 10 also shows that the wall with a lower quality of fill (i.e., smaller friction angle) had a flatter critical surface.

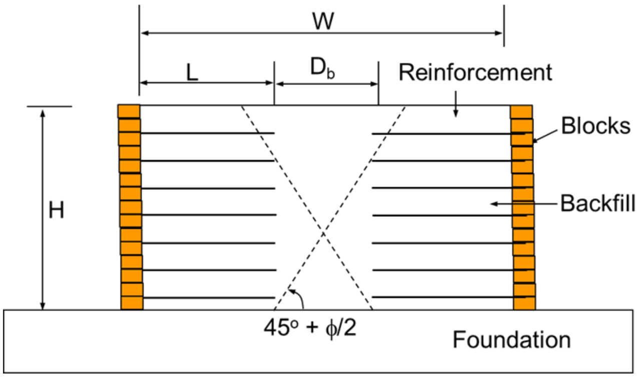


Fig. 9. Geometry and definitions of geosynthetic-reinforced back-to-back fill wall

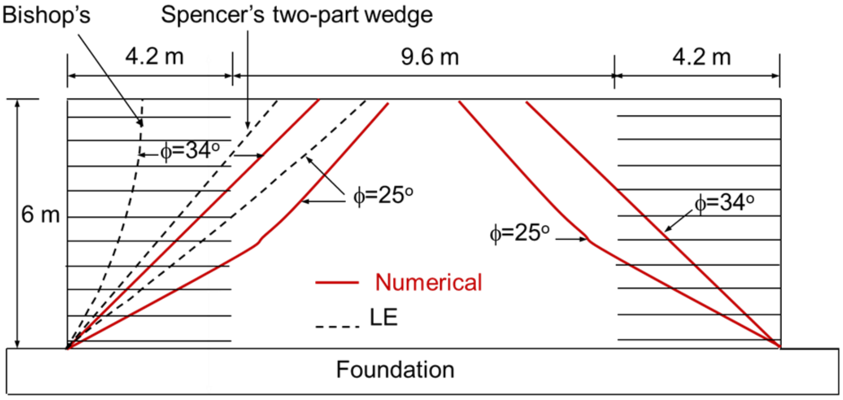


Fig. 10. Critical surfaces in the back-to-back fill walls with a wall width to height ratio of 3.0 (modified from Han and Leshchinsky [12])

Figure 11 presents the required tensile strengths of reinforcement to each limit equilibrium state at different wall width to height ratios for two types of fill quality. It is reasonable that the high-quality fill (i.e., ϕ = 34°) resulted in much lower required tensile strengths than the low-quality fill. The tensile strength from the limit equilibrium method was independent of the offset distance because the limit equilibrium method could not consider the interaction between two walls. The numerical results show that the interaction between two walls reduced the required tensile strength of the reinforcement. In other words, the limit equilibrium method is conservative in predicting the maximum required tensile strength of the reinforcement. Figure 11 also shows that the limits for no interaction for both types of fill recommended by Berg et al. [4] are much smaller than those indicated by the numerical method; however, these limits yield a conservative result in terms of the required maximum tensile strength of the reinforcement.

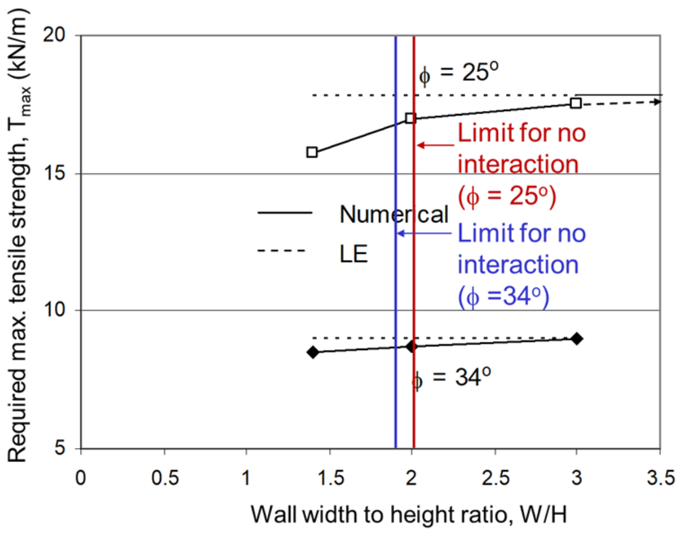


Fig. 11. Required maximum tensile strengths of reinforcement at different wall width to height ratios (modified from Han and Leshchinsky [12])

Based on the numerical results, Han and Leshchinsky [12] found that the lateral earth pressure behind the reinforced fill decreased with the reduction of the offset distance between two walls. However, still 85% and 70% lateral earth thrust existed behind the reinforced fill for high-quality (ϕ=34°) and low-quality (ϕ=25°) fill, respectively even when the offset distance was equal to zero. El-Sherbiny et al. [6] shows a similar result. Leshchinsky and Han [12] also showed that at the offset distance equal to zero, connecting the reinforcement layers from two sides reduced the required maximum tensile strength as compared with that without any connection. The numerical (finite element method) result from Benmebarek et al. [3] also showed that overlapping the reinforcement layers at the middle by approximately 0.25H was equivalent to a continuous reinforcement.

3.4 Limited space fill walls

Geosynthetic-reinforced limited space fill walls (also called narrow or shored walls) have been increasingly used and researched in the recent years (e.g., Leshchinsky et al. [18]; Morrison et al. [23]; Yang et al. [26]; Lawson et al. [16]; Kakrasul et al. [15]). This type of wall is used in front of a stable medium (e.g., bedrock, anchored wall, or nailed wall). Berg et al. [4] required that a geosynthetic-reinforced fill wall should have a reinforcement length not shorter than 0.7 times the wall height, H.

When the available space behind the wall facing cannot accommodate this minimum reinforcement requirement, it is considered limited space. However, Kakrasul et al. [15] found from their experimental tests that only when the available space behind the wall facing was less than 0.5H, the geosynthetic-reinforced fill wall had much larger lateral deformations and a much lower load capacity to support a footing than the wall with a reinforcement length of 0.7H.

Leshchinsky et al. [18] conducted limit equilibrium and numerical analyses of geosynthetic-reinforced limited space fill walls as shown in Figure 12. One reinforcement was used in this study to simplify the analyses. This study investigated three key influence factors: the base width, B, the stable medium slope (m:1), and the fill friction angle, ϕ. The Bishop method was first used to determine the required tensile strength to maintain the limit equilibrium (FoS = 1.0) for each case and then the same values were input into the finite difference software to calculate the FoS for each case. Leshchinsky et al. [18] showed that the FoS values calculated by the numerical method ranged from 0.96 to 1.06, practically the same as 1.0 as calculated by the limit equilibrium method.

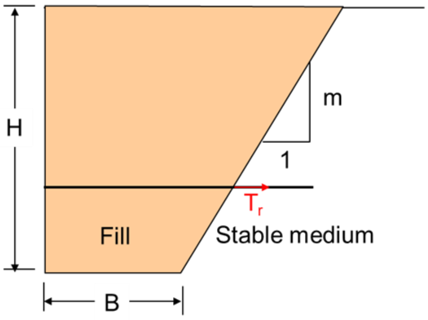


Fig. 12. Geometry and definitions of geosynthetic-reinforced limited space fill wall

Based on the required tensile strength of the reinforcement, T_r, calculated by the limit equilibrium method, the modified coefficient of lateral earth pressure, K_a', following the Rankine earth pressure distribution assumption and considering the limited space was calculated by $2T_r/(\gamma H^2)$ (γ = unit weight of fill). Figure 12 shows that the back-calculated modified coefficient ratio of lateral earth pressure, K_a'/K_a (K_a = coefficient of Rankine's active earth pressure), matched well with the centrifuge tests obtained by Frydman and Keissar [7]. Figure 12 also shows that when H/B = 1.0, K_a'/K_a = 1.0; in other words, the modified coefficient of lateral earth pressure, K_a' was equal to the coefficient of Rankine's active earth pressure, K_a, indicating the available space did not have any effect on the lateral earth pressure. However, when H/B increased, the K_a'/K_a ratio decreased; in other words, the limited space reduced the lateral earth pressure behind the wall facing. Lawson et al. [16] found that the effect of the limited space depended on the fill friction angle. When the fill friction angle is large, the required space to have a limited space effect on the lateral earth pressure becomes smaller.

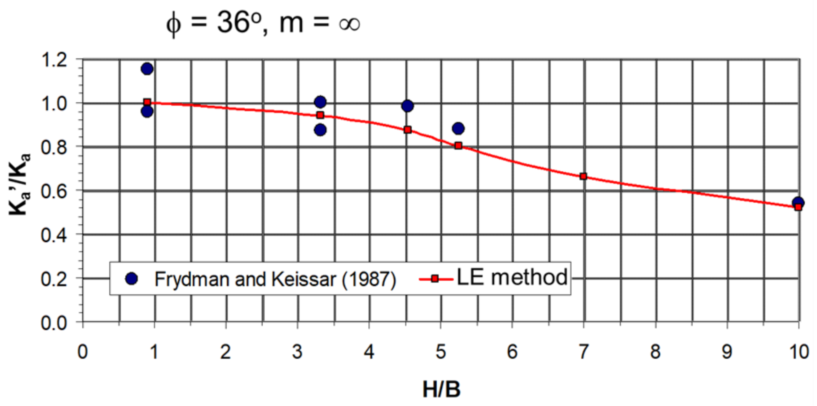


Fig. 13. Back-calculated lateral earth pressure coefficients by the limit equilibrium method versus centrifugal test results [18]

3.5 Fill walls with secondary reinforcement

When a geosynthetic-reinforced fill wall gets taller, the force at the connection between reinforcement and facing units becomes larger. At the same time, due to the use of a lighter compactor and the difficulty in compaction near wall facing, the density of the fill close to the facing is often lower. Therefore, possible connection failure and wall facing deformation may develop. To overcome this problem, secondary reinforcement has been proposed to be placed between primary reinforcements to reduce the connection force and the wall facing deformation as shown in Figure 14. Jiang et al. [13, 14] performed a field study and a numerical analysis to evaluate the performance of the geosynthetic-reinforced fill wall with secondary reinforcement. Both field and numerical studies show that the use of secondary reinforcement reduced the connection force between reinforcement and facing units. So far, there is no well-established analytical method to analyse or design geosynthetic-reinforced fill walls with secondary reinforcement except for the limit equilibrium method, which will be discussed in next section.

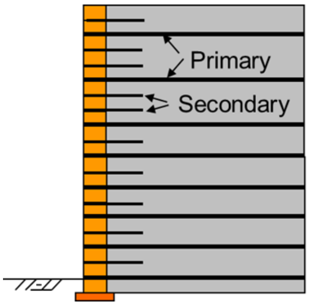


Fig. 14. Illustration of geosynthetic-reinforced fill wall with secondary reinforcement

4 Limit Equilibrium Design Approach

The preceding section verified that geosynthetic-reinforced fill walls under normal and special conditions can be analysed using the limit equilibrium and numerical methods. These methods require a known cross section and geosynthetic layout as well as a trial and error process to determine the required tensile strength of reinforcement to achieve a required factor of safety. This procedure is not convenient for design. This section presents a limit equilibrium design approach for geosynthetic-reinforced fill walls under normal and special conditions. This approach has more advantages than other methods for dealing with geosynthetic-reinforced fill walls under special conditions.

4.1 Required tensile strength of reinforcement

Based on limit equilibrium, Han and Leshchinksy [11] proposed the follow formula to determine the required tensile strength, T_{RI}, for a reinforcement to maintain the stability of the wedge at the limit equivalent state (i.e., FoS = 1.0) by assuming a planar slip surface as shown in Figure 15:

$$T_{RI} = \frac{\gamma H_1^2 \left(\sin\theta_I - \tan\phi\cos\theta_I\right)}{2\tan\theta_I\left(\tan\phi\sin\theta_I + \cos\theta_I\right)} \tag{1}$$

where γ = unit weight of the fill, H_1 = height of the wedge, ϕ = friction angle of the fill, and θ_1 = inclination of the failure plane. In Figure 15, W_1 is the weight of the wedge, N is the normal force applied on the failure plane, Q is the shear force applied on the failure plane, and T_{R1} is the required tensile force or strength of the reinforcement at the intersection between the reinforcement and the failure plane.

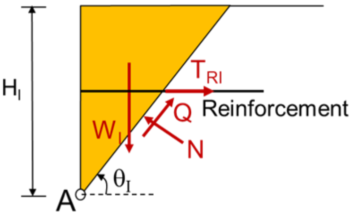


Fig. 15. Force equilibrium of a wedge with one reinforcement layer

When a fill wall is designed for a specific FoS, the friction angle in Equation (1) can be replaced by a mobilized friction angle as follows:

$$\phi_m = tan^{-1}\left[tan\left(\frac{\phi}{FoS}\right)\right] \tag{2}$$

4.2 Required tensile strength distribution along reinforcement

To ensure the upper portion of the fill wall with a height, H_1, at the limit equilibrium state, different possible failure planes should be examined and determine the required tensile strength for the reinforcement at the interaction between the reinforcement and the failure plane for each reinforcement using Equation (1). As a result, the required tensile strength distribution with different inclination angles, θ_I can be obtained as shown in Figure 16. Figure 16 shows no tensile strength is needed from the reinforcement at the failure plane angle smaller than a certain value because the fill itself is strong enough to maintain the stability of the wedge with this failure plane. As a result, no further calculation is needed for flatter failure planes.

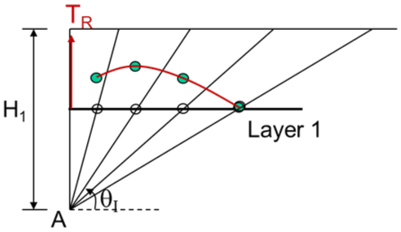


Fig. 16. Distribution of required tensile resistance in Layer 1 to satisfy limit equilibrium for fill height, H_1

When the fill wall has two reinforcement layers, the total required tensile strength calculated using Equation (1) with the wall height, H_2, for each slip surface should be equally divided for each reinforcement layer as shown in Figure 17. This is the basic principle of the limit equilibrium method with a potential slip surface.

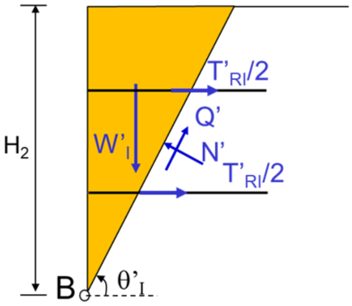


Fig. 17. Force equilibrium of a wedge with two reinforcement layers

Figure 18 shows the equal distribution of the total required tensile strength to both reinforcement layers.

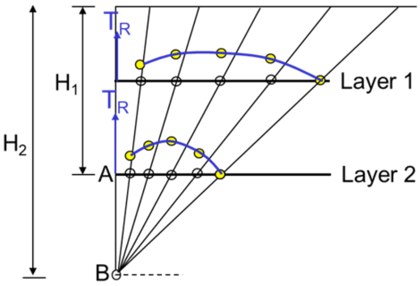
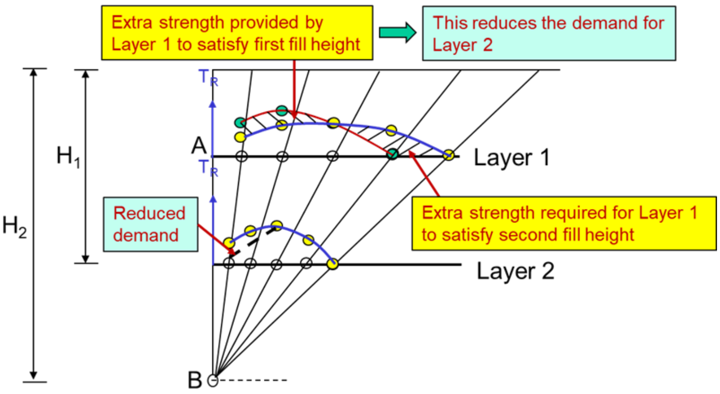


Fig. 18. Distributions of the total required tensile strength to Layers 1 and 2 to satisfy limit equilibrium for the fill height, H_2

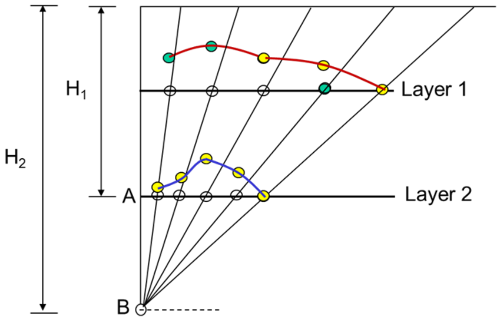
Figure 19(a) shows the required tensile strength distributions in Layers 1 and 2 to satisfy limit equilibrium for the fill heights, H_1 and H_2, respectively. To maintain the limit equilibrium of the fill wall at both heights, H_1 and H_2 at the same time, Layer 1 in the front (i.e., close to the facing) should provide a higher tensile strength to satisfy limit equilibrium for the first fill height, H_1 as shown in Figure 19(a). At the same time, Layer 1 in the rear should provide additional strength to satisfy limit equilibrium for the second fill height, H_2. Since Layer 1 provides a higher strength in the front, the demand for Layer 2 in the front should be reduced to satisfy force equilibrium. Figure 9(b) shows the adjusted, required tensile strengths in Layers 1 and 2 to maintain limit equilibrium of the fill wall at both heights, H_1 and H_2 at the same time. Since this design method starts the analysis of equilibrium from the top reinforcement layer to the bottom reinforcement layer, it is referred to as the top-down limit equilibrium design method. The design procedure continues until the tensile strength distributions of all layers are determined.

4.3 Required reinforcement design strength and length

The above procedure results in the minimum requirement for the required tensile strengths along each reinforcement. For practical applications, the required design tensile strength for each reinforcement should consider the allowable tensile strength of reinforcement material, the allowable connection strength in the front, and the allowable pullout capacity in the rear as shown in Figure 20. The design should consider the required connection strength between each reinforcement and wall facing units to ensure the local stability of wall facing. The required minimum connection strength can be determined by drawing a tangential line to the tensile strength demand envelope at Point A with a slope same as that for the pullout capacity line. The design should also require sufficiently long reinforcement to ensure the rear has sufficient tensile strength (depending on pullout capacity) to satisfy the limit equilibrium condition.



(a) Overlapped



(b) Adjusted

Fig. 19. Distributions of required tensile resistance in Layers 1 and 2 to satisfy limit equilibrium for both fill heights, H_1 and H_2

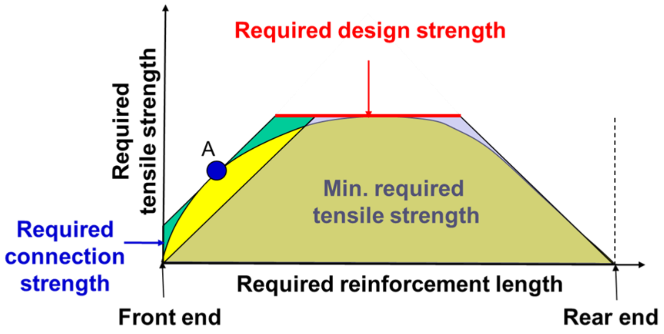


Fig. 20. Required reinforcement design strength and length (modified from Leshchinsky et al. [19])

Even though the above-discussed design framework is based on a planar failure surface, the design framework is also suitable for the circular (Bishop), log spiral, bilinear, or three-part wedge (Spencer) method as shown in Figure 21 where the log spiral failure surface was used for the design framework for geosynthetic-reinforced fill walls [19].

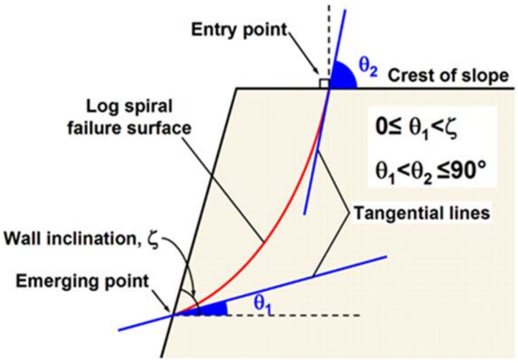


Fig. 21. Log spiral failure surface [19].

This design framework has been included in the US Federal Highway Administration (FHWA) design document entitled “Limit equilibrium design framework for MSE structures with extensible reinforcement” by Leshchinsky et al. [20].

4.4 Limit equilibrium design for special conditions

This design framework requires searching and adjustment of the required tensile strength at each location along the reinforcement from a top-down process; therefore, it involves extensive computation. Han and Leshchinsky [11] implemented the design method through the Microsoft Excel by assuming planar potential failure surfaces. However, when curved potential failure surfaces (i.e., log-spiral or circular) or two-part/three-part wedges are used, it is too complicated to use the Microsoft Excel; therefore, design software is needed to carry out the design of geosynthetic-reinforced fill walls using the limit equilibrium design framework. This design method shows clear advantages of dealing with geosynthetic-reinforced fill walls under special conditions over the lateral earth pressure method as discussed below for the design of a geosynthetic-reinforced tiered fill wall and a geosynthetic-reinforced fill wall with secondary reinforcement.

Figure 22 shows the outcome of a geosynthetic-reinforced tiered fill wall using this design framework incorporated in the ReSSA software (ADAMA Engineering). Based on the typical definition, the upper tier is a fill slope. The lateral earth pressure method is not valid for the combination of reinforced walls and reinforced slopes. However, this configuration can be easily designed using the top-down limit equilibrium method. Figure 22 clearly shows the distribution of the required tensile strengths along each reinforcement. The distribution of the required tensile strengths of the reinforcement layers in the lower tier was influenced by the upper tier and the uniform surcharge on the top of the upper tier. The reinforcement layers in the upper tier were long enough to avoid pullout failure; however, the upper four layers in the lower tier were controlled by the rear pullout capacities. The required connection strengths for all reinforcement layers at the wall facing were small.

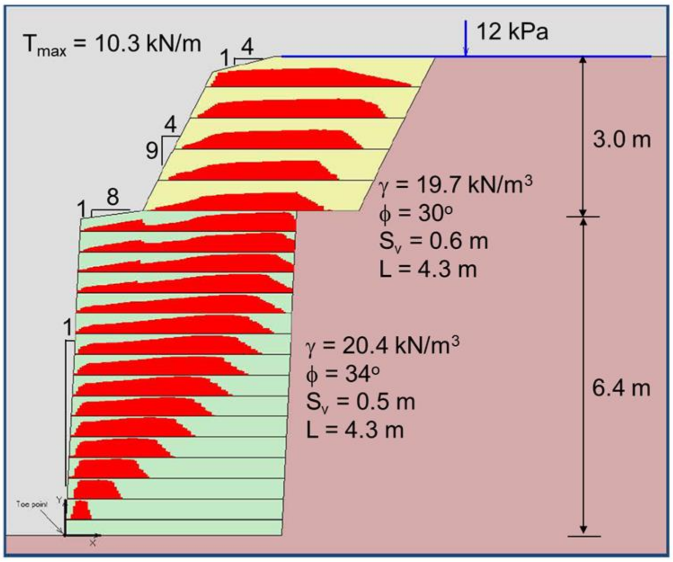


Fig. 22. Distributions of required tensile strengths in a geosynthetic-reinforced tiered fill wall [20]

Figure 23 shows the outcome of a geosynthetic-reinforced fill wall with secondary reinforcement. Clearly such a geosynthetic-reinforced fill wall cannot be designed using the lateral earth pressure method but can be easily designed by the limit equilibrium method. Figure 23 clearly shows that the distributions of the required tensile strengths for primary and secondary reinforcement layers were different. The short secondary reinforcement layers were controlled by rear pullout capacities except for the lowest layer. Leshchinsky et al. [20] also did the design for the same fill wall without any secondary reinforcement, which required the maximum tensile strength of 10.3 kN/m and the maximum connection of 6.4 kN/m, respectively. The use of secondary reinforcement layers reduced the required maximum tensile strengths and connection strengths to 7.0 and 3.0 kN/m, respectively for the primary reinforcement layers. This result is in agreement with that found in the field study [13].

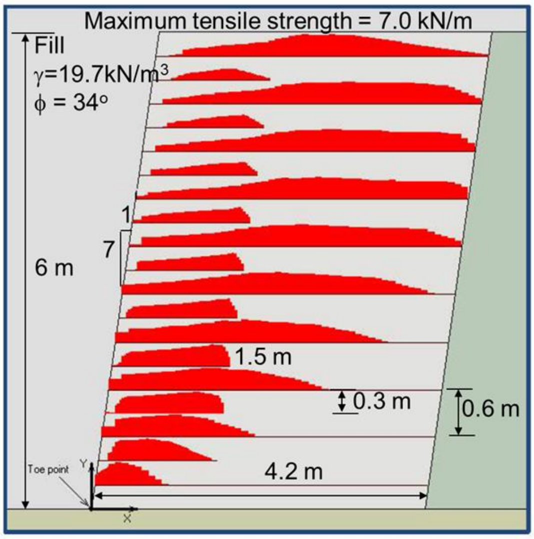


Fig. 23. Distributions of required tensile strengths for primary and secondary reinforcement layers in a geosynthetic-reinforced fill wall [20]

5 Summary

This paper presents the progressive failure of a fill wall to a fill slope due to the reduction of lateral support and failure of the unstable wedge and demonstrates the relationship between the fill wall and the fill slope. The comparisons show that the factor of safety, the critical surface, and the required tensile strength of reinforcement determined by the limit equilibrium method were similar to those calculated by the numerical method and/or measured by the experimental test. The limit equilibrium method showed the clear advantages of analysing and designing geosynthetic-reinforced fill walls under special conditions over the lateral earth pressure method. The special conditions examined in this paper include: (1) tiered walls, (2) back-to-back fill walls, (3) limited space fill walls, and (4) fill walls with secondary reinforcement. This limit equilibrium method can address the tensile strength requirements for geosynthetic reinforcement material, front connection, and rear pullout.

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