45TH IGS ANNUAL LECTURE

RELIABILITY AND RISK ANALYSIS IN GEOTECHNICAL AND GEOENVIRONMENTAL ENGINEERING

Dr. G L Sivakumar Babu
Professor, Department of Civil Engineering
Indian Institute of Science
Bengaluru

Presented at

Indian Geotechnical Conference 2023
December 14-16, 2023

Organized by

INDIAN GEOTECHNICAL SOCIETY
1948 - 2023
मृदा रक्षति रक्षित:

INDIAN GEOTECHNICAL SOCIETY
ROORKEE CHAPTER

In association

INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

CBRI
केंद्रीय भवन अनुसंधान संस्थान,रुड़की
Central Building Research Institute, Roorkee
Safe & Sustainable Habitat

CV of Dr. G L Sivakumar Babu

Dr. G L Sivakumar Babu completed his Ph.D. (Geotechnical Engineering) in 1991 from the Indian Institute of Science, Bangalore, India, after a Master's Degree (SMFE) in 1987 from Anna University, Madras, and B.Tech. (Civil Engineering) in 1983 from Sri Venkateshwara University, Tirupati. India. He worked as Visiting Scholar from 1995 to 1996 in Civil and Environmental Engineering at Purdue University, Lafayette, for a year and as Humboldt Fellow and Visiting Scientist in Landesgewerbeanstalt Bayern (LGA) Nuremberg, Germany, in 2000. He worked at the Central Road Research Institute and International Airports Authority of India New Delhi from May 1992 to November 1996. He joined the Indian Institute of Science in December 1996 as an Assistant Professor and became a full professor in 2009 and is currently Professor (HAG).

His international recognitions are 1) being the first person from the Indian Geotechnical Society to give an invited lecture at the International Conference on Soil Mechanics and Geotechnical Engineering in 75 years of IGS and he presented it during ICSMGE(2017) in Seoul, 2) John Booker Medal from International Association for Computer Methods and Advances in Geomechanics (IACMAG), USA for the year 2017, 3) Member of the Scientific Committee for International School on "LAndslide RiskAssessment and Mitigation for 2018-2024, University of Salerno, Italy, 4) Excellent Contributions Award of International Association for Computer Methods and Advances in Geomechanics (IACMAG), USA in the year 2010 and 5) Alexander von Humboldt Fellowship in 2000. He received five best paper awards for the papers published in ASCE journals.

He is an Honorary Fellow of the Indian Geotechnical Society and received the IGS Kuckelman Award 2020-2021 for his excellent contributions to Geotechnical Engineering. He also received several awards for his work, such as best Ph.D. thesis in Geotechnical Engineering in India from the Indian Geotechnical Society for the year 1992, DST BOYSACAST Fellowship, Young Engineer from Central Board of Irrigation and Power, New Delhi and seven best paper awards from Indian Geotechnical Society.

He served as the Chairman, International Technical Committee (TC-302) on Forensic Geotechnical Engineering (FGE) of International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE) for two terms (2013-2017 and 2017-2021; served as its secretary from 2005-2009 and 2009-2013. He conducted three international conferences on Forensic Geotechnical Engineering in 2010 and 2013-2016 and special technical sessions in ARCs in Fukuoka and Taipei and ICSMGE 2017. He was a member of the International Technical Committee (TC-32) on Risk Assessment in Geotechnical Engineering from 1997-2001 and 2001-2005 and conducted an International Workshop on Risk Assessment in Site Characterization and Geotechnical Design, (GEORISK 2004). He represented the Indian Geotechnical Society as its representative in ISSMGE and ARC meetings from 2013 to 2020.

He served as the Governor of Region 10, American Society of Civil Engineers from 2014 October and served as President, India Section from 2013 to 2014 and received the Distinguished Service Medal, Region 10 ASCE Award 2021. He is serving as a Council Member of the International Geosynthetics Society.

He served as Editor-in-chief of the Indian Geotechnical Journal for six years (2010-2016) and is Associate Editor of 1) Journal of Hazardous, Toxic, and Radioactive Waste, ASCE, 2) International Journal of Geomechanics, ASCE, Editorial Board member of 3) International Journal of Georisk -Assessment and Management of Risk for Engineered Systems and Geohazards, Taylor & Francis Group, and 4) Geosynthetics International, Journal of Institution of Civil Engineers, UK.

His teaching and research activities include courses on Geomechanics, Soil investigations, risk and reliability applications in Civil Engineering, Pavement Engineering, Geosynthetics, Ground Improvement and Geoenvironmental Engineering. He guided a number of students for research degrees (21 Ph.D. and 5 M. Sc (Engg. Total 26) and six students are in advanced stages for submission for Ph.D. He wrote a book on soil reinforcement and geosynthetics, edited six books and proceedings, and has several publications (International and national Journals over 200, International and national conf. more than 160 Total over 360).

Reliability and Risk Analysis in Geotechnical and Geoenvironmental Engineering

Prof. G L Sivakumar Babu*

Abstract

Properties of geological materials vary significantly both spatially and temporally. Geotechnical design and practice are influenced by variations in properties and proper consideration of loads on geotechnical systems, and resistance provided by the geological systems represented by engineering properties, which are random variables/fields is necessary. Significant developments exist in the consideration of the variability of loads and resistances, and the reliability-based design of geotechnical systems has developed very well. The objective of the paper is to present some work done by the author and his students in this area, with reference to foundations, retaining walls, slopes, dams, landslides, pavements, buried pipes, tunnels, landfill engineering, and contaminant transport in geological media. It is emphasized that probabilistic considerations play a significant role in understanding the role of uncertainties in design and provide a rational and risk-informed approach for the analysis and design of geotechnical and environmental systems. A few perceived benefits of rational design and analysis are demonstrated in this paper.

Introduction

Characterization and analysis of the behavior of the geological medium consisting mainly of soils and rocks under the given loading & environmental conditions form the basis for geotechnical or geoenvironmental analysis and design. However, owing to various physical, chemical, and biological processes during their formation, these natural materials possess unique properties that vary significantly over expansive spatial and temporal scales. This phenomenon manifests as an inherent randomness (also known as aleatory variability) or heterogeneity in the geological medium. Further, in geotechnical and geoenvironmental designs, lack of reliable information which can occur due to inaccurate characterization of site-specific properties (i.e., measurement errors), the inadequacy of mathematical formulations in the

* Professor, Department of Civil Engineering,
Indian Institute of Science, Bengaluru
Email: gls@iisc.ac.in

predictive models (i.e., modelling errors) engenders uncertainties (also known as epistemic uncertainty) in the system. Ignoring these uncertainties poses a serious challenge to the consideration of a single deterministic design outcome in the form of factors of safety, allowable stresses, and deformations from the analysis and its implications in practice, which affects construction costs, failure costs, consequences, etc. Also, by ignoring the uncertainties, the system's response will be either underestimated or overestimated, which results in an undesired and uneconomical solution. Thus, there is a need to account for these uncertainties and quantify their impact on the design by employing effective probabilistic and reliability-based design techniques.
The characteristics of natural soil exhibit inherent variability that significantly influences decisions in geotechnical engineering design. This variability may stem from both the intrinsic nature of the soil itself and variations introduced during the measurement of soil properties in field or laboratory tests, as well as errors in models. The variability in soil properties is commonly expressed in statistical terms such as mean, variance, and autocorrelation functions. These statistical measures are essential for incorporation into probability and reliability-based analyses and designs. In reliability-based design, the primary metric used is the reliability index or probability of safety, conversely expressed as the probability of failure (p_f). This index serves as a probabilistic measure of the assurance regarding the performance of a structure. The fundamental goal of reliability-based design in geotechnical engineering is to quantitatively assess the probability of failure or reliability of a given geotechnical system. This assessment takes into account the inherent variability present in design parameters and associated safety considerations.

Figure 1 shows the nature of variability in soils [1]. Variability of soil properties is due to data scatter and systematic errors. Data scatter can be attributed to spatial or temporal variations or random errors. Systematic errors are due to errors in the estimation of trends and bias in measurements. The spatial variability arises due to inherent variability, whereas measurement uncertainty contributes to data scatter. Bias in measurements arises from transformation uncertainty. Spatial correlation of soil properties due to inherent variability is an important geological consideration and the properties at a point are well correlated to those in nearby locations rather than those at a distance, indicating that the properties are correlated over distance and with distance the correlation diminishes.

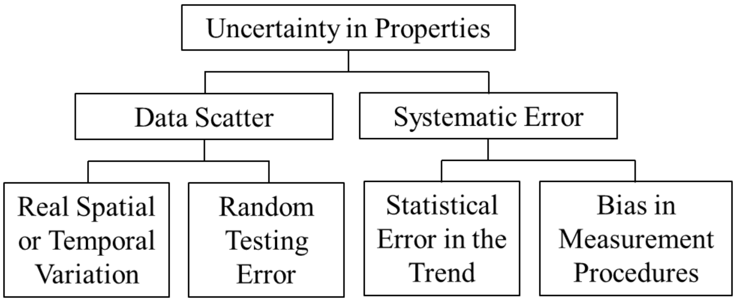


Fig.1. Variability in soil properties (redrawn from Christian et al. [1])

The spatial correlation is often expressed in terms of the auto-covariance functions and the exponential form of the auto-correlation function is widely used. Many researchers, such as Vanmarcke [2], Fenton and Griffiths [3], Jaksa et al. [4], etc., made notable contributions in this area. The average properties over areas or volumes influence the response of the structures in terms of displacements, strain mobilization, flow fields, permeability variations, pore pressure response, etc., and lead to variations in response.

Codes

Guidelines for reliability-based design and analysis are encouraged by USACE [5], emphasizing different performance levels along with associated probabilities of failure (p_f) and reliability index (β). The widespread adoption of reliability-based design (RBD) in structural engineering is supported by ISO2394 [6], which provides general principles on reliability for structures. Annex D of ISO 2394 outlines provisions for geotechnical reliability-based designs. ISO 2394 Clause 8.4 suggests that the target p_f should consider factors such as the consequence and nature of failure, economic losses, social inconvenience, environmental effects, sustainable use of natural resources, and the effort and expense required to reduce the p_f.

Geotechnical variabilities are traditionally addressed in deterministic design methods using the factor of safety (FOS), leading to either inefficient overdesign or underdesign with an unknown safety level. Load and resistance factor design (LRFD) employs partial factors calibrated to achieve a target reliability index across various design scenarios. However, fixed partial factors may not cover all scenarios involving different levels of soil property variation, potentially leading to inefficient designs [7]. Advanced mathematical techniques, like probabilistic methods, offer a systematic way to consider these variabilities.

According to Phoon [8], geotechnical reliability is a comprehensive methodology enhancing decision-making across various life-cycle stages, including design, construction, operation, maintenance, retrofit, and decommission/reuse. This methodology, leveraging probabilistic models for richer data characterization, extends beyond engineered systems to cover risk assessment and management of geohazards such as earthquakes and landslides. Its applications span different stages of engineered geotechnical systems, requiring thorough data collection, analysis, design, construction, and maintenance for robust engineering decision-making.

Methods of reliability analysis

The reliability analysis requires mean values and variances for random variables, along with autocorrelation functions for random fields. Empirical functions are commonly used to fit autocovariance functions, and normalizing the autocovariance by the variance yields the autocorrelation function. Various autocorrelation functions are available in the literature [2], with the autocovariance function relying on the correlation distance, defined as the distance over which a property demonstrates a strong correlation. Reliability analysis methods, including the First Order Reliability Method (FORM), partial factors, Monte Carlo simulations (MCS), quantile values, and system reliability, are grounded in probabilistic considerations, drawing from extensive literature [9,10].
Performing probabilistic analysis often demands numerous simulations, which can be computationally prohibitive with numerical models. Therefore, surrogate modelling techniques are used to approximate numerical models. The Response Surface Method (RSM) and Stochastic Response Surface Method (SRSM) serve as tools for developing surrogate models. When RSM falls short, and SRSM encounters limitations in approximating nonlinear functions, advanced kriging methods, such as active learning kriging or adaptive kriging, enhance the applicability of surrogate models in design and optimization. These methods contribute to computational efficiency by reducing the number of numerical simulations, concentrating on the area of interest.

Additionally, the implementation of subset simulation (SS), an advanced sampling method, proves beneficial for estimating the probability of events with very small failure probabilities [11]. Subset simulation is recognized for its computational efficiency and employs the Markov Chain Monte Carlo (MCMC) algorithm to sample from conditional distributions. The core idea of subset simulation involves representing a small failure probability as a series of more frequently occurring higher failure probabilities. This method has gained recognition as one of the most computationally efficient techniques for modelling rare events [12-14].

There has been a general reluctance to use reliability-based designs in geotechnical engineering despite its obvious advantages, whereas design in structural engineering uses reliability analysis better. Some reasons are that the resistance factors for geologic materials depend on coefficients of variation or variability, which are larger compared to those in the case of materials such as steel and concrete and correlations exist among geotechnical properties, which makes evaluation of p_f quite involved. Information from load variations such as rainfalls and earthquakes are not used commonly leading to failures in dams, pavements, embankments, landslides, etc. The research work carried out by the author and his students highlights the practical application of adopting the probabilistic methodologies in the areas of geotechnical and geoenvironmental engineering, which are presented in the following sections.

(i) Shallow foundations

Due to the inherent nature of soil formation and depositional processes, soils display variability in both horizontal and vertical directions, leading to anisotropic correlation structures of soil properties, with greater variability typically observed in the vertical direction [15]. Acknowledging contributions from inherent variability, measurement uncertainty, and transformation uncertainty, comprehensive analyses are essential. Economic considerations, exploration speed, equipment limitations, and time constraints often result in the collection of borehole data predominantly from the vertical direction.

Murthy and Sivakumar Babu [16] have appropriately considered these factors in their analyses. The work delves into the impact of assumptions regarding the spatial correlation of cone tip resistance on the performance of shallow strip foundations using shear criteria. Results indicate that an isotropic correlation structure based on vertical autocorrelation distance tends to underestimate design parameter variability. Conversely, perfect correlation in horizontal or vertical directions, or both, tends to overestimate design parameter variability, leading to conservative estimates of allowable bearing capacity. Notably, the transformation model significantly influences the degree of variability in design parameters.

Sivakumar Babu et al. [17] presented results on the bearing capacity of shallow foundations on cohesive soil, Sivakumar Babu and Murthy [18] examined the effect of spatial correlation of cone tip resistance on the bearing capacity of shallow foundations, Sivakumar Babu and Srivastava [19] focused on using the response surface method to evaluate allowable bearing pressure, and

Geetha Manjari et al. [20] investigated the evolution of settlement of footings on cohesionless soil under increasing loads as a stochastic process, employing a tri-level homogeneous Markov chain model.

(ii) Pile foundations

The spatial correlation structure of soils plays a crucial role in estimating ultimate load capacity and strain mobilization, as demonstrated in the case of variable soil around a pile foundation discussed below. The probabilistic study in question involves treating undrained shear strength as a random variable, employing random field theory for analysis. In this approach, inherent soil variability is considered a source of variation, and the field is modelled as a two-dimensional non-Gaussian homogeneous random field. The Cholesky decomposition technique is utilized to simulate the random field within a finite difference program, with Monte Carlo simulation being the chosen approach for the probabilistic analysis. The investigation focused on assessing the impact of undrained shear strength's variance and spatial correlation on the ultimate capacity, which is expressed as the sum of ultimate skin friction and end-bearing resistance of the pile.

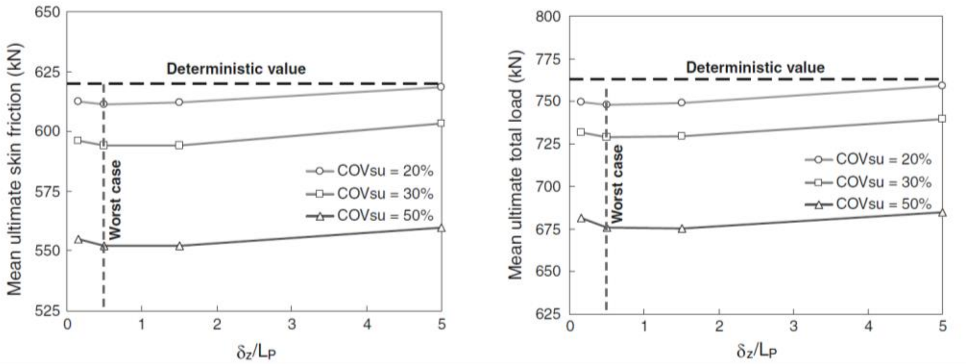


Fig. 2. Variation of Mean ultimate skin friction and Mean ultimate load of vertical pile with the CoV of undrained strength and scale of fluctuation.

Results, methodology, and insights provided by Haldar and Sivakumar Babu [21] emphasize the importance of properly considering the spatial variability of soils. This consideration is essential not only for determining allowable loads but also for understanding the strains mobilized within the soil medium due to variability. Figure 2 illustrates a comparison between values of ultimate skin friction and ultimate total load obtained from both deterministic and probabilistic analyses. Notably, as the coefficient of variance (CoV) of undrained strength increases, mean values decrease, while an increase in correlation distance results in higher mean values.

Figures 3(a) and 3(b) show the differences in strain mobilization in deterministic soil and the soil medium with different correlation structures defined in terms of coefficient of variance (CoV) and spatial correlation distance. For example, close to the pile tip, the soil strains in the range of 0.008 in the case of uniform soil and in the case of soil medium with high CoV (50%) and low correlation distance of 1.5 m has higher magnitudes of strain in the range of 0.012 at the tip compared to other cases.

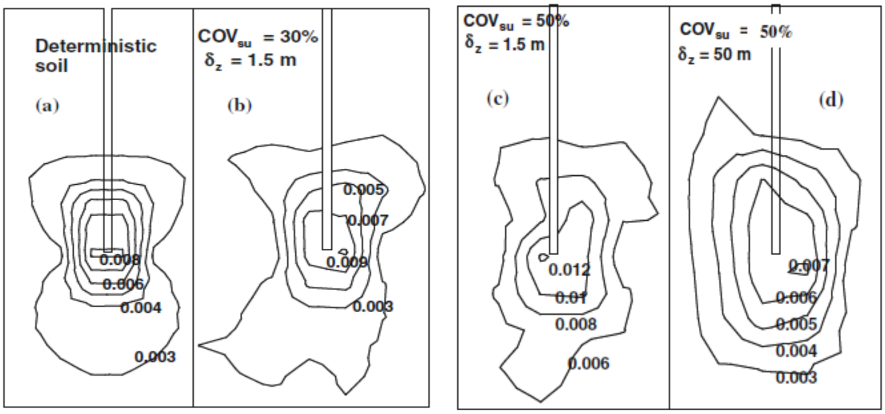


Fig. 3. Strain mobilization in the case of a) vertical pile in uniform soil medium and b) spatially variable medium.

Haldar and Sivakumar Babu [22] reported a similar study on the lateral capacity of the piles, and the results are presented in Figures 4a and 4b. Figure 4a shows the variation of mean lateral load and mean maximum moment of lateral pile with CoV of undrained strength and scale of fluctuation. It can be noticed that the ultimate lateral load decreases with an increase in CoV of undrained strength and increases with correlation distance. In Fig. 4b, the strain mobilization for a lateral pile is presented, revealing that the maximum strain in a spatially variable deposit is twice that observed in a uniform deposit.

Furthermore, various studies have been conducted to assess the reliability of pile foundations. Notably, Haldar and Sivakumar Babu [23] explored the estimation of reliability using Cone Penetration Test (CPT) data. Additional investigations by the same authors cover probabilistic analyses of load-displacement responses from pile load tests [24], LRFD (Load and Resistance Factor Design) of piles [25], and the design of pile foundations in non-liquefiable soils [26]. These references provide detailed results for further exploration.

Nazeeh and Sivakumar Babu [27] presented a critical appraisal of foundation design codes and the role of a reliability-based approach and offer valuable insights. The results from their analysis emphasize that the p_f is a relative measure, providing a basis for comparing different designs. Christian et al. [1] highlight the effectiveness of reliability analysis in establishing design values for factors of safety, especially when assessing consistent risks associated with different failure types. They suggest that probabilistic methods are most useful when estimating relative probabilities of failure and revealing the effects of uncertainties in the parameters. The optimization of foundation design considering variability is explored as a useful alternative, with an illustrative example presented in the following section.

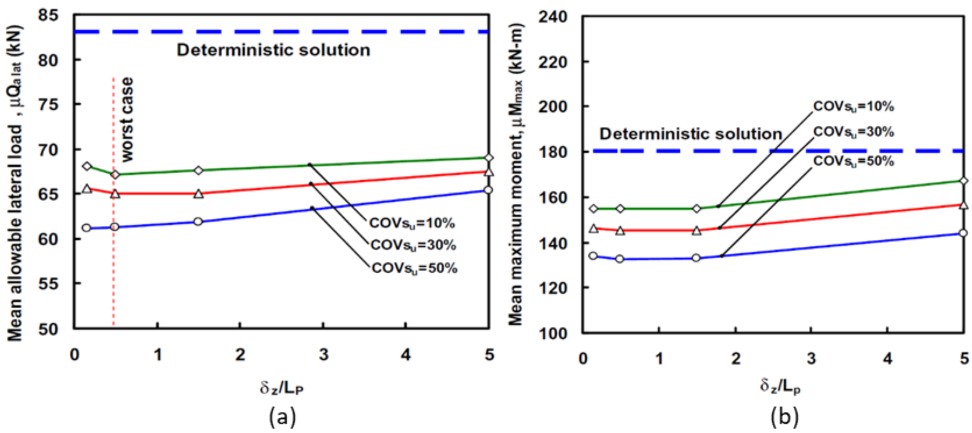


Fig. 4a. Variation of (a) mean lateral load and (b) mean maximum moment of lateral pile with CoV of undrained strength and scale of fluctuation [22]

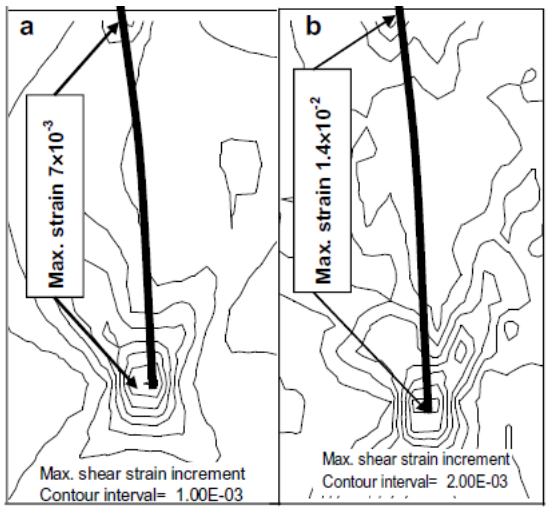


Fig. 4b. Maximum strain values in the case of a) homogeneous soil and spatially variable soil (undrained strength CoV= 30% $\delta_z/L_p = 0.15$) [22]

(a) Foundation design optimization

In geotechnical design, ensuring that no relevant limit state is exceeded is a fundamental requirement. The economic optimization limit state (EOLS), along with the ultimate limit state (ULS) and serviceability limit state (SLS), constitutes the three foundational requirements in foundation design. The ULS and SLS are specifically associated with the risks of shear failure and excessive settlement, respectively. The optimization process aims to minimize construction costs, treating design parameters as optimization variables and design requirements as constraints. The final design, which satisfies the target failure probability, is determined as the least-cost design through the comparison of cost and failure probability among available designs.

Fathima et al. [28] demonstrated a design methodology for shallow foundations using numerical modelling with reliability-based optimization. Computational efficiency is enhanced by incorporating a kriging model into the design methodology, tapping into the potential of numerical modeling. The process is illustrated using a shallow foundation example. The analysis involves the development of kriging-based surrogate models for various foundation dimensions, considering different CoV and correlation values. Design optimization is then executed by evaluating the costs of different designs and selecting the least-cost design that meets the design requirements. Figure 5 illustrates a typical result of this optimization process.

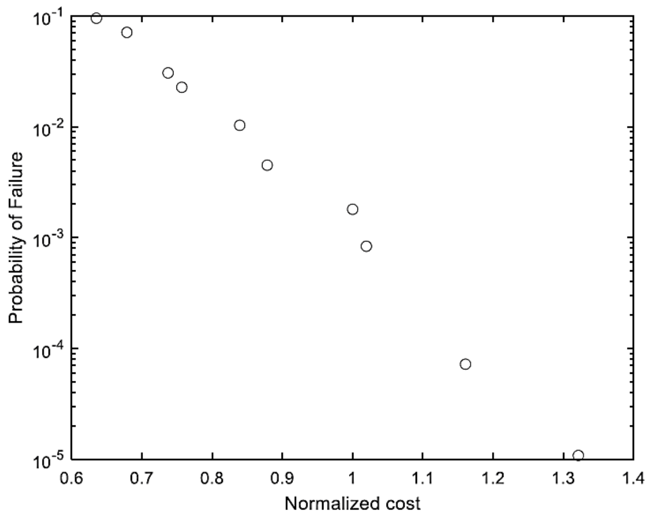


Fig. 5. Normalized cost for different failure probabilities

The kriging-based surrogate modeling technique has demonstrated its efficiency as a substitute for computationally expensive numerical models in predicting system performance accurately. An R^2 value of 0.99 proves the precision of the predictions. The study reveals that, using kriging-based probabilistic design and optimization, a foundation area of 12.25 m^2 can be achieved at a cost of 1.1x for a target probability of failure (p_f) value of 1.00 $\times 10^{-3}$. Foundation costs tend to increase when targeting lower probabilities of failure within the range of 0.001. For comparison, values for 99% and 95% reliability yield areas of 8.75 m^2 and 10.5 m^2, respectively. The methodology also facilitates the determination of partial factors, with factors identified for 95%, 99%, and 99.9% reliable designs. Specifically, factors for the angle of friction, unit weight, and Young's modulus are found to be 0.85, 0.74, 0.72; 0.99, 0.99, 0.98; and 1, 1, 0.61, respectively, corresponding to 95%, 99%, and 99.9% reliability. The costs associated with 95%, 99%, and 99.9% reliable designs are determined as 0.73x, 0.88x, and 1.1x, respectively.

Nazeeh and Sivakumar Babu [29] extended this work to reinforced soil foundations using geogrids (GG), and a representative result for a raft foundation is shown in Figure 6.

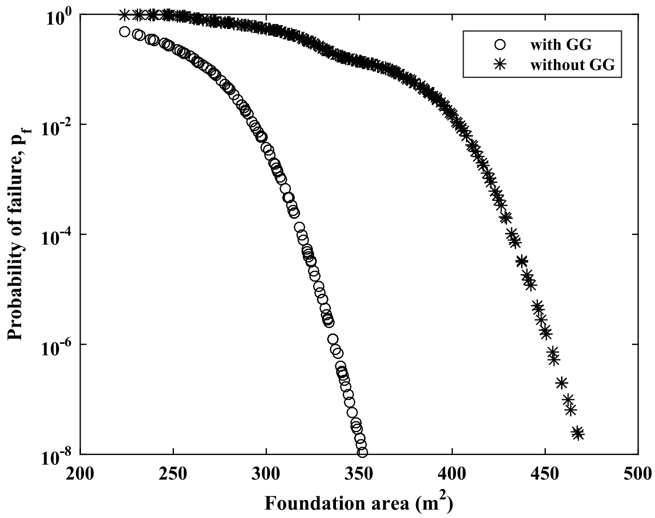


Fig. 6. Comparison of p_f of unreinforced and reinforced soil foundations with foundation area [29]

Comparing the bearing capacity of reinforced and unreinforced soil, the improvement ratios range from 1.10 to 1.40, accounting for variations in soil properties and foundation sizes. If greater deformation is permissible, these values can increase to approximately 2 to 3 for ultimate bearing capacity. In the case of a typical isolated shallow foundation, kriging-based design, considering soil variabilities, indicates a foundation area of 14 m^2 on unreinforced soil. This area is reduced to 12 m^2 on geogrid-reinforced soil, representing a reduction of around 15%. For a raft foundation, an initial area of 420 m^2 is obtained without ground improvement using geogrids. This area is then reduced to 307.5 m^2 on geogrid-reinforced soil, resulting in a reduction of approximately 17% (refer to Fig. 6). The kriging-based methodology developed in this study provides a versatile approach to designing and optimizing shallow foundations based on any numerical model.

(iii) Retaining walls and bridge abutments

The design of retaining structures hinges on critical factors, including the load transferred from backfill soil, external loads, and structural resistance each of which is characterized by random variables. Sivakumar Babu and Basha [30] and Basha and Sivakumar Babu [31-41] formulated reliability-based design frameworks for various retaining structures and bridge abutments. These encompass (i) cantilever sheet pile walls, (ii) cantilever retaining walls, (iii) gravity retaining walls, and (iv) reinforced soil structures.

For cantilever sheet pile walls, Basha and Sivakumar Babu [31,32] determined the design penetration depth and section modulus, accounting for the desired target stability and considering rotational failure modes about the base point and the flexural failure mode of the sheet pile. In the case of anchored cantilever sheet pile walls, the study considered rotational, sliding, and flexural failure modes, determining penetration depth, anchor force, and section modulus corresponding to targeted reliability indices. The investigation highlighted that the optimum penetration depth and section modulus of the pile are notably influenced by the anchor pull and the soil-pile interface.

For cantilever retaining walls, Sivakumar Babu and Basha [30] demonstrated that suitable optimal wall proportions can be achieved based on targeted component reliability indices, taking into account safety and economic requirements. Upper and lower bounds of the system reliability index were obtained for a series system with statistically dependent component failures. The study proposed optimal wall proportions for varying coefficients of variation of the friction angle of the backfill soil (5% to 10%) and cohesion of the foundation soil (5% to 20%), corresponding to different values of lower bounds of system reliability indices. The impact of the correlation coefficient between failure modes on upper and lower bounds of the system probability of failure was also explored. The results indicated that utilizing reliability-based design for optimized proportions could lead to cost reductions compared to conventional designs. Figure 7 provides a representation of typical results.

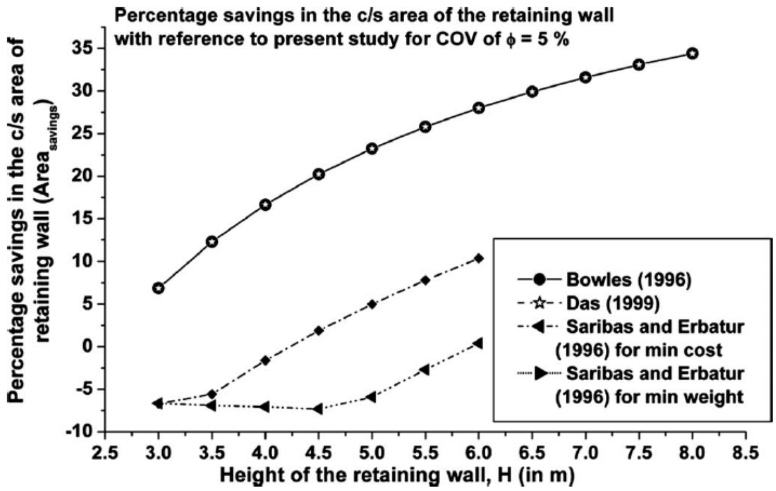


Fig. 7. Percentage savings in the cross-sectional area of retaining wall with reference for CoV_{ϕ}=5%

Basha and Sivakumar Babu [36,37] reported findings related to reliability-based design optimization of gravity wall bridge abutments under seismic conditions. The study employed reliability analysis to assess the p_f across three modes: sliding failure of the wall at its base, overturning failure about its toe, and eccentricity failure of the resultant force, as well as bearing failure of the foundation soil below the base of the wall. Both backfill and foundation soil properties beneath the abutment's base were treated as random variables. Furthermore, uncertainties associated with characteristics of earthquake ground motions, including horizontal seismic acceleration and shear wave velocity propagating through backfill soil, were considered. Optimal abutment proportions required to ensure stability were determined against these three modes of failure by targeting various component and system reliability indices. A summary of typical results is provided in Table 1.

(iv) Reinforced soil structures

Basha and Sivakumar Babu [32] introduced a method for assessing the external stability of reinforced soil walls under earthquake conditions, utilizing the pseudo-dynamic method. The seismic reliability of the wall was evaluated, taking into account various potential failure modes, including sliding along the base, overturning about the toe point of the wall, bearing capacity, and the eccentricity of the resultant force. The analysis considered random variables such as properties of the reinforced backfill, foundation soil below the wall's base, length of the geosynthetic reinforcement, and

Table. 1. Optimum Abutment Width to Height Ratios (B/H) for $\phi = 30°$–$45°$ and k_h =0–0.3

	Abutment width to height ratio (B/H) for $\beta_{sli} \geq 3.0$, $\beta_e \geq 3.0$, $\beta_b \geq 3.0$, and $\beta_{sys} \geq 3.0$,			
k_h	$\phi = 30°$	$\phi = 35°$	$\phi = 40°$	$\phi = 45°$
0.00	0.48	0.42	0.32	0.29
0.05	0.54	0.48	0.38	0.34
0.10	0.60	0.53	0.43	0.39
0.15	0.67	0.60	0.48	0.44
0.20	0.74	0.66	0.54	0.49
0.25	0.80	0.72	0.60	0.54
0.30	0.87	0.78	0.65	0.59

characteristics of earthquake ground motions, such as shear wave and primary wave velocity. The study determined the optimal length of reinforcement necessary to maintain stability against these four modes of failure by targeting various component reliability indices. The comparison between the pseudo-static and pseudo-dynamic methods was emphasized, highlighting that the pseudo-dynamic method yields more realistic design values for the length of geosynthetic reinforcement under earthquake conditions. This difference arises due to the limitations of pseudo-static analysis, such as neglecting the effects of time and phase differences resulting from finite shear wave and primary wave velocities, as well as the amplification of seismic accelerations. A representative result is depicted in Fig. 8.

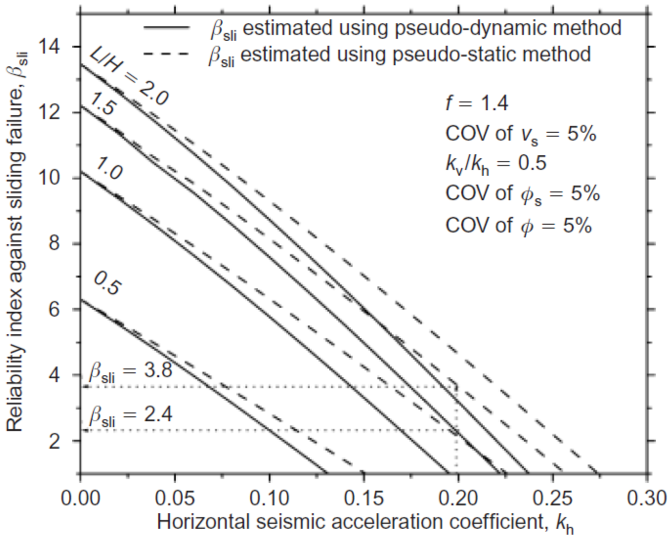


Fig.8. Comparison of reliability indices based on pseudo-static and pseudo-dynamic methods.

Basha and Sivakumar Babu [40] proposed an approach for target component and system reliability-based design optimization to assess the safety of the internal seismic stability of geosynthetic-reinforced soil structures. The analysis considers three failure modes: tension failure of the bottom-most layer of reinforcement, pullout failure of the topmost layer of reinforcement, and total pullout failure of all reinforcement layers. The backfill properties and geometric and strength properties of the reinforcement are treated as random variables. The study determines the optimal number of reinforcement layers and pullout length necessary to maintain stability against tension failure, pullout failure, and total pullout failure for different coefficients of variation of the friction angle of the backfill, design strength of the reinforcement, and horizontal seismic acceleration coefficients, targeting various system reliability indices. The results offer guidelines for the total length of reinforcement required, considering the variability of backfill and seismic coefficients. An illustrative example is provided to explain the evaluation of reliability for internal stability of reinforced soil structures using the proposed approach.

Mukherjee and Sivakumar Babu [42] conducted a probabilistic evaluation to analyse and design the uplift capacity of transmission tower foundations reinforced with a horizontal anchor, employing the radial basis function (RBF) based response surface method. The deterministic uplift capacities of horizontal anchors in both unreinforced and reinforced soils were determined through a finite difference numerical approach. Response surface models based on RBF were developed using the observed uplift resistance obtained from the deterministic numerical models. The foundation's reliability under uplift forces was evaluated through Monte Carlo simulation, considering uncertainties associated with soil and geogrid properties. The impact of safety factors on the failure probability of both unreinforced and reinforced foundations was demonstrated. Additionally, the variation of the p_f with different CoV of input variables was explored. Among the parameters studied, the reinforcement stiffness emerged as the most influential, exerting a notable effect on the failure probability of the reinforced foundation. The study revealed a substantial enhancement in uplift capacity and a reduction in the p_f of the foundation when a reinforced anchor was employed, underscoring the effectiveness of reinforcement on the anchor plate. Results indicated that, for an equivalent failure probability, reinforced anchors necessitate a shallower depth compared to unreinforced anchors. Figure 9 illustrates a representative outcome. This noteworthy reduction in p_f can be attributed to the increased uplift capacity facilitated by geogrid reinforcement atop the anchor plate.

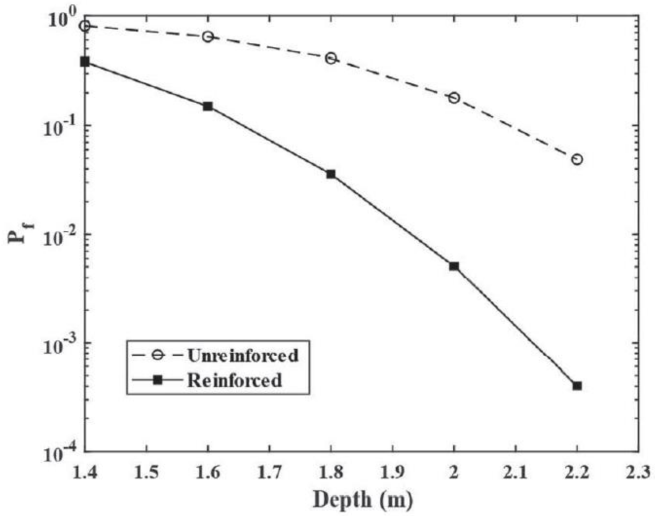


Fig. 9. Variation of p_f with depth of embedment of anchor plate with anchor and geosynthetic reinforced anchor.

Pramanik et al. [43] employed a surrogate model-based response surface method to anticipate the maximum displacement of the wall facing in geosynthetic-reinforced soil segmental walls. The multivariate adaptive regression splines were utilized for this prediction. Synthetic datasets were generated through Latin hypercube sampling for both training and testing purposes. For each input dataset, the finite difference software FLAC was used to assess the maximum wall-facing displacement. An expression, involving basis functions, was proposed to represent the maximum wall-facing displacement. The model's performance was evaluated by comparing the ratio of predicted and simulated maximum wall-facing displacement, and the results indicated satisfactory performance. The study highlighted the significant influence of the soil friction angle on predicting the maximum wall-facing displacement. A comparison with other soft computing techniques revealed that the proposed model demonstrated minimal errors. Furthermore, a probabilistic analysis was conducted concerning the normalized maximum wall-facing displacement. The outcomes demonstrated that the probabilistic prediction of the maximum wall-facing displacement using the proposed model fell within the specified limits.

(v) Soil nailed walls

Sivakumar Babu and Singh [44-48] conducted extensive research on the reliability analysis of soil-nailed walls, examining four failure modes under both static and seismic conditions. Key findings from the reliability-based evaluation of a typical soil-nailed wall in the study include: (a) the variability of the angle of internal friction in the in-situ soil is more critical to all failure modes compared to unit weight and cohesion, (b) the correlation among in-situ soil parameters significantly influences the reliability-based assessment of soil-nailed wall stability, (c) accounting for the vertical component of seismic loading leads to an overestimation of global stability and an underestimation of other failure modes, and (d) the susceptibility order of failure modes in seismic conditions is arranged as follows: Sliding > Tensile > Pullout > Global. Moreover, Sivakumar Babu and Singh [48] introduced a load and resistance factor design (LRFD) methodology for soil-nailed walls. The process for determining reliability-based load and resistance factors is outlined with respect to six limit strength states (i.e., failure modes) for a typical soil-nailed wall. A comparison of the proposed methodology's load and resistance factors is made with those available in other soil nailing design codes. The study emphasizes the necessity for separate design factors (i.e., load and resistance factors) for each limit state.

Sivakumar Babu and Singh [44] demonstrated the application of a 2^k factorial design of experiment methodology for developing regression models through numerical simulations. These models predict the global stability and lateral displacement of soil nail walls. The proposed methodology offers a straightforward equation to estimate site-specific displacements, considering in-situ soil parameters from geotechnical investigations and the typical range of soil wall heights in local practice. Reliability analyses using regression models revealed that variations in in-situ soil properties, especially soil friction angle and unit weight, play a crucial role in soil nailing stability criteria. Additionally, using soil modulus E_s as a design factor yielded a more appropriate displacement response.

Pramanik and Sivakumar Babu [49] presented a load and resistance factor design (LRFD)-based reliability assessment of soil nail walls against facing failures. Analysis covered three facing failure modes: flexural, punching shear, and headed-stud tensile limit states. Both default and improved Federal Highway Administration (FHWA) load models, along with only the default FHWA resistance model, were considered for formulating limit states against facing failures. Variations in design parameters across the depth of the wall were examined for different target reliability indices and levels of variability in random input variables. Estimated load and resistance factors for different target reliability indices indicated that adopting the improved FHWA load model minimizes the maximum required design parameters. The study underscored the significant influence of input variable variability on load and resistance factors, emphasizing the correlation coefficient between soil parameters as a key factor.

(vi) Pavements

Deepthi et al. [50,51] conducted a reliability analysis on a flexible pavement section, assessing fatigue cracking and subgrade rutting failure criteria using the first-order reliability method (FORM), second-order reliability method (SORM), and crude Monte Carlo simulation. In the sensitivity analysis, the surface layer thickness was identified as the most critical parameter influencing design reliability for both fatigue and rutting failure criteria [52]. The study emphasizes a significant dependence between the two failure modes, indicating a high probability of simultaneous occurrences compared to individual component failures. This highlights the importance of considering system reliability in pavement analysis. The research suggests that improving pavement performance should prioritize reducing the likelihood of simultaneous failure rather than focusing solely on the more critical failure mode. Additionally, the probability of simultaneous failures is observed to rise with slight increases in mean traffic loads, resulting in wider system reliability bounds.

Deepthi and Sivakumar Babu [53] investigated the impact of spatial variability of resilient moduli. The study revealed that reducing the spatial correlation length led to a slight increase in the mean values of critical strains. This suggests that modeling pavement layers as homogeneous, without considering spatial variability, may underestimate critical strains. As the correlation length decreases, the probability density functions (PDFs) of critical strains become less spread out (ref. Fig. 10). This reduction in variance is attributed to the spatial averaging phenomenon. Moreover, higher variability in resilient moduli, indicated by a higher CoV, results in increased variability in pavement responses.

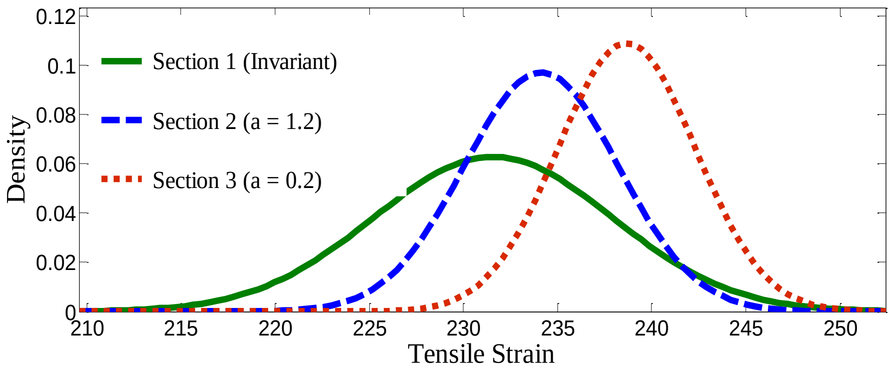


Fig. 10. Effect of the Correlation length on the PDF of the Tensile strain

Deepthi and Sivakumar Babu [54] explored the impact of anisotropy within each pavement layer on the pavement responses, focusing on identifying the layer where introducing anisotropic characteristics has the most significant effect on critical strains. The findings indicate that anisotropy in the base layer has a notable and varied influence on critical strains. Specifically, the compressive strain tends to be substantially higher compared to the isotropic section (refer to Fig. 11), while the mean value of tensile strains decreases with the introduction of base-layer anisotropy (see Fig. 12). Additionally, anisotropy in the asphalt layer tends to decrease the critical tensile strain, with minimal impact on the critical compressive strain.

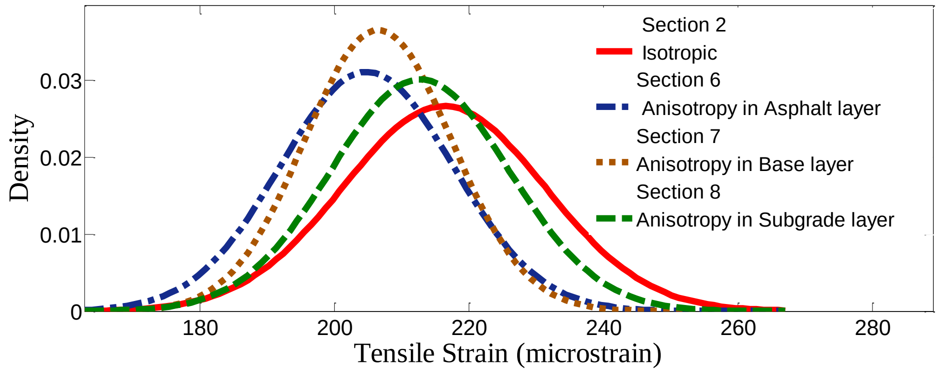


Fig. 11. Effect of Anisotropy on the PDF of the Critical Tensile Strain

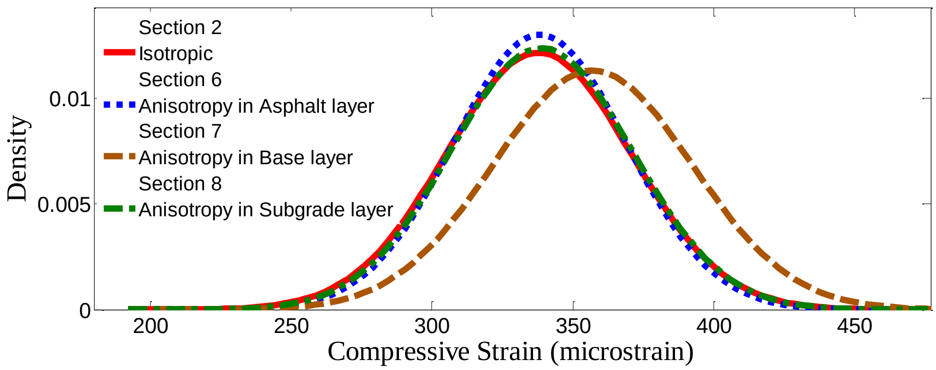


Fig. 12. Effect of Anisotropy on the PDF of the Critical Compressive Strain

Nevertheless, as compared to the spatially invariant section, the introduction of spatial variability and anisotropy in each of the layers sees a significant reduction in the design lives (Table 2). Not considering the spatial variability of the layers may, therefore, be one of the reasons for the premature failure of pavements observed worldwide.

Table 2. Influence of Spatial Variability on Design Life

Design Life	Spatially Invariant	Isotropic	Layer Anisotropy (ax=1m; ay=0.7)		
			Asphalt	**Base**	**Subgrade**
Fatigue (msa)	100	50	60	66	55
Rutting (msa)	450	300	300	250	320

The inclusion of time-dependent factors, such as loading, external environmental conditions, damage, and maintenance practices, is crucial in understanding the causes of early failures in pavements. These factors contribute to the deterioration of structural resistance over the design life. Deepthi et al. [55] conducted a study focusing on the degradation of the surface layer modulus and its impact on pavement fatigue reliability. The decrease in modulus over time is modelled as a function of accumulated damage from repeated loading, as illustrated in Fig. 13. Figure 13a demonstrates the degradation effect under various traffic loadings, while Figure 13b highlights the difference in reliability when degradation is considered. The pavement section, designed for a 15-year period at an 80% reliability level, exhibits a p_f of approximately 50% after 8 years and 75% after 10 years when accounting for strength degradation. This highlights the importance of incorporating temporal characteristics of materials and loading in a time-dependent reliability analysis framework.

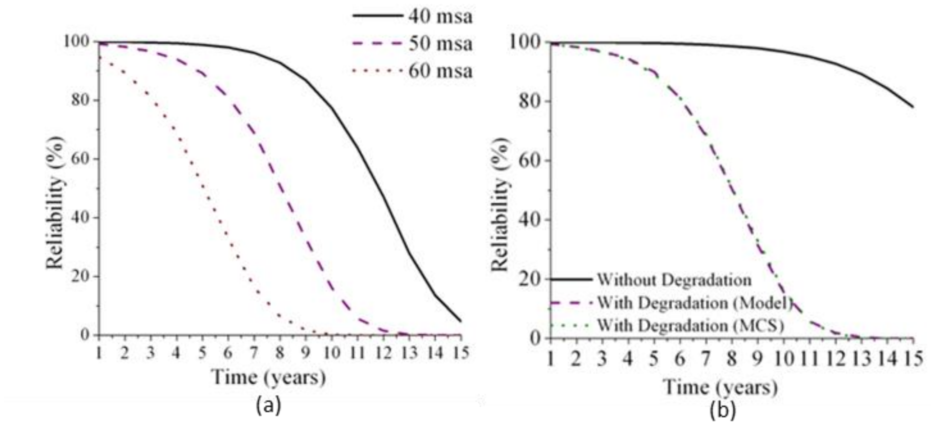


Fig. 13. Time-variant reliability with asphalt modulus degradation for a) Different traffic loadings b) Effect of modulus degradation

Deepthi and Sivakumar Babu [56] introduced a Reliability-Based Design Optimization (RBDO) approach to enhance the cost-effectiveness of flexible pavements by adjusting combinations of design parameters, such as thickness and resilient moduli. The optimal solutions reveal that for pavement sections initially designed with thin asphalt layers and thicker granular base/sub-base layers, achieving the same target levels of reliability is possible by increasing the asphalt layer thickness, particularly when fatigue governs pavement failure (refer to Fig. 14). The augmented asphalt thickness is compensated by significant reductions in the base layer thickness, leading to overall thinner sections and lower construction costs. These findings are confirmed across different cost models, accounting for regional cost variations and resource availability.

Deepthi and Sivakumar Babu [57] extended the study to System Reliability-Based Design Optimization (SRBDO), aiming to address correlations between different pavement failure modes. This approach seeks to optimize pavement layer thicknesses and moduli, considering target reliability levels, traffic demand, and subgrade strength. It seamlessly integrates economic analysis with the Mechanistic-Empirical procedure within a system reliability framework. To enhance efficiency, two meta-modeling approaches were explored: the second-order adaptive Response Surface Model (RSM) and adaptive Polynomial-Chaos-based Kriging (PC-Kriging) meta-models. For the numerical examples, model uncertainty was observed to be around 1% for three-layer sections and below 2.5% for four-layer pavements.

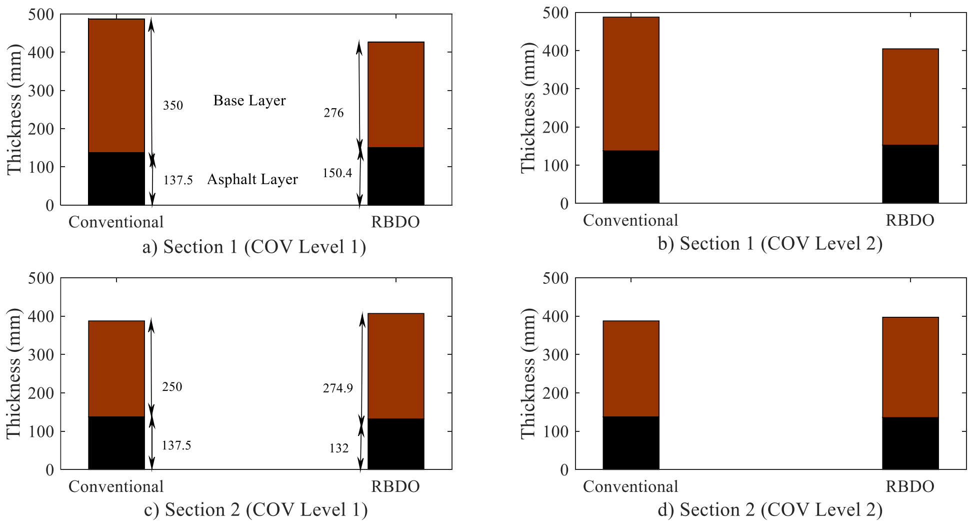


Fig. 14. Comparison of conventional and RBDO based pavement sections

Deepthi et al. [58] introduced the 'Simplified Effective Random Dimension Quantile Value Method' (Simplified ERD-QVM) for the reliability-based design of flexible pavements. Current reliability-based approaches in pavement design guides often rely on overall reliability factors, while simulation-based probabilistic techniques can be computationally intensive. The Simplified ERD-QVM bridges this gap, offering an efficient technique without surrogate models. Verified through Monte Carlo Simulations for various design scenarios, it provides significant computational time savings, utilizing familiar techniques for practitioners following the Mechanistic-Empirical Pavement Design Guide. This comprehensive reliability analysis approach considers both design parameter and model uncertainty. The study highlights that the design traffic, computed to prevent exceeding the critical rut threshold, is lower when parametric uncertainties are considered (refer to Fig. 15). Essentially, failure probability is underestimated if both design parameter and model uncertainties are not considered in reliability-based design. The paper also outlines the methodology for integrating Simplified ERD-QVM with existing flexible pavement design codes.

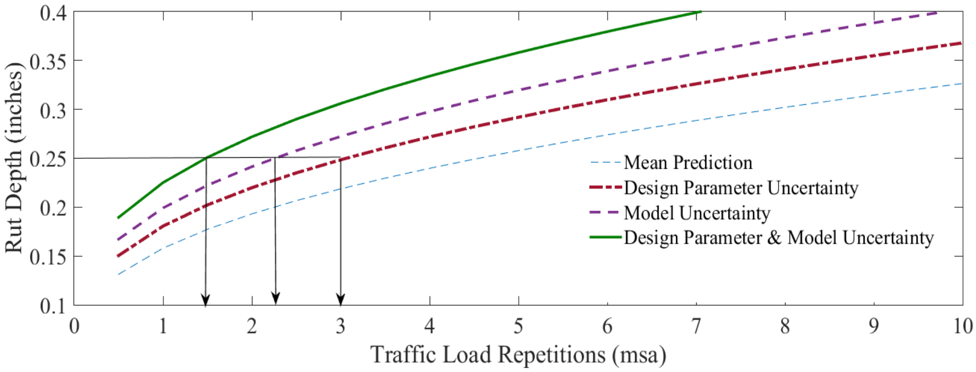


Fig 15. Simplified ERD-QVM for Rut Depth Prediction

Kalore et al. [59] introduced a framework for assessing and mitigating risks associated with the drainage layer in a pavement subsurface drainage system. The framework suggests strategies to reduce risks and enhance system performance, acknowledging dependencies on inflow characteristics, aggregate gradation, unsaturated soil drainage properties, and pavement section geometry. The system's demand is defined as the required permeability, estimated based on total inflow and geometric section properties, while the discharge capacity relies on the hydraulic conductivity of the drainage layer. Capacity-demand models are employed to scrutinize the system's design, and a rational methodology accommodating variations in demand due to rainfall and capacity in the form of permeability is proposed. Recognizing the complexity and stochastic nature of determining exact demand and capacity, a more realistic evaluation involves considering uncertainty, and incorporating the p_f and associated risks. The system's performance is analyzed by assessing the sensitivity of the design to various variables. Optimal enhancement of the system's performance can be achieved by increasing layer thickness and/or employing coarser gradation with stabilization. Illustrative results are provided in Fig. 16 and Fig. 17.

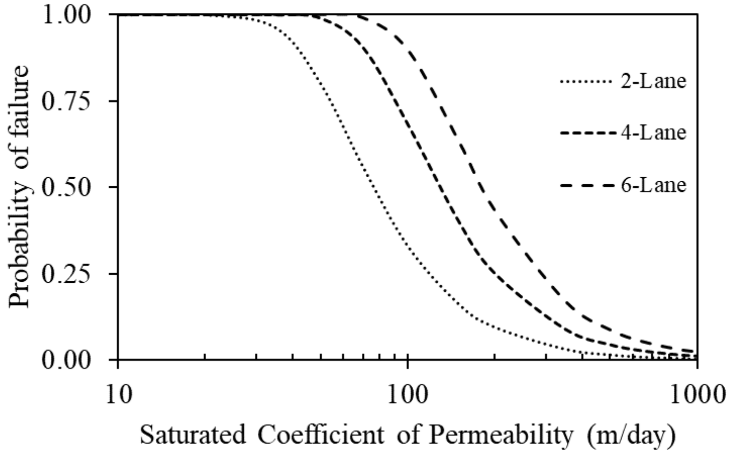


Fig. 16. Probability of failure versus saturated coefficient of permeability

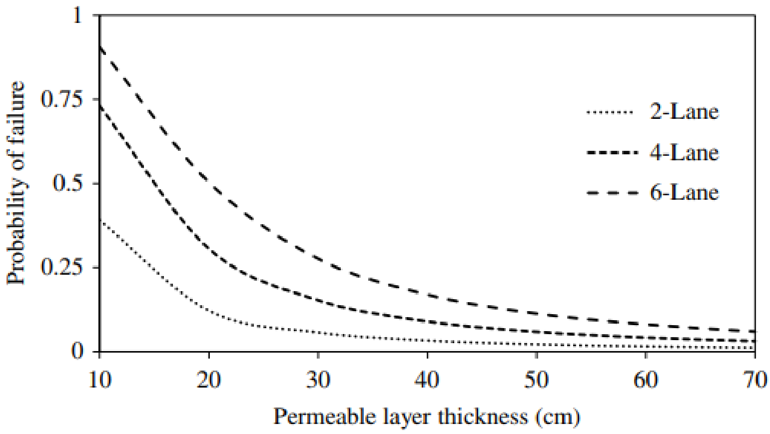


Fig. 17. Probability of failure versus drainage layer thickness

(vii) Stability of slopes, landslides, and dams

Soil slopes in general are unsaturated and typically exhibit negative pore pressure or suction for most of the time. The stability of slopes decreases as suction diminishes due to the infiltration of moisture. This reduction in stability, a time-dependent phenomenon, contributes to rainfall-induced landslides in the Himalayan region. Sivakumar Babu and Murthy [60] conducted a reliability analysis on a typical slope in a region prone to rainfall-induced landslides. Their analysis effectively captures the slips occurring in the area due to rainfall, highlighting that variations in suction and permeability govern landslide occurrences.

Srivastava et al. [61] explored the impact of spatial variability in soil permeability on slope stability and seepage. They considered the permeability parameter as a spatially correlated log-normally distributed random field. Using Monte Carlo simulations for a 5.0 m high cohesive–frictional soil slope with a 30° incline, they conducted parametric studies. The studies included a range of CoV in permeability values (60 to 90%) and various correlation distances (0.5–15 m) (ref. Figs. 18 and 19). Their investigation focused on the effects of stochastic soil permeability on seepage flow statistics, strain and deformation patterns, and slope stability assessed in terms of the factor of safety (FS). The results indicated that mean seepage discharge tends to decrease as the CoV in permeability increases, and an increase in correlation distance leads to an increase in mean seepage discharge. Additionally, variability in permeability decreases the factor of safety, while an increase in correlation distance results in an increase in the factor of safety. Similar patterns were observed by Griffiths and Fenton (1993) on seepage beneath water-retaining structures founded on spatially random soil.

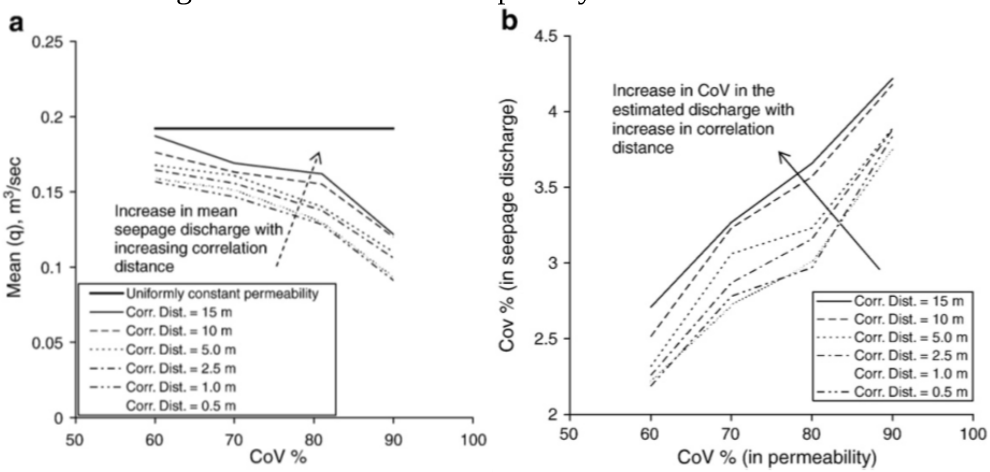


Fig. 18 Effect of CoV of permeability on the a) estimated mean of total flow rate (q, m^3/s) and b) estimated CoV of mean flow rate.

Risk-based design procedures for dam safety analysis are gaining prominence (ICOLD [62]; USBR [63]). The Bhuj earthquake in 2001 led to the failure or severe damage of numerous earth dams in Gujarat, India. Dams in Zone V were redesigned for revised earthquake loading, prompting a re-evaluation using reliability analysis. Sivakumar Babu and Srivastava [64] presented reliability analyses of four rehabilitated earth dam sections (Chang, Tapar, Rudramata, and Kaswati) under pseudo-static loading conditions. Utilizing response surface methodology, first-order reliability method, and numerical analysis, reliability index values were obtained and compared with a conventional factor of safety values. The impact of variability in soil shear strength parameters, horizontal seismic coefficient, and reservoir full-level location on stability assessment was discussed in a probabilistic framework. Comparisons with Monte Carlo simulations and limit equilibrium approach results indicated that the considered earth dam sections are reliable and expected to perform satisfactorily.

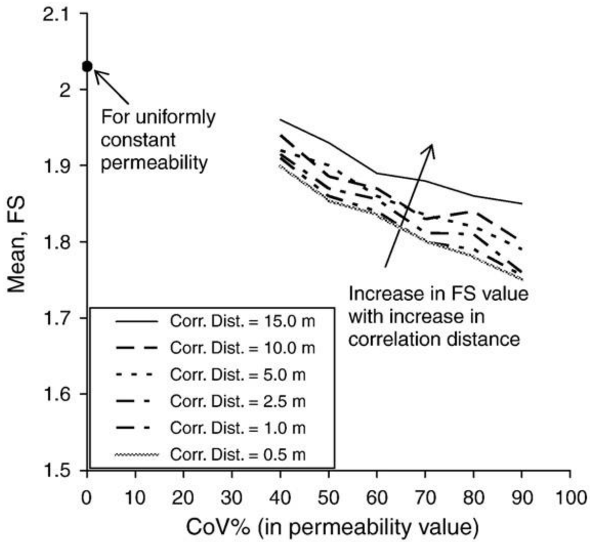


Fig. 19 Influence of variable permeability on the stability of slope under steady seepage condition

Ering and Sivakumar Babu [65] conducted a probabilistic back analysis of the Malin landslide, integrating flow and mechanical modelling to understand fluid-mechanical interactions in unsaturated soils. Results suggested that antecedent rainfall, intensity, and duration affect slope stability. Probabilistic back analysis based on Bayesian analysis considered uncertainties in soil parameters, pore pressures, field observations, and the method of analysis. The method identified a decrease in matric suction triggering landslide initiation, revealing a 100% decrease in matric suction along the slip circle and increased hydraulic conductivity as the main mechanisms.

Incorporating spatial variability, Ering and Sivakumar Babu [66] performed a probabilistic back analysis for slope failure using Bayesian analysis and random field theory. The method efficiently back-analysed the slope failure, providing confidence in parameter values for post-failure slope design and highlighting the importance of considering spatial variability in avoiding uneconomical slope remediation design.

Table. 3 Posterior statistics of cohesion and friction angle for different correlation lengths.

Autocorrelation length (m)	Prior mean	Posterior mean	Prior variance	Posterior variance
1	36	34.34	51.84	48.9
2	36	31.4	51.84	43.0
5	36	26.72	51.84	28.28

Characterization of critical rainfall is crucial for selecting suitable slope correction measures. Ering and Sivakumar Babu [67] introduced a method to identify critical rainfall thresholds, emphasizing the importance of incorporating relevant physical phenomena in the analysis. In a landslide-prone area, FLAC was employed to perform infiltration analyses using FLaIR model-identified rainfall values, offering insights into landslide kinematics. The destabilizing impact of rainfall was quantified in terms of landslide acceleration. Notably, the 2012 landslide exhibited a substantial acceleration rate of 0.508 m/s^2, causing extensive damage compared to other events.
In regions susceptible to both earthquakes and heavy rainfall, common slope stability analyses typically assess the effects of these hazards separately. However, to accurately gauge the threat posed by their potential interaction, it is imperative to consider multiple processes simultaneously. Sivakumar Babu and Ering [68] proposed a systematic methodology to predict landslide initiation under the combined influence of earthquakes and rainfall events. This probabilistic approach involves deterministic analyses in FLAC, encompassing seismic stability and infiltration analysis. Rainfall loads, treated as random variables, are incorporated using the Intensity-Duration-Frequency relationship (IDF). The study focuses on the Guwahati region, utilizing IDF curves specific to the area.

Figure 20 illustrates the density function of slope displacement, considering the interplay between earthquake and rainfall loads. It is evident that rainfall infiltration modifies slope displacement, with the extent of modification contingent on the rainfall pattern. Rainfall pattern 1 (T = 5 years) induces minimal changes, whereas pattern 2 (T = 10 years) and pattern 3 (T = 20 years) exhibit substantial modifications in the density function.

The failure probability attributed to earthquakes surpasses that of rainfall. As the return period increases, the p_f due to rainfall also rises. Importantly, the conditional failure probability, considering the interaction between earthquake and rainfall events, exceeds the failure probability of individual events. This emphasizes the necessity of accounting for the potential interaction between earthquake and rainfall events in landslide risk assessments to avoid underestimating the landslide risk.

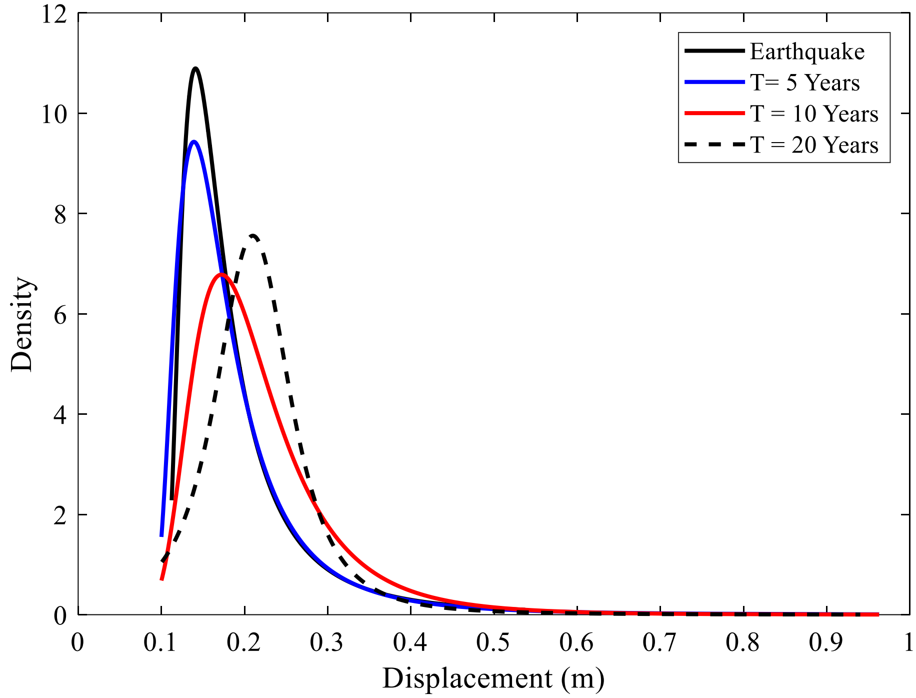


Fig.20 PDF of slope displacement due to earthquake load and combination of earthquake and rainfall

Rana and Sivakumar Babu [69] proposed a methodology employing multi-output least square support vector regression (MLS-SVR) to replicate a numerical model for slopes under precipitation at the Malin landslide site. This approach incorporates a multi-objective genetic algorithm and Bayesian analysis to update statistics of soil parameters based on observed slope data. Matric suction emerges as a significant factor influencing slope behavior under rainfall, and continuous updating of observations reduces uncertainties in soil parameters. Calculated safety factor values using updated parameters align with observed slope failures in the field.

Rana et al. [70] introduced a probabilistic back analysis methodology to estimate uncertainties in soil parameters, considering observed slope responses under seismic loading. The method involves a support vector regression (SVR) model, generated from numerical simulations of slopes under seismic loading using FLAC 2D. Probabilistic back analysis using Markov Chain Monte Carlo (MCMC) simulation demonstrates that updated soil parameters have less variability than prior distributions. Model uncertainty's impact on posterior statistics of soil parameters is investigated through a parametric study.

To Address the impact of rainfall infiltration on the response of unsaturated embankments, Showkat et al. [71] implemented the Barcelona Basic Model (BBM) to simulate the effect of varying rainfall intensities. BBM accurately models the loss of suction upon saturation, resulting in larger deformations compared to the Mohr-Coulomb model. Reliability analysis highlights the importance of probabilistic assessments for slope stability, emphasizing suction and hydraulic conductivity as critical parameters for unsaturated soil slope embankments.

Showkat and Sivakumar Babu [72] studied the introduction of geocomposites to prevent embankment failures by acting as drains. Numerical analyses, using both deterministic and probabilistic approaches, revealed that the geocomposite layer reduces surface displacements by serving as a capillary barrier and drainage layer. Probabilistic analysis, utilizing parameters such as hydraulic permeability and soil water characteristic curve (SWCC) as random variables, showed that embankments with geocomposites have lower probabilities of failure compared to those without, considering rainfall infiltration. This is illustrated in Fig. 21.

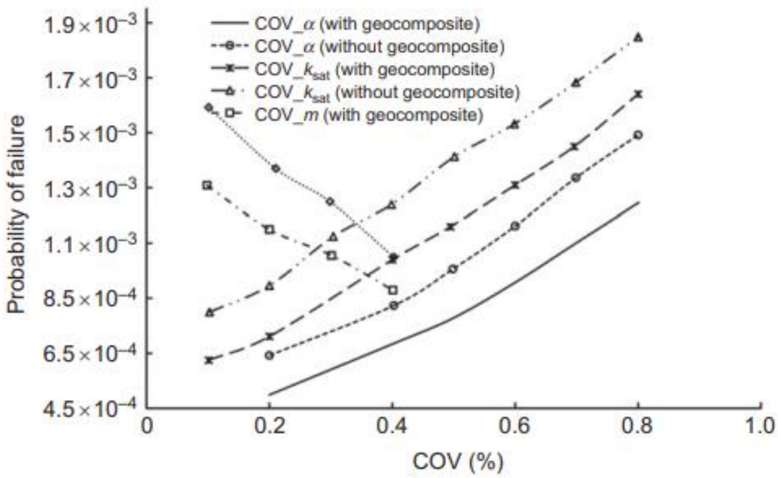


Fig. 21 Variation of the probability of failure with CoVs of different parameters

(viii) Buried pipes

Design methods and codes for flexible pipes, such as Spangler's formula for deflection and Luscher's buckling formula, are often semi-empirical and do not adequately consider the complex soil-pipe interaction, which is significantly influenced by material properties and soil constitutive behavior. Moser [73] highlighted that flexible steel pipes exhibit curvature reversal at 20%, while buried polyvinyl chloride pipes show reversal at 30%. Moser [73] suggested reducing these values by a factor of 4 to determine allowable deflections. While adherence to these limits is practical, assessing the reliability associated with these limits through rational considerations is valuable. Sivakumar Babu and Rajaparthy [74] demonstrated that variations in design parameters, such as soil modulus and bulk density of the fill impact the performance of buried pipe system. The reliability index decreases with an increase in the CoV of soil modulus and bulk density, while it increases with an increase in the correlation coefficient between these variables. It is feasible to obtain a central factor of safety (CFS) value based on the target reliability and variations in the design parameters. The commonly used factor of safety of 4 in codes for deflection calculations appears to be conservative, as indicated by the results presented in Fig.22 concerning a typical buried pipe installation.

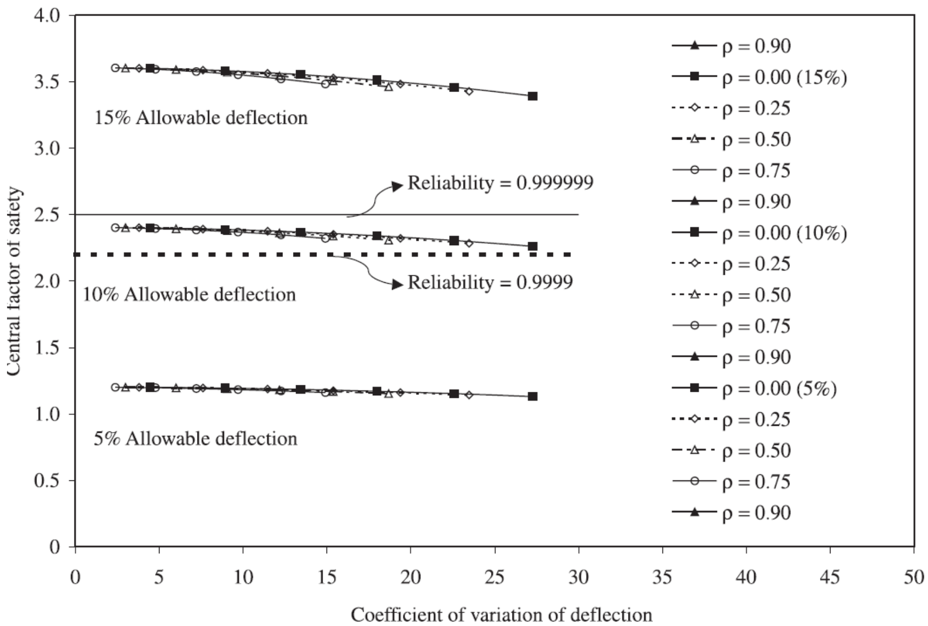


Fig. 22 Variation of Central Factor of Safety (CFS) with CoV of deflection for 5%, 10%, and 15% allowable deflection.

Pipe failure manifests through three key indicators: 1) excessive deflection, 2) actual buckling pressure surpassing critical buckling pressure, and 3) elevated tensile stress resulting from over pressurization. Extensive research has delved into these failure modes, yielding closed-form solutions and finite element analysis outcomes documented in existing literature. Sivakumar Babu and Srivatsava [75] employed response surface methodology to establish approximate functional relationships for various limit states, drawing from both analytical solutions and numerical analysis results. Their reliability analysis unveiled insights, with a comparison of buried pipe-soil system reliability analysis results suggesting that a probabilistic approach, considering input parameter variability, enhances understanding. Notably, numerical analysis yielded higher reliability index values than those obtained from available analytical equations, challenging the conservatism of conventional approaches.

(ix) Rock slopes and Tunnels

Traditional approaches to reliability-based design typically prioritize selecting the least costly design that meets safety requirements. However, these designs can be sensitive to variations in input parameters (noise parameters) and may prove inadequate if the CoV is underestimated. In addressing this concern, Pandit and Sivakumar Babu [76] proposed a reliability-based robust design (RGD) for reinforcing jointed rock slopes, demonstrated through the reinforcement of rock slopes using end-anchored rock bolts. This approach ensures the selection of a cost-effective and safe design that minimizes the sensitivity of the probability of failure (p_f) to noise parameters. The reliability-based RGD approach involves evaluating p_f for various designs considering different noise parameters. Due to the computational expense of finding p_f for complex geotechnical structures, an augmented radial basis function-based response surface serves as a surrogate to the finite element model of the rock slope. This efficient response surface is then used with the minimum distance algorithm to obtain a cost-effective and robust design. A cost comparison between two robust designs for different target probabilities of failure illustrates the method's advantages. The simulation of field conditions for seismically induced slope failures incorporates model uncertainties, accounting for differences between simulated and observed slope behavior.

Pandit [77] explored heterogeneity effects in rock masses using random field approaches characterized by the statistics/moments of rock mass properties and auto-correlation structure. The auto-correlation structure includes the correlation function and the scale of fluctuation (SOF) parameter. Realizations of isotropic and anisotropic random fields of the unconfined compressive strength (UCS) of rock with different SOF values are presented. For example, with SOF = 2 m, UCS variation is correlated over short distances, resulting in a rough spatial distribution. In contrast, with SOF = 64 m, increased spatial correlation leads to a more uniform spatial distribution of UCS values. The study found that different failure mechanisms arise due to these variations in spatial correlation. For instance, a lower SOF (2 m) results in a tortuous and longer slip surface, yielding approximately similar factors of safety (FOS) values for different realizations. Conversely, a higher SOF (64 m) leads to a smoother slip surface, causing significant FOS variations (ref. Fig. 23). This change in mechanism results in lower p_f for lower SOF (2 m) and higher p_f for higher SOF (64 m). This observation is supported by the study of an anisotropic random field with SOF_x = 64 m and SOF_y = 4 m, which shows a relatively tortuous slip surface and lower p_f (order of 10^{-5}) compared to the random variable method (p_f of the order of 10^{-4}).

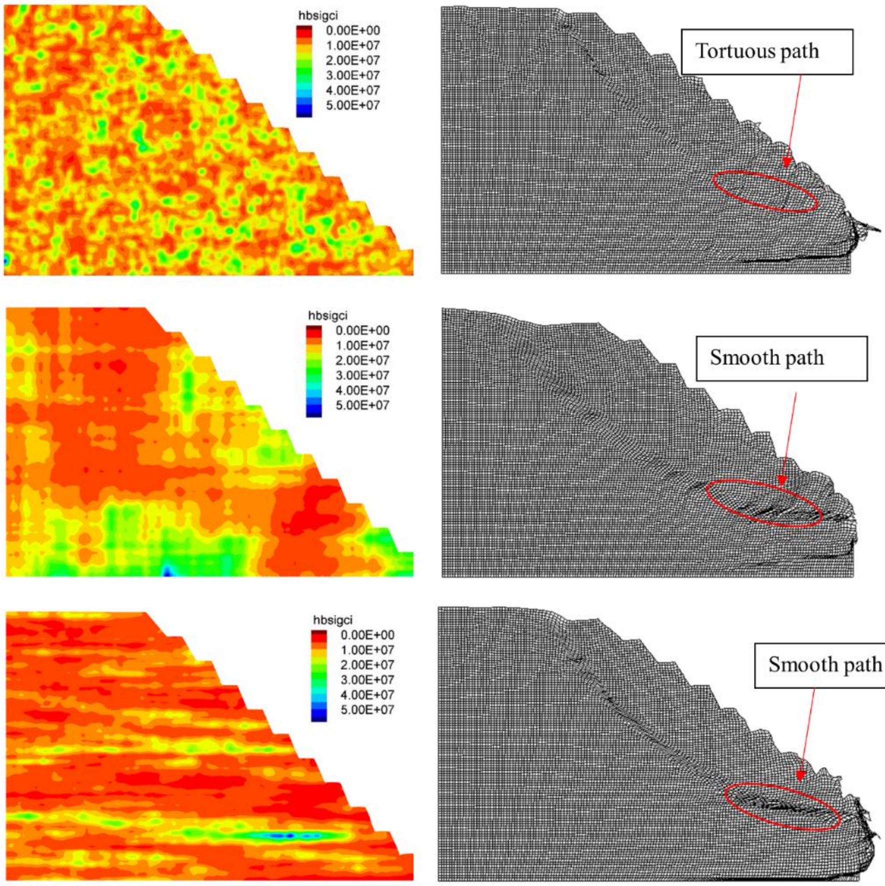


Fig. 23. Random field realization and failure mechanism with SOF [x,y]: [2 m,2 m] (top), [64 m, 64 m] (middle) and [64 m, 4 m] (bottom)

Pandit and Sivakumar Babu [78] investigated the behavior of a horseshoe-shaped tunnel excavated in weak rock mass through deterministic, random variable, and random field methodologies. The tunnel's performance in probabilistic analysis was evaluated by establishing three limit states: maintaining tunnel convergence below a safe threshold, ensuring proper embedding of rock bolts beyond the depth of yielded rock mass, and preventing the load induced on the liner support from exceeding its capacity. Both unsupported and supported tunnels were subjected to analysis using deterministic, random variable, and random field approaches. To discretize the random fields, the Fourier series method was applied, and Monte Carlo simulations were employed for random finite difference analysis. The study explored the influence of the scale of fluctuation (SOF), varying isotropic SOF for random fields, and horizontal and vertical SOF ratios for anisotropic random fields on the tunnel's performance and associated failure mechanisms. The findings highlighted the significant impact of SOF on output statistics and the p_f for the defined limit states. Notably, it was observed that the random variable approach tended to underestimate the tunnel-support system's performance. However, in the absence of data required for random field characterization, this approach could be considered as conservative option.

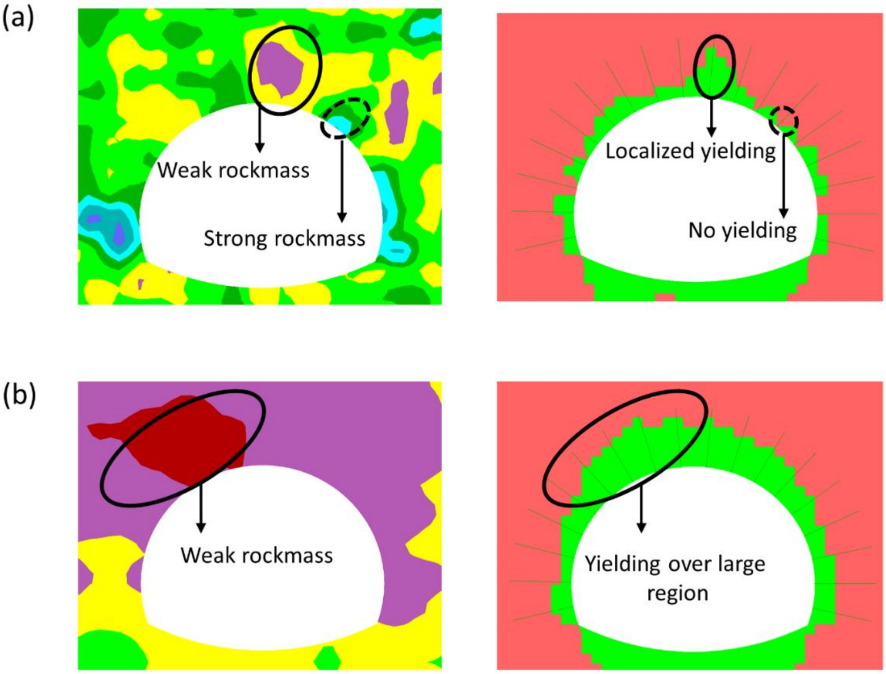


Fig. 24 (a) Localized yielding observed for SOF = 3 m (probability of occurrence of this type of yielding is high) (b) Yielding of rock mass over a large region for SOF = 60 m (probability of occurrence of this type of yielding is low)

The p_f in tunnels due to insufficient bolt length (p_f^B) shows the opposite trend compared to the rock slopes case, i.e., p_f^B decreases with an increase in the SOF. Figure 24 explains the reasons behind this phenomenon and attributes it to localized yielding along the tunnel boundary for low SOF values, which is observed in several realizations increasing the p_f^B. For higher SOF, global yielding takes place but in fewer realizations, leading to lower p_f^B. Further studies on probabilistic analysis of tunnels considering uncertainty in peak and post-peak strength parameters and analysis of tunnel support requirements using deterministic and probabilistic approaches in average quality rock mass are conducted by Tiwari et al. [79,80].

(x) Landfill engineering

Variabilities in material properties and model uncertainties significantly influence the design and analysis processes in landfill engineering. Municipal Solid Waste (MSW) introduces higher variability in geotechnical properties compared to geological materials, with influences stemming from the composition of MSW and time-dependent factors such as creep and biodegradation. Given the potential high costs and consequences associated with the failure of MSW slopes in landfills, stability analysis methods must account for this variability, making reliability analysis essential. Sivakumar Babu et al. [81] presented the outcomes of a probabilistic slope stability analysis conducted on a typical MSW landfill slope. The analyzed slope, with a height of 30 m and a slope ratio of 1V:3H, considered the spatial variation of geotechnical properties within the MSW, comparing scenarios of a single depth layer to a multilayered depth configuration. Figure 25 depicts a typical section of the landfill, and Table 4 outlines the properties in each layer of MSW, taking into account various degrees of decomposition during the analysis.

The published data on the geotechnical properties of MSW is used to define statistics of spatial variation of geotechnical parameters. The application of random field theory, coupled with the finite difference numerical code FLAC (Fast Lagrangian Analysis of Continua), generates a two-dimensional non-Gaussian homogeneous random field through the Cholesky decomposition technique. Monte Carlo simulations are conducted to determine the stability statistics of the MSW landfill slope, specifically in terms of the factor of safety. This information is then employed to evaluate performance within a probabilistic framework. The findings are compared and discussed in relation to a conventional factor of safety approach, where geotechnical parameters are assumed to be uniformly constant. The overall results indicate a decrease in reliability indices with an increase in MSW property variation, emphasizing the necessity to consider a multilayered MSW profile. These considerations resulted in reduced reliability indices compared to the results obtained by considering a single MSW layer for the entire depth.

Table. 4 Properties in Each Layer of MSW with Degree of Decomposition

Layer	Depth to mid-layer (m)	Degree of decomposition (%)	Unit weight (kN/m^3)	c (kPa)	ϕ (°)ξ
10	1.5	0 (fresh)	7.5	10.0	30.0
9	4.5	20	8.7	19.2	28.3
8	7.5	30	9.3	23.8	27.5
7	10.5	40	10	28.3	26.6
6	13.5	50	10.6	32.9	25.8
5	16.5	60	11.2	37.5	25.0
4	19.5	70	11.8	42.1	24.1
3	22.5	80	12.4	46.7	23.3
2	25.5	90	13.0	51.3	22.4
1	28.5	100	13.6	55.8	21.6

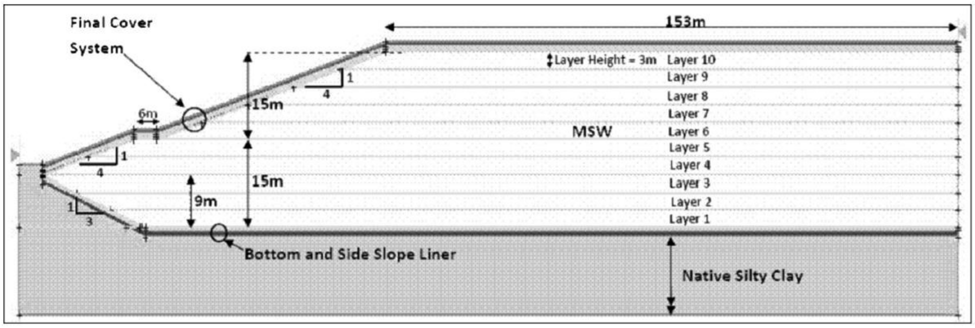


Fig. 25. Cross-sectional details of MSW landfill

In the context of bioreactor landfills, leachate recirculation has become a crucial operational component due to its environmental benefits aligned with sustainability considerations. Reddy et al. [82] conducted an analysis of leachate distribution in the Orchard Hills Landfill, Davis Junction, Illinois. Using a two-phase flow model, the study assessed the influence of hydraulic conductivity variability on the effectiveness of the leachate recirculation system through reliability analysis. Numerical modeling, employing finite-difference code, accounted for the spatial variation of hydraulic conductivity within the MSW, assuming an inhomogeneous and anisotropic waste condition for a realistic representation. The reliability analysis involved dividing the landfill into 10 MSW layers with varying mean values of vertical and horizontal hydraulic conductivities, and parametric studies considered CoV's ranging from 50% to 200%. Monte Carlo simulations provided statistical information on output parameters, including the wetted area of the MSW, maximum induced pore pressure, and leachate outflow. The results of the reliability analysis were instrumental in determining the influence of hydraulic conductivity on leachate recirculation effectiveness. The study identified two critical parameters defining the efficiency of the leachate recirculation system: i) the percentage area of influence and ii) the ratio of excess porewater pressure to total stress. Comparisons between deterministic cases, spatial variability modeling, and Monte Carlo simulations underscored the significant impact of spatial variation in hydraulic conductivity on various output parameters. These findings emphasize the importance of considering such variations when assessing the performance of bioreactor landfills. The reliability analysis results provided guidelines for enhancing the bioreactor landfill's performance, suggesting that the percentage area of influence of MSW should not be less than 60%, and the ratio of pore pressure to total stress can be considered as 0.52 for configuring the leachate recirculation system and determining injection rates.

Sivakumar Babu et al. [83] introduced an innovative approach for utilizing landfill settlements to formulate closure plans based on the variability of design parameters, including compression index and two parameters, each associated with creep and biodegradation. Incorporating these five design variables into the constitutive model developed by Sivakumar Babu et al. [84], multi-linear equations were derived through response surface methodology (RSM). These equations, specific to boundary conditions, mean property values, and CoV, are invaluable for reliability calculations. While not easily generalized, these equations offer precision in predicting likely settlements and contribute to the development of landfill closure plans. The proposed methodology is illustrated with a typical example, showcasing the integration of variability in parameters and reliability analysis. The bioreactor concept, coupled with a gas collection system (GCS) for leachate recirculation, serves as an effective strategy to stabilize landfills and expedite settlements while harnessing gas for practical purposes. Despite numerous studies on bioreactors in the literature, the detailed performance of a bioreactor landfill with a GCS remains largely unexplored. Given the temporal and spatial variabilities in waste properties, accounting for these variations is crucial in GCS design. Parameswaran et al. [85] investigated the GCS performance for a prototype bioreactor using kriging surrogate models to accommodate waste property variability. Numerical models implemented through TOUGH3 EOS7CA three-dimensional simulations calibrated the kriging models. The radius of influence (ROI), a critical parameter influencing GCS design, was estimated using methane generation rate (MGR), suction pressure (Sp), absolute permeability (k), and depth of the extraction well (D) as input parameters.

Gas pressure distributions and the ROI inside the bioreactor were determined, considering input parameter variabilities through Monte Carlo simulations on the Kriging model. Initial observations of gas pressures within the prototype bioreactor, equipped with a single gas extraction well (GEW) system, raised safety concerns due to pressure values exceeding atmospheric levels. A sensitivity analysis emphasized the critical role of Sp and MGR in controlling the ROI. Consequently, the GCS design was optimized with two GEWs, and safety assessments varied only the critical parameters using another developed kriging model. Gas pressures within the safe range of atmospheric pressure confirmed the effectiveness of the design. A relation between Sp and MGR was established, enabling a 90% methane recovery at the GEWs. The p_f for the prototype bioreactor, considering this developed relation, was estimated to be low (1.23×10^{-4}), affirming the safety of the design. The methodology was extended to an actual landfill, enhancing comprehension of the design, as depicted in Fig. 26, showcasing the gas collection system (GCS) plan.

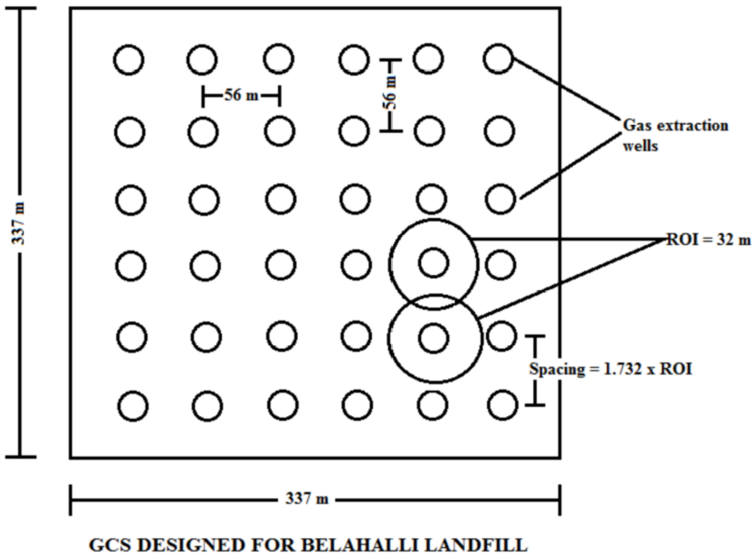
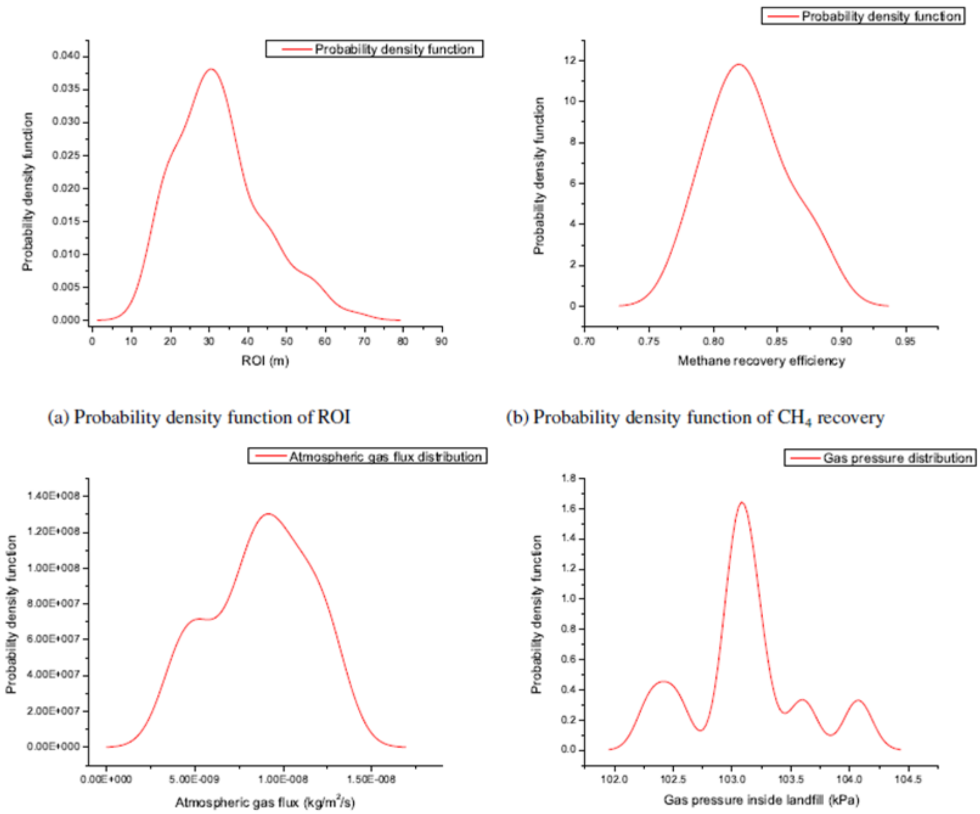


Fig. 26 Plan showing the gas collection system

To optimize gas extraction efficiency, designing a GCS involves modeling three-dimensional, multiphase, multicomponent gas migration in a landfill, considering spatio-temporal variabilities in geotechnical waste properties (i.e., heterogeneity) and uncertainties through stochastic models. Parameswaran et al. [86] proposed a method for calculating the radius of influence (ROI) of a gas extraction well (GEW) and designing a GCS for such landfills. The method utilizes TOUGH simulator to model three-dimensional gas migration, and stochastic models are employed to estimate geotechnical properties for simulator calibration. Gas flows at specific spatiotemporal locations are determined, and Kriging is utilized to obtain gas flow values at remaining locations, facilitating ROI calculation. To ensure comprehensive gas collection across the landfill area, 36 gas extraction wells (GEWs) are strategically placed, adhering to established heuristic guidelines. This study validates the rationale behind these guidelines. Under heterogeneity, a passive GCS achieves a gas recovery efficiency of 0.82 and a gas pressure of 103 kPa. Figure 27 illustrates typical results. In contrast, a homogeneous design yields an efficiency of only 0.56, highlighting the significance of the heterogeneous model. With an active GCS, the efficiency improves to 0.86. The framework introduced in this study can enhance gas flow estimation in large landfills, facilitating the design of secure GCS in landfills to minimize methane emissions.



(a) Probability density function of ROI (b) Probability density function of CH_4 recovery

Fig. 27. Probability density functions of ROI, Methane gas recovery, atmospheric gas flux and gas pressure inside landfill

(xi) Contaminant Transport in soils and rocks

Santhosh and Sivakumar Babu [87] presented an assessment approach for evaluating the risk associated with a landfill liner system contributing to groundwater contamination due to chloride migration in proximity to a landfill site. Response Surface Methodology (RSM) was employed, taking into account the properties of Municipal Solid Waste (MSW), primary liner systems, and the attenuation layer for the analysis. The considered variables included initial chloride concentration, clay diffusion coefficient, liner soil porosity, aquifer porosity, and base outflow velocity. The study encompassed two cases: Case I incorporated different CoV values based on literature data, representing realistic scenarios, while Case II assumed a uniform CoV of 5% for all variables, a common practice in reliability calculations. This approach aimed to investigate the variation of regression coefficients in the two cases and their impact on reliability analysis results, as depicted in Fig. 28, showing a typical set of results.

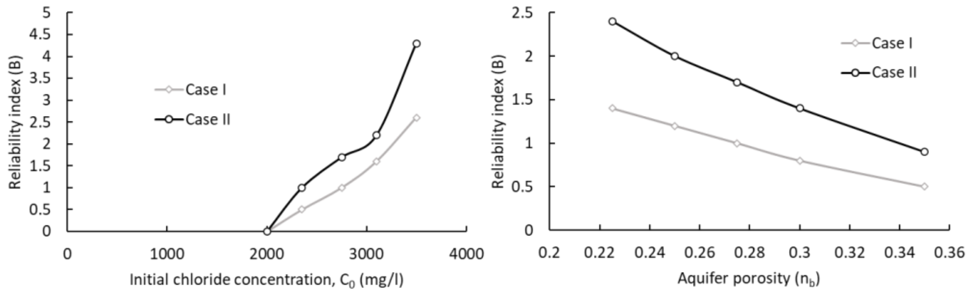


Fig. 28. Variation of β with the variation of initial chloride concentration and aquifer porosity

It is crucial to highlight that liner porosity (n) emerges as the critical variable, followed by initial chloride concentration (C_0), clay diffusion coefficient (D), aquifer porosity (n_b), and base outflow velocity (V_b). Proper estimation of these parameters as variables is essential to accurately assess the risk of liner system failure and subsequent groundwater contamination. Additionally, the choice of permissible chloride concentration, which varies among different countries, can influence the reliability index. It's important to note that the developed response surface equations are valid only for the conditions considered in the analysis, and computational complexity may increase with an expanding number of variables.

Santhosh et al. [88] presented a reliability-based approach to evaluate the performance of a landfill, considering uncertainties associated with the hydraulic properties of various system components. The model domain, comprising a final cover system and a municipal solid waste (MSW) layer, is treated as an integrated system for analysis. Water balance is computed using the Hydrologic Evaluation of Landfill Performance (HELP) model to construct response surfaces for reliability analysis. The probability of leachate head accumulation at the bottom of the MSW layer above an allowable threshold serves as the criterion for reliability estimates in this study. The results are discussed in terms of the reliability index, and the impact of variations in (a) hydraulic characteristics of various layers and (b) defects and placement conditions of the geomembrane (GM) on system performance are explored. The analysis reveals that the hydraulic conductivity of the compacted clay layer (CCL) in the cover component, along with the porosity and saturated hydraulic conductivity of the MSW layer, are significant parameters influencing reliability. Furthermore, the study includes an exploration of the impact of model uncertainty and parametric analysis of significant variables, with the results presented in Fig. 29 outlining the approach followed for the analysis.

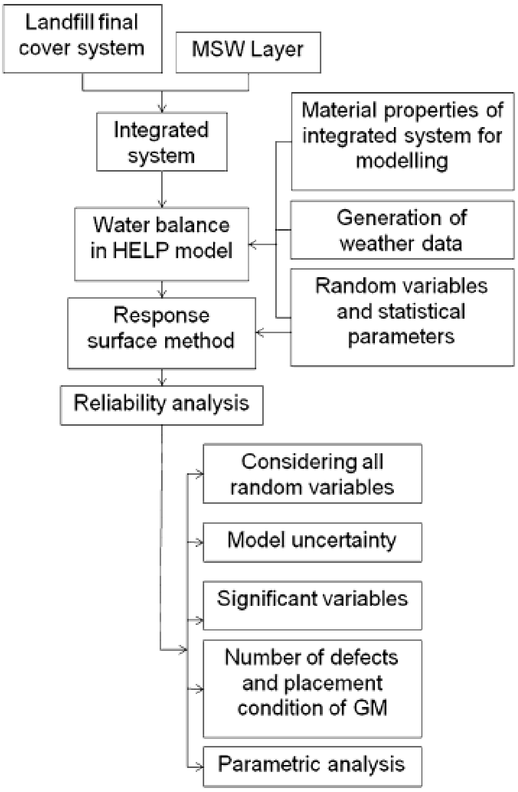


Fig. 29. Flow chart of the analysis approach

Figure 30a shows a typical result. In the scenario with a well-functioning geomembrane (GM) (Fig. a), β values decrease with a leachate head increase. The acceptable value of β = 3 is attained at h_{ac} ≈ 37.5 cm, resulting in a corresponding p_f for the integrated system of 0.13%. This value aligns reasonably closely with the permissible limit of 30 cm according to USEPA [89]. Turning to Fig. 30b, portraying the case of a defective GM, β values decrease linearly with increasing values from 0 to 70 cm. It is noteworthy that values are lower for uncorrelated random variables and higher for correlated random variables. For uncorrelated variables, the leachate head is 52 cm, while for correlated variables, it is 57 cm, both corresponding to β = 3.

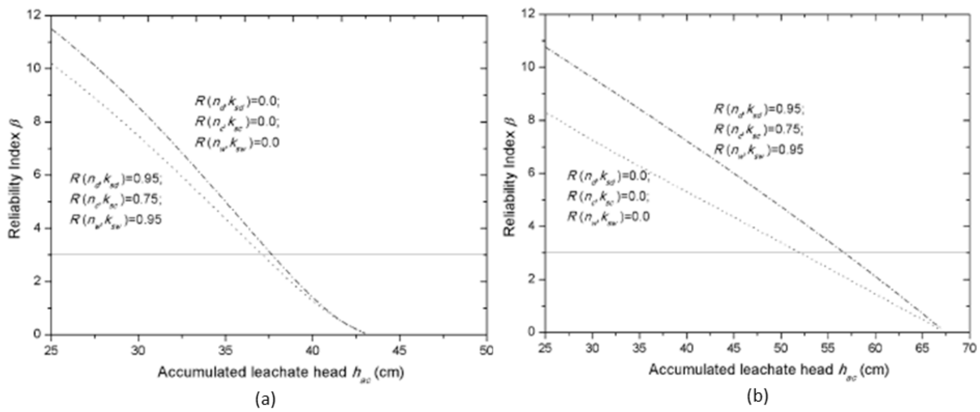


Fig. 30. Variation of β with accumulated leachate head for good GM and defective GM

Additionally, this investigation explores the impact of the number of geomembrane (GM) defects and their placement conditions on reliability, accounting for correlated random variables. Various scenarios encompassing poor, good, and excellent placement conditions of the GM, coupled with varying numbers of defects, are examined. Each defect is assumed to be a 1 cm^2 hole in the GM. The outcomes of these scenarios are detailed in Fig. 31.

The results reveal a significant reduction in β values with an increasing number of defects. For achieving β = 3, a maximum of 26 defects is allowable under poor placement conditions, and this threshold increases for cases with good and excellent placement conditions of the geomembrane (GM). Therefore, the number of permissible defects should be less than 26, 180, and 450 in poor, good, and excellent placement conditions, respectively. The volume of leachate generation (expressed in m^3/ha/yr) due to precipitation percolation through the defective GM is a critical consideration. Poor placement of the GM demonstrated a comparatively high leachate volume (4.64 × 103 m^3/ha/yr), escalating to 10.2 × 103 m^3/ha/yr with an increase in the number of defects from 7 to 1000. Under good and excellent placement conditions, the GM generated leachate in the range of 3.57–6.92 × 103 m^3/ha/yr and 3.38–6.43 × 103 m^3/ha/yr, respectively. Reliability analysis results for the integrated system indicate that a Compacted Clay Liner (CCL) overlain by a well-placed GM, especially in excellent contact, effectively reduces infiltration, leading to a more reliable system, a conclusion also supported by Rowe et al. [90].

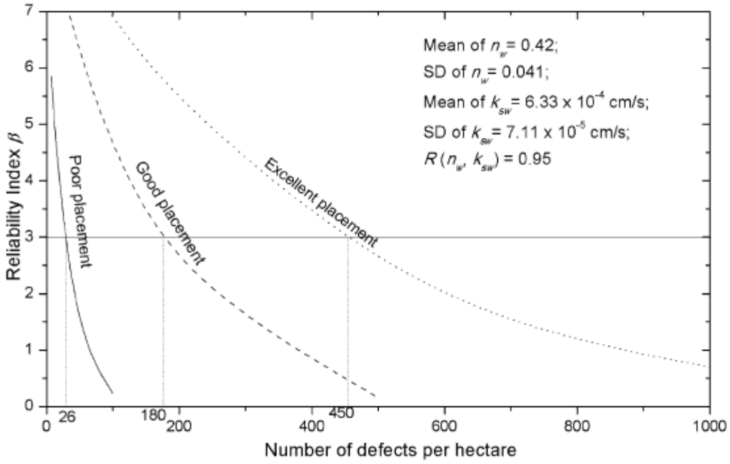


Fig. 31. Variation of reliability index β with number of defects and placement condition of GM

(xii) Transport of radionuclides

The effectiveness of near-surface disposal facilities (NSDFs) in preventing the migration of radionuclides for low-level radioactive waste is contingent on the materials used in the barrier system construction, the geological formations surrounding the facility, and the type of waste containment system. Addressing the impact of these factors, Sujitha et al. [91] formulated a mathematical model to assess radionuclide migration using a contaminant transport model from NSDFs to the nearest geosphere, providing a comprehensive evaluation of the entire system's performance. The model considers scenarios during the dumping period, after the dumping period, and after the closure of the NSDF. The annual radiation dose values resulting from radionuclides through the drinking water pathway (for both single and multiple dump modes) are calculated using the model. A thorough analysis is conducted to estimate the radiation doses for the radionuclides ^{3}H, ^{14}C, ^{59}Ni, ^{99}Tc, ^{129}I, ^{237}Np, and ^{239}Pu in various disposal modes. The results, depicted in Fig. 32, illustrate concentration profiles extending over significant temporal scales. Among the considered radionuclides, radioactive carbon (^{14}C) exhibits the highest concentration in groundwater. To account for parameter variability in the model and quantify uncertainties arising from inherent variability, the presence of a heterogeneous medium, and variability associated with long-time scales of interest, a surrogate modelling technique known as collocation-based stochastic response surface method (CSRSM) is employed. This method approximates a complex analytical equation with a higher-order polynomial, utilizing polynomial chaos expansion (PCE).

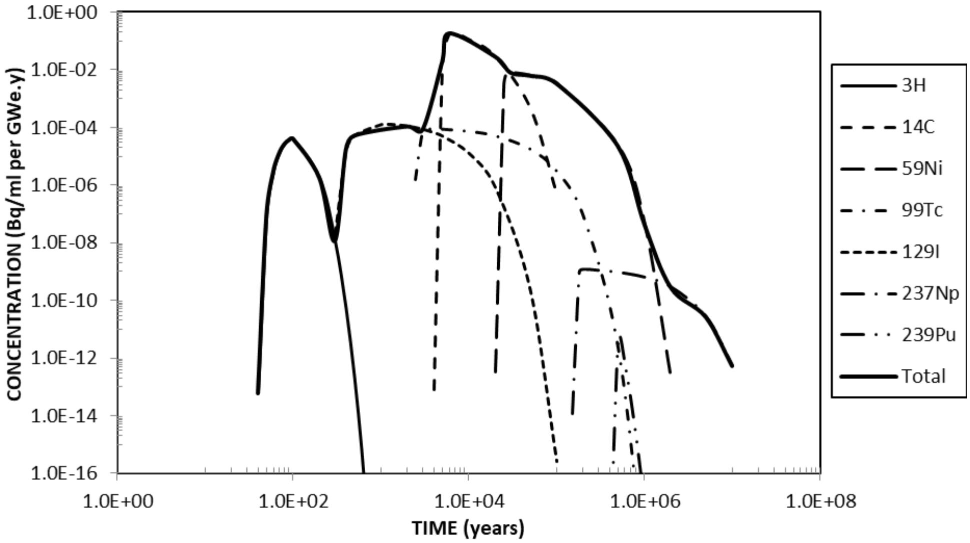


Fig. 32. Time history of radionuclide concentration in groundwater at 1.6 km parallel to flow

For the probabilistic analysis of radioactive carbon (^{14}C), groundwater velocity, thickness of the unsaturated zone, dispersivity, and distribution coefficient were treated as random variables. In all scenarios, a third-order polynomial was found to provide the best fit for the model, with a coefficient of determination (R^2) reaching 0.99 for the third-order polynomial. Reliability analysis was conducted, and the p_f for an annual radiation dose of ^{14}C exceeding the permissible limits was estimated across various scenarios and results are presented in Table 5.

The results presented in Table 5 highlight the computational efficiency of the surrogate model, requiring only a few seconds to execute Monte Carlo simulations for reliability analysis. Additionally, it is noteworthy that the probability of system failure in all scenarios is very low, confirming the adequacy of the system. Sujitha et al. [92] developed a time-dependent reliability analysis for radionuclide migration in groundwater in a near-surface disposal facility was performed using the enhanced Monte Carlo method. The Enhanced Monte Carlo (EMC) method was compared with the Subset Simulation (SS) method for estimating the first-passage probability, achieving computational times of 20 minutes and 42 minutes, respectively. Examining potential failure modes of a barrier system for a Near-Surface Disposal Facility (NSDF), Sujitha and Sivakumar Babu [93] identified failure of the top cover, failure of the waste container, degradation of the waste form, and failure of the bottom cover as independent failure events. The study emphasized the importance of considering system reliability in NSDF, revealing a high degree of dependence between failure modes and demonstrating the probability of simultaneous failures. The study recommended the use of optimization techniques for evaluating the p_f, providing a better estimate, as validated by results obtained from Monte Carlo simulations.

Table 5. Comparison of p_f for different cases

Dumping mode	Probability of failure (p_f)	Reliability Index (β)	Time for Computation using mathematical equation (seconds)	Time for Computation using CSRSM equation (seconds)
Single-dump 1D	0.0075	3.432	2182.4	1.05
Single-dump 2D	0.0083	3.77	2357.5	0.9
Multiple-dump 1D	0.0085	3.387	2863.5	1.2
Multiple-dump 2D	0.0026	3.807	3332.9	1.3

Geetha Manjari and Sivakumar Babu [94] developed a three-dimensional groundwater contaminant transport model with a decaying source to determine radiation doses at different points of interest for short-lived (Strontium (^{90}Sr), Caesium(^{137}Cs)) and long-lived radionuclides (Carbon(^{14}C) and Iodine(^{129}I)). The study considered uncertainties in input parameters, propagating them using CSRSM technique based on Polynomial Chaos Expansion (PCE). The potential risk to a critical group through the drinking water pathway was calculated for all the radionuclides, with maximum values observed lower than the risk due to industrial accidents and natural catastrophes ($1x10^{-3}$–$1x10^{-4} y^{-1}$). The developed code using the Python interface of FEFLOW made the model computationally efficient, and surrogate models were employed to reduce the computational effort, presenting concentration trends over time for radionuclides Carbon and Strontium at various observation points in Fig. 33.

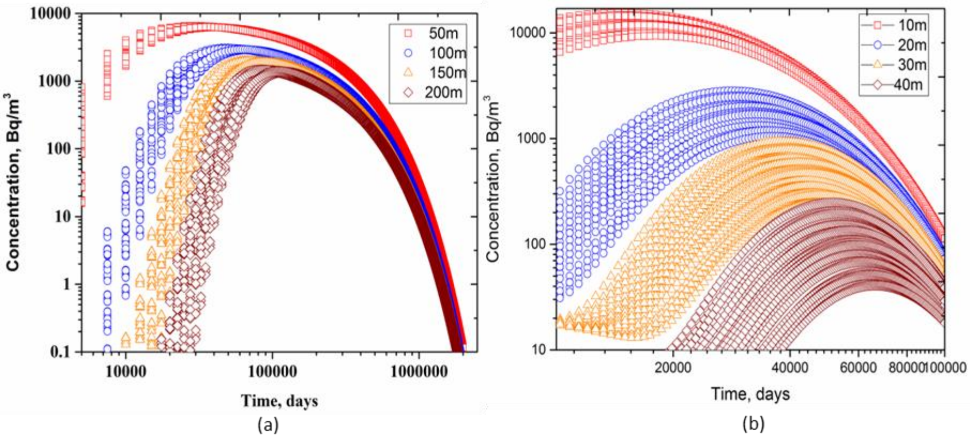


Fig. 33. Concentration trends evolving over time for radionuclides (a) carbon and (b) Strontium at various observation points from the source

Table 6. Probability of failure values for short-lived and long-lived radionuclides

Sr. no.	Radionuclide	Permissible limit (mSv/yr)	Probability of failure (p_f)	Computational time
1	Caesium	0.4	2E-10	10
2	Iodine	0.7	8E-11	12

Hence, a more accurate understanding of the most influential parameters significantly reduces the model prediction uncertainty. Additionally, surrogate models generated from CSRSM were employed for reliability analysis. Using the Subset Simulation method, the study estimated the probability of the radiation dose exceeding the permissible threshold through the drinking water pathway, with the results presented in Table 6.

The consistently low values of failure probability (as presented in Table 6) indicate that the system is secure from the risk associated with radiation through the drinking water pathway. Geetha Manjari and Sivakumar Babu [95] focused on the long-term safety of radioactive waste disposal facilities by developing probabilistic performance assessment models. The assessment included endpoints such as radiation dose and risk due to disposal practices. A two-dimensional radionuclide transport model with a decaying source was numerically modeled to compute the radiological impact caused by radionuclide iodine (^{129}I) in the biosphere. The performance assessment model addressed inherent uncertainties and variabilities in the system, including inherent spatial variability in the geological medium. The study treated the hydraulic conductivity of the medium as a log-normal random field and discretized it using Karhunen-Loeve (K-L) series expansion to account for inherent spatial variability. Contaminant concentration profiles were compared between stochastic spatial variable cases and deterministic cases, as illustrated in Fig. 34. In the deterministic case (homogeneous medium), the radionuclide showed a maximum concentration value of 8000 Bq/m^3, whereas in the spatially variable case (heterogeneous medium), the maximum concentration value increased to almost 10000 Bq/m^3 at its mean. The collection of concentration profiles obtained from the random field realizations, represented by grey lines, cover a range of possible concentration profiles. Ignoring the fluctuations in concentration trends (i.e., in the deterministic case) can lead to an underestimation or overestimation of the system's performance.

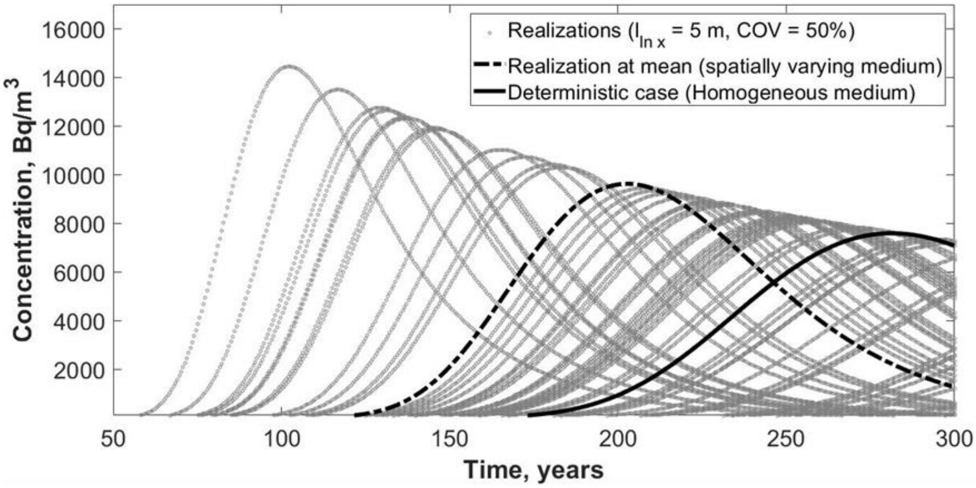


Fig. 34. Concentration versus time for various cases

Further, Fig. 35 displays contours of radionuclide concentration in the domain for both homogeneous and different spatially variable cases. The observation point is situated 80 meters from the source, marked as "×" in the figure. Concentration contours for a homogeneous medium are shown in Fig. 35(a), while various cases of spatially varying media are presented in Figs. 35(b–d). In Fig. 35(a), the contours are more spread, indicating greater dispersion and concentration at the endpoint compared to other cases [Figs. 35(b–d)]. These results suggest that the migration of radionuclides is faster in the homogeneous medium than in the spatially varying medium. This is because the flow of radionuclides is mainly driven by advection, and factors contributing to the advective process influence the final concentration at the endpoint. A higher hydraulic conductivity, indicative of high seepage velocity, increases the advective and dispersive transport processes. As the autocorrelation length increases (indicating a stronger correlation of hydraulic conductivity), the mean rate of flow of the radionuclide in groundwater also increases. Therefore, in Figs. 35(b and c), the concentration front advances quicker in the latter case due to a higher correlation length. Additionally, a slight effect of the CoV on the concentration front was observed by comparing Figs. 35(c and d). These results emphasize the importance of considering spatial variability in the assessment of contaminant transport problems.

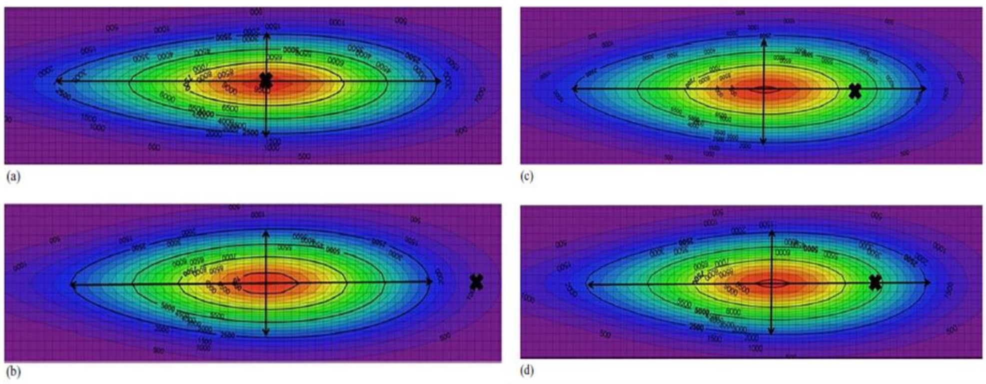


Fig. 35. Concentration contours in (a) homogeneous medium; (b) spatially varying medium ($l_{\ln x} = 2$ m; CoV = 50%); (c) spatially varying medium ($l_{\ln x} = 5$ m; CoV = 50%); and (d) spatially varying medium ($l_{\ln x} = 5$ m; CoV = 10%)

In addition, a reliability analysis was conducted to assess the impact of spatial variability in hydraulic conductivity on the performance of the disposal system. The probability of the radiation dose exceeding the design limit was estimated using the subset simulation method. The study also investigated the effect of autocorrelation length and the CoV of hydraulic conductivity on the p_f and the migration behavior of radionuclides. Thus, the results of the reliability analysis for various cases are illustrated in Fig. 36.

In the figure, it can be observed that the range of p_f lies between $10^{-9} – 10^{-1}$. Trends in the data reveal that an increase in the CoV of the random field corresponds to an increase in the probability of system failure. Additionally, the p_f rises with an increase in the auto-correlation length. Notably, increasing the auto-correlation length from 2 m to 5 m results in a reduction in p_f by a factor of 10^4. Moreover, an increment in the auto-correlation distance from 5 m to 10 m and beyond leads to a 10 times reduction in p_f, although this reduction is not highly significant. The influence of auto-correlation length appears to be most prominent between 2 m and 5 m. In erratic (small correlation length) conditions, the scale of fluctuation in conductivity values increases along the length, causing slower radionuclide travel. Conversely, in smoother (large correlation length) conditions, the medium becomes less heterogeneous, allowing the radionuclide to move faster. The p_f values for homogeneous soil are higher than those for spatially varying soil. These results underscore the importance of a probabilistic framework that considers spatial variability in the geological medium for assessing the performance of radioactive waste disposal facilities.

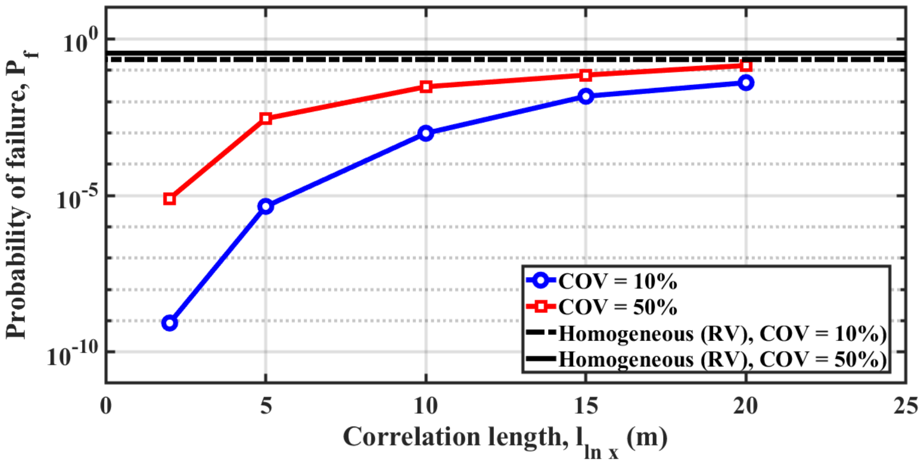


Fig. 36. Probability of failure versus correlation length for different cases

Moving beyond soils, fractured rock masses represent natural geological formations consisting of a complex network of fractures and the intact rock matrix. Geetha Manjari and Sivakumar Babu [96] developed a two-dimensional numerical model using the discrete fracture network approach to study groundwater flow and contaminant transport behavior in fractured rock masses. The model incorporates the effect of aperture variation along fractures as an additional feature. Simulating contaminant transport through a heterogeneous fracture network and intact rock matrix, the model computes contaminant concentration over spatial and temporal scales. The results of contaminant plume movement through a 0°-45°-90° fractured rock network is presented in Fig. 37.

The contaminant plume exhibits distinct shapes along the x-direction and y-direction, indicating that the complex network of fractures introduces significant heterogeneity, resulting in an irregular pathway for contaminant migration. Additionally, the fracture network and its connectivity serve as the primary pathway for the flow and movement of contaminants. Therefore, factors such as fracture geometry, fracture orientations, and local aperture variations in fractures significantly influence contaminant movement through fractured rock masses. Further, the probability of contaminant concentration exceeding the permissible limit is estimated using the subset simulation method. The investigation into the influence of stochasticity in the fracture network reveals that each fracture arrangement (i.e., each realization) has a unique failure probability, indicating its significance on the contaminant transport process in the fractured rock mass.

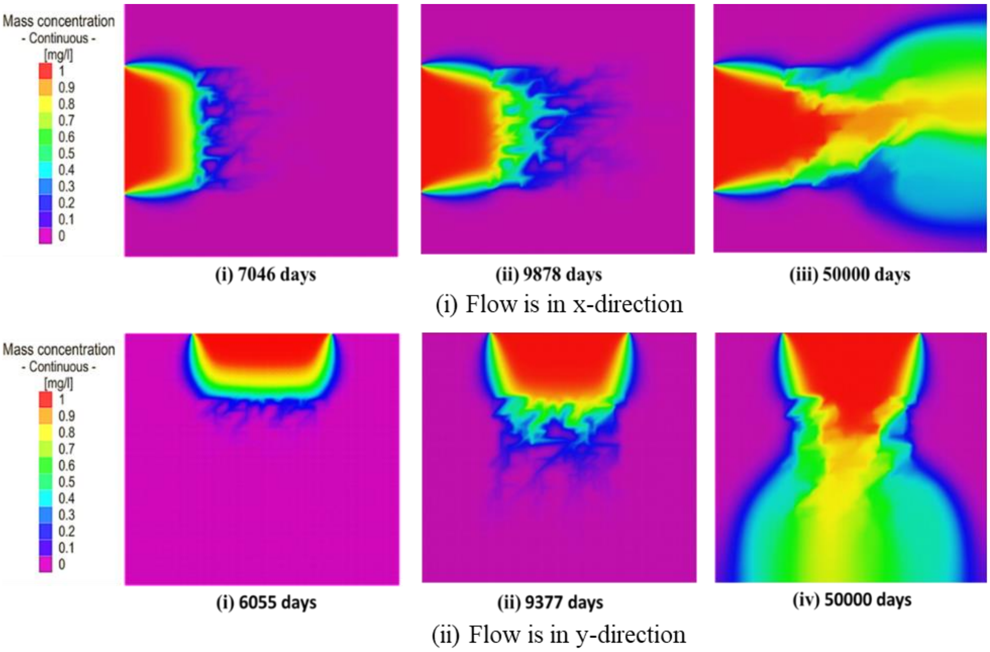


Fig. 37. Snippets of concentration plume evolving in time for 0° - 45° - 90° fracture set

Moreover, p_f values are estimated for various fracture sets, as presented in Table 7. The results indicate that for the 45° fracture network, p_f is very high. This is attributed to the presence of a series of long, parallel fractures that conduct the flow of contaminants toward the endpoint, resulting in a high probability of contamination. In contrast, for two and three fracture sets, the range of p_f values is relatively low. These findings highlight the significant influence of fracture geometry, including orientation and the number of sets, on the model's performance.

Table 7. Probability of failure for different fracture sets (CoV of 40%)

Fracture set	**45°**	**45° - 90°**	**0° - 45° - 90°**	**0° - 90°**	**0° - 45° - 135°**
Probability of failure	0.6	3.2×10^{-3}	1.3×10^{-3}	4.6×10^{-4}	1.3×10^{-5}

The findings highlight the importance of incorporating variations in aperture sizes and addressing their uncertainties when modeling contaminant transport in fractured rocks. The probabilistic framework established through this research offers valuable insights for engineering practitioners. It aids them in identifying critical areas that demand utmost attention when planning contaminant site remediation or designing waste disposal systems. This comprehensive approach acknowledges the inherent variability and uncertainty in the geological medium, contributing to more robust and reliable assessments in geoenvironmental engineering practices.

Probabilistic nature of geomaterials

The macro variability observed in geotechnical materials in practice can be attributed to the inherent micro-level variability at the grain size, encompassing different distributions of sizes, shapes, particle contacts, bonding, and contact medium. This micro-level variability is substantial and undergoes a significant reduction at the macro level. Parameters such as stiffness, strength, permeability, average void ratio, and other fundamental engineering properties exhibit variations at the macro level. This situation is distinct from materials like steel or concrete, which are treated as continua, with observed variabilities being considerably less compared to geomaterials. Therefore, incorporating probabilistic considerations in deterministic analyses proves highly beneficial.

Kalore et al. [97] illustrated this concept in the context of granular filters. Granular filters must meet two crucial requirements: retention and hydraulic conductivity. Traditional design approaches rely on representative grain sizes to satisfy these requirements. Kalore et al. [97] proposed a probabilistic assessment criterion for retention requirements by considering grain size and constriction size as random variables. For hydraulic conductivity requirements, a probabilistic assessment criterion was defined based on the variability of hydraulic conductivity and a semi-analytical model for saturated hydraulic conductivity. The limit states for these criteria were established using published experimental data.

The filter grain size distribution (GSD) is designed according to ICOLD [62] guidelines to match the averaged base soil GSD, shown in Fig. 38. The designed filter GSD for the averaged base soil GSD will meet the requirements for retention and hydraulic conductivity. However, the variability in the base soil GSD introduces uncertainty regarding effective filtration performance. The extremely fine base soil GSD is critical for retention, while the extremely coarse base soil GSD is critical for hydraulic conductivity function. The risk associated with the performance or failure of granular filters in filtration is determined by the variability in the GSD of the base soil; higher variability corresponds to a greater risk of failure. It is important to note that the variability in base soil is inherent, whereas the variability in filter GSD is governed by the manufacturing process. This example illustrates the importance of considering soil GSD and its spatial variability within a risk assessment framework.

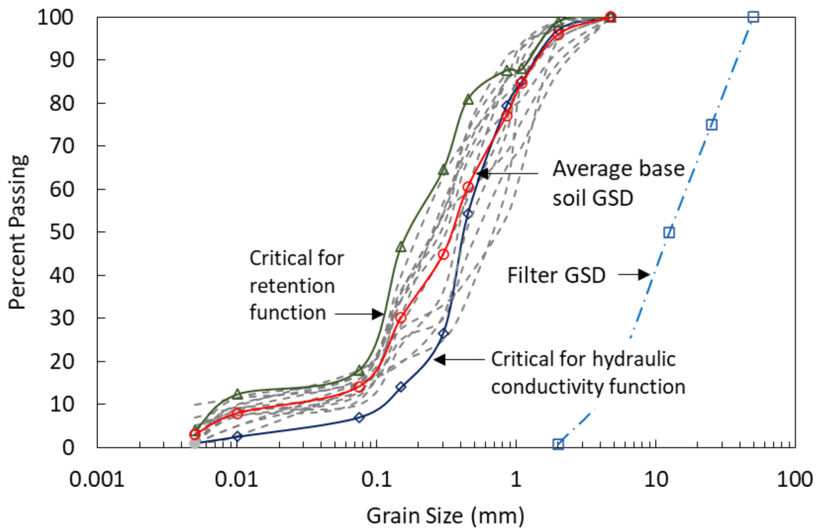


Fig. 38 Example illustrating variability in the grain size distribution of base soil

In another example, the consideration of the distribution of fibers in soils as random, following a uniform distribution in fiber-reinforced soil, provided valuable insights into stress diffusion due to the random reinforcement effect of fibers compared to preferred orientations (Sivakumar Babu et al. [98]).

The application of Discrete Element Method (DEM) based approaches has proven valuable in addressing uncertainty in geotechnical studies. An example involves using discrete fracture networks (DFN) to model the geo-mechanical behavior of fractured rocks, as demonstrated by Pandit [78] (ref. Fig. 39). In DFN, fractures are treated as straight lines, and their attributes (orientation, trace length, etc.) are considered as random variables with specific probability distributions. To quantify the uncertainty induced solely by the fracture networks, Uniaxial Compressive Strength (UCS) tests on synthetic rock masses are conducted for 30 realizations of the stochastic DFN. The rock matrix, represented by an assembly of triangular elements, is calibrated at the mean value of the scale-corrected UCS value (i.e., 149.6 MPa).

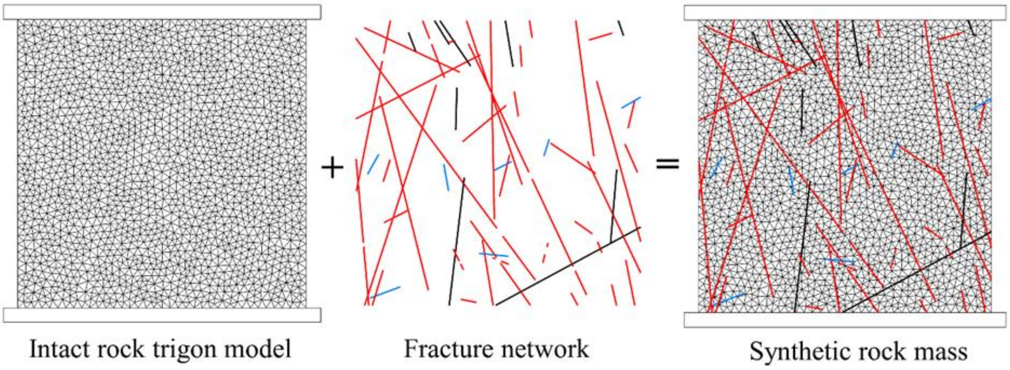


Fig. 39 Model of stochastic Discrete Fracture Networks

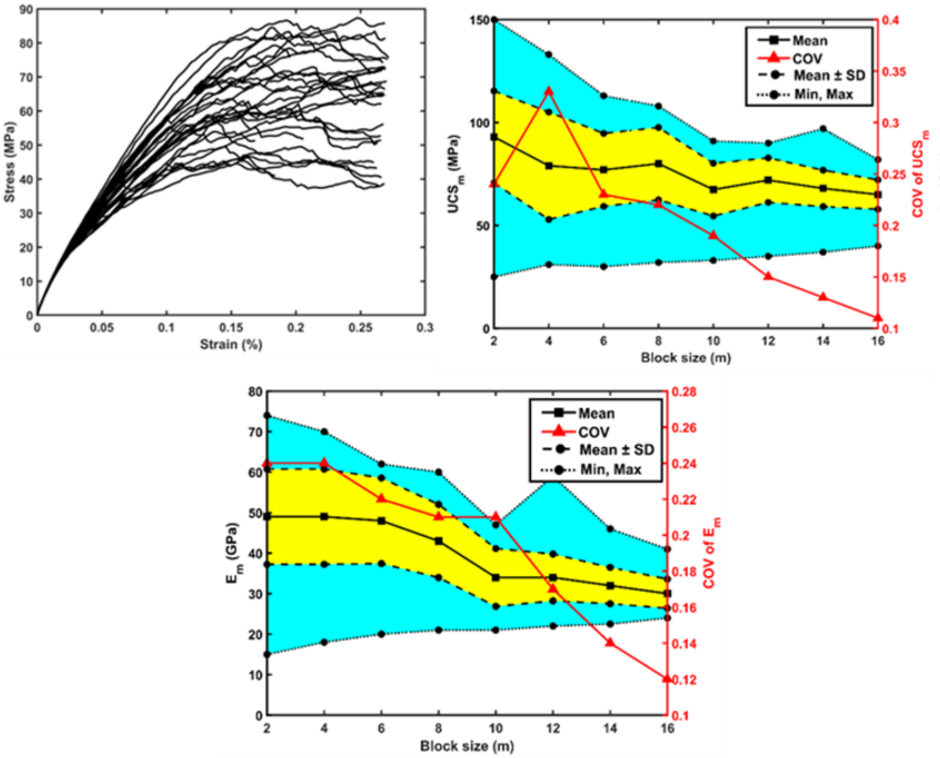


Fig. 40. (a) Simulating synthetic rock mass (b) Identification of representative elemental volume (REV) using mechanical indicators (c) Results of UCS simulation in UDEC on synthetic rock block of REV size i.e., 10 m

For the estimation of Representative Elementary Volume (REV), synthetic rock masses were generated with increasing block sizes after the calibration of micro parameters of intact rock. Uniaxial Compressive Strength (UCS) tests were conducted in UDEC to quantify the variability in strength and deformation of increasing block sizes, considering 30 random realizations of stochastic Discrete Fracture Networks. The results shown in Fig. 40, including peak strength and deformation modulus, indicate a decrease in their CoV values with an increase in the size of the synthetic rock block. For a block size of 10 m, the CoV goes below 20%, and therefore, 10 m is considered as the REV for the given rock mass conditions. Within the determined REV block size, variability of the intact rock material was introduced. Using the point estimate method, the uncertainty in rock mass strength and deformation was quantified. The results showed that the contribution of intact rock material variability in the uncertainty of rock mass strength and deformation is minimal, confirming that micro features can contribute to significant variability. Additionally, a correlation coefficient of 0.5 was found between uniaxial strength and deformation of the rock mass.

The application of DEM and similar tools, including imaging techniques, to capture micro-variability is being developed to link micro features to responses at the macro level. However, addressing geotechnical challenges in practice remains an ongoing pursuit. The calculation of a factor of safety involves considerations and conservatism to reduce failure probability and costs. For example, if a design has a factor of safety equal to 3, its relevance may not be universally understood. However, expressing this information in probabilistic terms, such as stating a 35 percent chance that the levee will fail in a 50-year period, could enhance comprehension among a broader audience.

Professor Milton Harr [99] made significant contributions to the probabilistic approach in examining the response of particulate media, emphasizing the importance of probabilistic thinking in addition to continuum approaches. He also authored a book on the applications of reliability in civil engineering [100]. In his Rankine lecture in 1977, De Mello [101] highlighted the need for the use of statistics, probability, and Bayesian thinking in dam engineering.

Christian [102] and Baecher [103], in their recent Terzaghi lectures, eloquently emphasized the need for probabilistic considerations in geotechnical engineering. They discussed several case studies addressing large-scale engineering problems such as dams and natural hazard mitigation, presenting many ideas and suggestions for implementation in practice that the profession should embrace.

Concluding Remarks

Geological materials are highly variable compared to other civil engineering materials, and hence, the role of variability in material properties as well as loads need to be considered for rational design, analysis, and decision-making in geotechnical engineering. This contribution highlights a few applications of these methods. With ever-increasing methods of better understanding and analysis, including machine learning and artificial intelligence, the benefits of applications of these tools are humongous. Characterization of the variability of materials is essential to understand the design implications as well as behavioural responses of geotechnical structures. Design consistency in treating both the loads and resistances from soils using probabilistic methods provides a rational way in design and practice. To understand, reduce, and consider uncertainties in design, proper material testing procedures following the physics of failure of geomaterials under variable loads are required. Sampling procedures and guidelines need to represent the design domain and properly represent geological features and stratification; an adequate number of sampling points are required. Methods of reliability analysis varying from simple approaches to complex models are available, and often consideration and use of means, variances, co-variances, and also performing even a simple reliability analysis is beneficial to justify design decisions, examine if the guidelines and codes are satisfactory, and modify them if required. A few examples are cited in this paper to give an insight into the applications and use of reliability analysis in practice. There is a vast scope for work towards a better understanding of design procedures, failure processes, and consequences of failures and provide sufficiently safe designs in geotechnical practice.

Acknowledgments

I wish to thank the Indian Geotechnical Society (IGS), New Delhi for bestowing the honour of Society's 45th Annual lecture and IGS Roorkee Chapter for providing the facilities to give the lecture during the Indian Geotechnical Conference at Roorkee. I wish to remember Prof. M. E. Harr of Purdue University, West Lafeyette, Indiana, USA for introducing me to the need for probabilistic thinking and analysis in geotechnical engineering during our discussions in 1996 while I was a visiting scholar at Purdue and was associated with Late Prof. G A Leonards, a strong supporter of IGS.

I thank the Indian Institute of Science, Bangalore for providing an excellent eco system for nurturing the research and efficient support system. I wish to thank my past and present colleagues in the Department of Civil Engineering and Centre for Sustainable Technologies for the support and valuable discussions. The work presented is based on the excellent work done in the area by my past and present students, Dr. Seshagiri Rao R, Prof. D S N Murty, Prof. A K Vasudevan, Prof. Sumanta Haldar, Prof. Munwar Basha, Prof. Vikas Pratap Singh, Prof. Amit Srivatsava, Dr. Sandeep K Chouskey, Dr. M D Deepthi, Dr. P Lakshmikanthan, Dr. Lekshmi Jaidev Nair, Dr. L G Santhosh, Dr. Pinom Ering, Dr. K Geetha Manjari, Prof. P Sughosh, Dr. Bharadwaj Pandit, Dr. Himansu Rana, Dr. K M Nazeeh, Dr. Shubham Arun Kalore, Sougata Mukherjee, Rakshanda Showkat, Prathima. P, Kalyani K, N. Anusree, and Prince Kumar. Thanks are due to M D Mukesh, Dr. Awdesh Kumar, Dr. Parameswaran, Dr. Rajarshi Pramanik, S Sujitha, Dr. Asha Nair and many others who were associated with me in research projects. The work received financial support from many funding agencies such as Council of Scientific & Industrial Research, (CSIR), Department of Science and Technology (DST), Ministry of Road Transport and Highways, Board of Research in Nuclear Sciences Government of India (BRNS), Public Works Department of Arunachal Pradesh, Ministry of Power, Science and Engineering Research Board (IMPRINT and NPDF schemes), and many others. I wish to specially thank Dr. Shubham Arun Kalore, Dr. Geetha Manjari, Dr. M D Deepthi and Prof. Munwar Basha for help in review and preparation of the manuscript.

I thank Smt. K. Himabindu, my wife, children Deepti and Vennela for providing a wonderful atmosphere of happiness and joy at home which significantly contributed to my progress.

References

1. Christian JT, Ladd CC, Baecher GB (1994) Reliability applied to slope stability analysis. Journal of Geotechnical Engineering, 120(12), 2180–2207. https://doi.org/10.1061/(ASCE)0733-9410(1994)120:12(2180)
2. Vanmarcke EH (1977) Probabilistic modeling of soil profiles. Journal of Geotechnical Engineering, 103(11), 1227–1246. https://doi.org/10.1061/AJGEB6.0000517
3. Fenton GA, Griffiths DV (1996) Statistics of free surface flow through a stochastic earth dam. Journal of Geotechnical Engineering, 122(6), 427-436.
4. Jaksa M, Goldsworthy J, Fenton G, Kaggwa G, Griffiths D, Kuo Y, Poulos H (2005) Towards reliable and effective site investigations. Geotechnique, 55(2), 109-121.
5. US Army Corps of Engineers (USACE) (1997) Engineering and design: Introduction to probability and reliability methods for use in geotechnical engineering. ETL Rep. No. 1110-2-547. Washington, DC: Dept. of the Army.
6. ISO (2015). International Organization for Standardization. ISO 2394:2015 General principles on reliability for structures. ISO/TC 98/SC 2. 4th ed.
7. ISSMGE (International Society of Soil Mechanics and Geotechnical Engineering) (2021) TC304 Engineering practice of risk assessment & management. Accessed December 6, 2021. http://140.112.12.21/issmge/tc304.htm
8. Phoon KK (2021) What Geotechnical Engineers Want to Know about Reliability. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering. https://doi.org/10.1061/AJRUA6.RUENG-1002
9. Haldar A, Mahadevan S (2000) Probability, Reliability and Statistical Methods in Engineering Design. John Wiley & Sons, Inc., New York, 2000.
10. Baecher GB, Christian JT (2005). Reliability and statistics in geotechnical engineering. John Wiley & Sons.
11. Au SK, Beck JL (2001) Estimation of small failure probabilities in high dimensions by subset simulation. Probabilistic Engineering Mechanics, 16(4), 263-277.
12. Cadini F, Avram D, Pedroni N, Zio E (2012) Subset simulation of a reliability model for radioactive waste repository performance assessment. Reliability Engineering & System Safety, 100, 75-83.

13. Nazeeh KM, Sivakumar Babu GL (2018) Reliability analysis of near-surface disposal facility using subset simulation. Environmental Geotechnics, 6(4), 242-249.
14. Manjari KG, Sivakumar Babu GL (2022) Reliability and sensitivity analyses of discrete fracture network based contaminant transport model in fractured rocks. Computers and Geotechnics, 145, 104674.
15. Uzielli M, Vannucchi G, Phoon KK (2005) Random field characterization of stress-normalized cone penetration testing parameters. Geotechnique, 55(1), 3-20.
16. Murthy DSN, Sivakumar Babu GL (2008). Reliability analysis of allowable pressure of strip footing in cohesionless soil. Geotechnical Engineering Journal, South East Geotechnical Society, Bangkok, 39(2), 77-85.
17. Sivakumar Babu GL, Srivastava A, Murthy DSN (2006) Reliability analysis of bearing capacity of shallow foundation resting on cohesive soil. Canadian Geotechnical Journal, 43(2), 217-223.
18. Sivakumar Babu GL, Murthy DSN (2007) Effect of spatial correlation of cone tip resistance on the bearing capacity of shallow foundations. Geotechnical and Geological Engineering Journal, 26, 37-46.
19. Sivakumar Babu GL, Srivastava A (2007) Reliability analysis of allowable pressure on shallow foundation using response surface method. Computers and Geotechnics, 34(3), 187-194.
20. Geetha Manjari K, Balaji Rao, Sivakumar Babu GL (2015) Stochastic model for settlement: footings on cohesionless soil. International Journal of Georisk - Assessment and Management of Risk for Engineered Systems and Geohazards, 8(4), 269-283.
21. Haldar S, Sivakumar Babu GL (2012) Response of vertically loaded pile in clay: a probabilistic study. Geotechnical and Geological Engineering, 30(1), 187-196.
22. Haldar S, Sivakumar Babu GL (2009) Design of laterally loaded piles in clays based on cone penetration test data: a reliability-based approach. Geotechnique, 59, 1-14.
23. Haldar S, Sivakumar Babu GL (2008) Reliability measures for pile foundations based on cone penetration data. Canadian Geotechnical Journal, 45, 1699-1714.
24. Haldar S, Sivakumar Babu GL (2008) Probabilistic analysis of load-settlement response from pile load tests. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 2(2), 79-91.
25. Haldar S, Sivakumar Babu GL (2008) Load resistance factor design (LRFD) of axially loaded pile based upon load test results. ASCE Journal of Geotechnical and Geoenvironmental Engineering, 134(8), 1106-1117.

26. Haldar S, Sivakumar Babu GL (2009) Probabilistic seismic design of pile foundations in non-liquefiable soil by response spectrum approach. Journal of Earthquake Engineering, 13, 737-757.
27. Nazeeh KM, Sivakumar Babu GL (2019) Critical Appraisal of Codes for Foundation Design and Role of Reliability-Based Approach. Indian Geotechnical Journal, 49, 467-477.
28. Fathima Sana VK, Nazeeh KM, Deepthi MD, Sivakumar Babu GL (2022) Reliability-based design optimization of shallow foundation on cohesionless soil based on surrogate-based numerical modeling. ASCE International Journal of Geomechanics, 22(2). https://doi.org/10.1061/(ASCE)GM.1943-5622.0002274
29. Nazeeh KM, Sivakumar Babu GL (2022) Reliability-based design of geogrid reinforced soil foundation using kriging surrogates. Geosynthetics International, 1-14.
30. Sivakumar Babu GL, Basha BM (2008). Optimum design of cantilever retaining walls using target reliability approach. ASCE Journal of International Journal of Geomechanics, 8(4), 240-252.
31. Basha BM, Sivakumar Babu GL (2008) Target reliability-based design optimization of anchored cantilever sheet pile walls. Canadian Geotechnical Journal, 45, 535-545.
32. Basha BM, Sivakumar Babu GL (2009) Seismic reliability assessment of external stability of reinforced soil walls using pseudo-dynamic method. Geosynthetics International, 16(3), 197-215.
33. Basha BM, Sivakumar Babu GL (2010) Reliability assessment of internal stability of reinforced soil structures: a pseudo-dynamic approach. Soil Dynamics and Earthquake Engineering, 30(5), 336-353.
34. Basha BM, Sivakumar Babu GL (2010) Load and resistance factor design (LRFD) approach for the reliability-based seismic design of bridge abutments. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 4(3), 127-139.
35. Basha BM, Sivakumar Babu GL (2010) Optimum design for external seismic stability of geosynthetic reinforced soil walls: a reliability-based approach. Journal of Geotechnical and Geoenvironmental Engineering ASCE, 136(6), 797-812.
36. Basha BM, Sivakumar Babu GL (2010) Optimum design of bridge abutments under high seismic loading using the modified pseudo-static method. Journal Earthquake Engineering, Taylor & Francis, 14(6), 874-897.
37. Basha BM, Sivakumar Babu GL (2010) Optimum design of bridge abutments under seismic conditions: a reliability-based approach. Journal Bridge Engineering ASCE, 15(2), 183-195.

38. Basha BM, Sivakumar Babu GL (2011) Reliability-based earthquake-resistant design for the internal stability of reinforced soil structures. Geotechnical and Geological Engineering, 29(5), 803-820.
39. Basha BM, Sivakumar Babu GL (2011) Seismic reliability assessment of internal stability of reinforced soil walls using the pseudo-dynamic method. Geosynthetics International, 18(5), 221-241.
40. Basha BM, Sivakumar Babu GL (2012) Target reliability-based optimization for internal seismic stability of reinforced soil structures. Geotechnique, 62(1), 55-68.
41. Basha BM, Sivakumar Babu GL (2014) Reliability-based load and resistance factor design approach for external seismic stability of reinforced soil walls. Soil Dynamics and Earthquake Engineering, 60, 8-21.
42. Mukherjee S, Sivakumar Babu GL (2023) Probabilistic Evaluation of the Uplift Capacity of Transmission Tower Foundations Using Reinforced Anchors. International Journal of Geomechanics, 23(11), 04023203.
43. Pramanik R, Mukherjee S, Sivakumar Babu GL (2022) Deterministic and probabilistic prediction of the maximum wall facing displacement of geosynthetic-reinforced soil segmental walls using multivariate adaptive regression splines. Transportation Geotechnics, 36, 100816. https://doi.org/10.1016/J.Trgeo.2022.100816
44. Sivakumar Babu GL, Vikas Pratap Singh (2009) Deformation and stability regression models for soil nailing. Proceedings of the Institution of Civil Engineers, Journal of Geotechnical Engineering, 162, 1-11.
45. Sivakumar Babu GL, Vikas Pratap Singh (2009) Reliability analysis of soil nail walls. Journal of Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 3(1), 44-54.
46. Sivakumar Babu GL, Vikas Pratap Singh (2009) Reliability-based study on the seismic stability of soil nail walls. Journal Of South East Geotechnical Society, Bangkok, 40(4), 237-246.
47. Sivakumar Babu GL, Vikas Pratap Singh (2010) Reliability analyses of a prototype soil nail wall using regression models. Geomechanics and Engineering, An International Journal, 2(2), 71-88.
48. Sivakumar Babu GL, Vikas Pratap Singh (2011) Reliability-based load and resistance factors for soil nail walls. Canadian Geotechnical Journal, 48(6), 915-930.
49. Pramanik R, Sivakumar Babu GL (2023) Reliability-Based Load and Resistance Factors for Soil Nail Walls Against Facing Failures. International Journal of Geosynthetics and Ground Engineering, 9(3), 27.
50. Deepthi MD, Sivakumar Babu GL (2013) A methodology for pavement design evaluation and back analysis using Markov chain Monte Carlo simulation. Journal of Transportation Engineering, ASCE, 139(1), 65-74.

51. Deepthi MD, Ravi P, Sivakumar Babu GL (2013) System reliability analysis of flexible pavements. ASCE Journal of Transportation Engineering, 139(10), 1001-1009.
52. Deepthi MD, Sivakumar Babu GL (2016) Methodology for global sensitivity analysis of flexible pavements in a Bayesian back-analysis framework. ASCE-ASME Journal of Risk Uncertainty Eng. Syst., Part A: Civ. Eng., 10.1061/AJRUa6.0000865, 04016002.
53. Deepthi MD, Sivakumar Babu GL (2014) Influence of spatial variability on pavement responses using Latin hypercube sampling on 2D random fields. ASCE Journal of Materials. Eng J. Mater. Civ. Eng., 2014, 26(11), 04014083.
54. Deepthi MD, Sivakumar Babu GL (2016) Influence of anisotropy on pavement responses using adaptive sparse polynomial chaos expansion. ASCE Journal of Mater. Civ. Eng., 2016, 28(1), 04015061.
55. Deepthi MD, Sivakumar Babu GL, Lekshmi S (2015) Time-Dependent Reliability Analysis of Pavement Structures under Fatigue Loading. Geotechnical Safety and Risk V; IOS Press: Amsterdam, The Netherlands (2015), 358-363.
56. Deepthi MD, Sivakumar Babu GL (2020) Reliability-based design optimization of flexible pavements using kriging models. Journal of Transportation Engineering, Part B: Pavements, 147(3), 04021046.
57. Deepthi MD, and Sivakumar Babu GL (2023) System reliability-based design optimization of flexible pavements using adaptive meta-modelling techniques. Construction and Building Materials, 367, 130351.
58. Deepthi MD, Nazeeh KM, Sivakumar Babu GL (2023) Reliability analysis of flexible based on the quantile-value method. International Journal of Pavement Engineering, 24(1), 2241109, doi: 10.1080/10298436.2023.2241109.
59. Kalore SA, Sivakumar Babu GL, Mallick RB (2019) Risk analysis of permeable layer in pavement subsurface drainage system. Pavements J. Transp. Eng., 145(3), 04019028.
60. Sivakumar Babu GL, Murthy DSN (2005) Reliability analysis of unsaturated slopes. Journal of Geotechnical and Geoenvironmental Engineering, 131(11), 1423-1429.
61. Srivastava A, Sivakumar Babu GL, Sumanta Haldar (2010) Influence of spatial variability of permeability property on steady state seepage flow and slope stability analysis. Engineering Geology, 110(3-4), 93-101.
62. ICOLD (International Commission on Large Dams) (1999) Paris: ICOLD.
63. USBR (United States Bureau of Reclamation) (2011) Embankment Dams. Design Standards. No. 13, chapter 5, revision 9.

64. Sivakumar Babu GL, Srivastava A (2010) Reliability analysis of earth dams. Journal Of Geotechnical and Geoenvironmental Engineering ASCE, 136(7), 995-998.
65. Ering P, Sivakumar Babu GL (2016) Probabilistic back analysis of rainfall-induced landslide- A case study of Malin landslide, India. Engineering Geology, 208, 154–164.
66. Ering P, Sivakumar Babu GL (2016). A Bayesian framework for updating model parameters while considering spatial variability. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards. doi: 10.1080/17499518.2016.1255760.
67. Ering P, Sivakumar Babu GL (2020) Characterization of critical rainfall for slopes prone to rainfall-induced landslides. ASCE Nat. Hazards Rev., 21(3), 06020003.
68. Sivakumar Babu GL, Ering P (2017) Integrating Rainfall Load into Remedial Design of Slopes Affected by Landslides, Geotechnics for Natural Disaster Mitigation and Management, 67–74.
69. Rana H, Sivakumar Babu GL (2022) Probabilistic back analysis for rainfall-induced slope failure using MLS-SVR and Bayesian analysis. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 1-14.
70. Rana H, Pandit B, Sivakumar Babu GL (2023) Estimation of Uncertainties in Soil Using MCMC Simulation and Effect of Model Uncertainty. Geotechnical and Geological Engineering, 1-15.
71. Showkat R, Mohammadi H, Sivakumar Babu GL (2022) Effect of rainfall infiltration on the stability of compacted embankments. International Journal of Geomechanics, 22(7). doi: 10.1061/(ASCE)GM.1943-5622.0002425.
72. Showkat R, Sivakumar Babu GL (2023) Reliability analysis of unsaturated embankment considering the effect of geocomposite under infiltration. Geosynthetics International, https://doi.org/10.1680/jgein.22.00268.
73. Moser A (1990) Buried Pipe Design, McGraw Hill Professional.
74. Sivakumar Babu GL, Rajaparthy RS (2005) Reliability measures for buried flexible pipes, Canadian Geotechnical Journal, 42(2), 541-549.
75. Sivakumar Babu GL, Srivastava A (2010) Reliability analysis of buried flexible pipe-soil systems. Journal of Pipeline Systems Engineering and Practice, ASCE, 1(1), 33-41.
76. Pandit B, Sivakumar Babu GL (2017) Reliability based robust design for reinforcement of jointed rock slope. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 12(2), 152–168.

77. Pandit B (2021) Reliability based analysis and design of slopes and tunnels in rock mass, Ph.D. thesis, Indian Institute of Science, Bangalore.
78. Pandit B, Sivakumar Babu GL (2021) Probabilistic stability assessment of tunnel-support system considering spatial variability in weak rock mass. Computers and Geotechnics, 137; 104242.
79. Tiwari G, Pandit B, Madhavi LG, Sivakumar Babu GL (2017) Probabilistic analysis of tunnels considering uncertainty in peak and post-peak strength parameters. Tunneling and Underground Space Technology, 70, 375-387.
80. Tiwari G, Pandit B, Madhavi LG, Sivakumar Babu GL (2018) Analysis of tunnel support requirements using deterministic and probabilistic approaches in average quality rock mass. International Journal of Geomechanics, 18(4), 04018017.
81. Sivakumar Babu GL, Reddy KR, Srivastava A (2014) Influence of spatially variable geotechnical properties of MSW on the stability of landfill slopes. Journal of Hazardous, Toxic, And Radioactive Waste, 18(1), 27–37.
82. Reddy KR, Kulkarni HS, Srivastava A, Sivakumar Babu GL (2013). Influence of spatial variation of hydraulic conductivity of municipal solid waste on the performance of bioreactor landfill. Journal of Geotechnical and Geoenvironmental Engineering, 139(11), 1968-1972.
83. Sivakumar Babu GL, Chouskey SK, Reddy KR (2013) Approach for the use of MSW settlement predictions in the assessment of landfill capacity based on reliability analysis. Waste Management, 33, 2029-2034.
84. Sivakumar Babu GL, Reddy KR, Chouksey SK (2010) Constitutive model for municipal solid waste incorporating mechanical creep and biodegradation-induced compression. Waste Management Journal, 30(1), 11 To 22.
85. Parameswaran TG, Nazeeh KM, Deekshith PK, Sivakumar Babu GL (2022) Probabilistic Design of Gas Collection Systems for a Prototype Bioreactor. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 8(4), 04022053.
86. Parameswaran T, Nazeeh KM, Deekshith PK, Sivakumar Babu GL, Chamindu DTKK (2023). Gas collection system design for a landfill via three-dimensional stochastic waste heterogeneity models and kriging. Journal of Environmental Chemical Engineering, 11, 110563.
87. Santhosh LG, Sivakumar Babu GL (2014) Reliability of the liner system using the response surface method. Environmental Geotechnics, 1(2), 71-80.
88. Santhosh LG, Lakshmikanthan P, Sivakumar Babu GL (2017) Reliability-based approach for the prediction of leachate head in MSW

landfills. International Journal of Geosynthetics and Ground Engineering, 3(4). https://doi.org/10.1007/S40891-016-0080-4.
89. USEPA (1993) Solid waste disposal facility criteria, subpart D-design criteria. U.S. Environmental Protection Agency, Washington, EPA530-R-93-017, 40 CFR § 258.40.
90. Rowe RK, Chappel MJ, Brachman RWI, Take WA (2012) Field study of wrinkles in a geomembrane at a composite liner test site. Canadian Geotech J, 49(10), 1196–1211.
91. Sujitha S, Manjari GK, Sampurna Datta, Sivakumar Babu GL (2015) Risk and reliability analysis of multi-barrier system for near-surface disposal facilities. ASCE J. Journal of Hazardous, Toxic, And Radioactive Waste, 20(2). https://doi.org/10.1061/HZ.2153-5515.0000284, 04015014.
92. Sujitha S, Deepthi DM, Sivakumar Babu GL (2016) Time-dependent reliability analysis for radionuclide migration in groundwater in near-surface disposal facility using enhanced Monte Carlo method. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards. https://doi.org/10.1080/17499518.2016.1229867.
93. Sujitha S, Sivakumar Babu GL (2017). System reliability analysis for near-surface radioactive waste disposal facilities. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 11(4), 2017.
94. Geetha Manjari K, Sivakumar Babu GL (2017) Probabilistic analysis of groundwater and radionuclide transport model from near surface disposal facilities. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 12(1), 60-73.
95. Geetha Manjari K, Sivakumar Babu GL (2021) Probabilistic analysis of radionuclide transport for near-surface disposal facilities in spatially varying soils. Journal of Hazardous, Toxic, and Radioactive Waste, 25(1), 04020059.
96. Geetha Manjari K, Sivakumar Babu GL (2022) Reliability and sensitivity analyses of discrete fracture network-based contaminant transport model in fractured rocks. Computers and Geotechnics, 145, 104674.
97. Kalore SA, Sivakumar Babu GL, Mahajan R (2021) Probabilistic design framework for granular filters. Doi: 10.1061/(Asce)Gt.1943-5606.0002674.
98. Sivakumar Babu GL, Vasudevan AK, Haldar S (2008) Numerical simulation of fiber-reinforced sand behavior, Geotextiles and Geomembranes, 26(2), 181-188.
99. Harr M (1977) Mechanics of particulate media: A probabilistic approach, McGraw-Hill.
100. Harr M (1987) Reliability-Based Design in Civil Engineering. McGraw-Hill Book Company.

101. De Mello VFB (1977). Reflections on design decisions of practical significance to embankment dams, 17th Rankine Lecture. Geotéchnique, 27(3), 281–355. https://doi.org/10.1680/geot.1977.27.3.281.
102. Christian JT (2004) Geotechnical engineering reliability: How well do we know what we are doing?. J. Geotech. Geoenviron. Eng., 130(10), 985–1003. https://doi.org/10.1061/(ASCE)1090-0241(2004)130:10(985).
103. Gregory BB (2021) Geotechnical Systems, Uncertainty, and Risk. January 2023Journal of Geotechnical and Geoenvironmental Engineering 149(1):03023001, DOI:10.1061/JGGEFK.GTENG-10201