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on

ASSESSMENT OF ENGINEERING PROPERTIES OF ROCK-MASS IN CIVIL ENGINEERING APPLICATIONS

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1.0 GENERAL

Geotechnical engineers often deal with underground structures like tunnels, large caverns, landslides, road cuts, and foundations of heavily loaded structures like dams, bridges etc. that are situated in or on rocks. Analysis and design of these structures needs a good understanding of physical and engineering properties of rocks. The engineering properties of the rock mass include the shear strength, compressibility and hydraulic conductivity. This report is primarily focussed on strength behaviour, however, some discussions are given for deformational behaviour as well. The physical properties of the rocks include dry, bulk and saturated unit weight, water content, colour, texture, porosity and specific gravity of rock grains. The engineering properties of the rock substance (intact rock) include uniaxial compressive strength (UCS), tensile strength and triaxial/ polyaxial strength. In addition, the modulus of deformation and Poisson's ratio are also required for assessing deformational behaviour of the rock. The UCS may be obtained by testing cylindrical specimens of rock cores under uniaxial loading conditions or through point load strength index tests conducted on irregular specimens. Brazilian tests may be conducted to get the tensile strength. Conventional triaxial strength tests, considering no effect of intermediate principal stress, may be performed to study the effect of confinement on rock. Recently, polyaxial tests are also being popularised to study the effect of both minor and intermediate principal stress on the strength of the rock. Comprehensive elaboration on how to determine engineering properties of intact rocks may be found elsewhere (Ramamurthy, 2014; ISRM, 1981)

The rocks encountered in geotechnical applications are invariably intersected by discontinuities like joints, foliations and bedding planes. The discontinuities induce planes of weakness in the rock mass. As compared to intact rock, the failure mechanism of jointed rock is highly complex as failure may occur due to sliding on pre-existing discontinuities, shearing of rock substance, translation and/or rotation of intact rock blocks. As a result, the jointed rock is quite incompetent and anisotropic in strength and deformational behaviour. In addition, the strength behaviour of jointed rock is highly non- linear with increase in confining pressure.

The engineering properties assessed from laboratory tests conducted on intact rocks cannot be directly applied for analysis and design in the field and the geotechnical engineers have to deduce the response of jointed rock by incorporating the effect of discontinuities on the intact rock properties. The present guidelines focus on assessing the shear strength response of jointed rock masses encountered in the field.

Broadly, the shear strength aspects of jointed rocks have been grouped into the following two major categories:

i) Shear strength along planar discontinuity (Discontinuity shear strength)

ii) Shear strength of jointed rock mass (Rock mass shear strength)

If a planar discontinuity is persistent, sliding may occur along the discontinuity. An example of a hillside failure along planar discontinuities is shown in Fig. 1. For stability analysis, the shear strength response of the discontinuity should be considered in this case.

Fig.1. Hill side failure along planar discontinuities

The second class of rock shear strength pertains to the rock mass as a whole (Fig. 2). The potential failure surface lies partly on discontinuity surfaces, and partly through the intact rock. The rock mass generally consists of large number of intact rock blocks separated by discontinuities. At the time of failure, these blocks may slide, translate, rotate, split or shear. Further, the rock mass may behave isotropically or anisotropically depending upon the number, orientation and spacing of discontinuities.

Fig.2. Heavily jointed rock mass below a bridge pier

2.0 DISCONTINUITY SHEAR STRENGTH

Various models are available for prediction of shear strength of a planar discontinuity at a prevailing normal stress. Some most commonly referred shear strength models are presented below.

2.1 Coulomb's Model

It is the most commonly used model for assessing the shear strength along the discontinuity surfaces. The shear strength parameters namely, cohesion c_j and friction angle ϕ_j are used to estimate the shear strength at normal stress existing on the joint plane. The shear strength parameters may be obtained by performing direct shear tests on the discontinuity surfaces. Portable field shear box may be used for this purpose. Direct shear tests are conducted under various normal loads and shear stress vs. shear displacement plots (Fig. 3a) are obtained. From these plots, values of peak and residual shear strength of the joint are obtained. The failure envelopes of peak and residual shear strength are then plotted (Fig. 3b). The shear strength of the discontinuity is defined as:

$$\tau_f = c_j + \sigma_n \tan\phi_j \qquad \text{(peak strength)} \qquad (1)$$

$$\tau_f = \sigma_n \tan\phi_r \qquad \text{(residual strength)} \qquad (2)$$

where τ_f is the shear strength along the discontinuity; σ_n is the effective normal stress over the discontinuity; ϕ_j is the peak friction angle of the discontinuity surface; c_j is the peak cohesion of the discontinuity surface, and ϕ_r is residual friction angle for discontinuity surface.

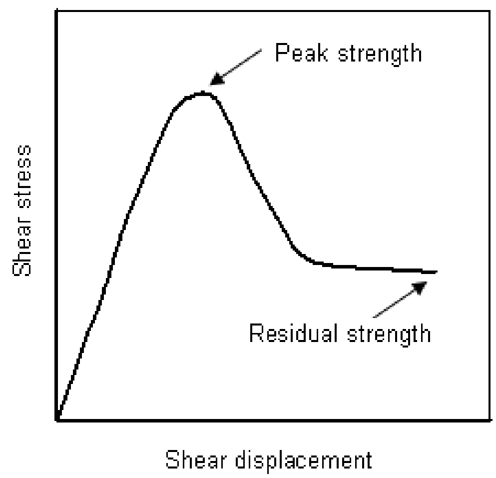

Fig. 3a. Typical shear stress-shear displacement plot from direct shear test on rough rock joints

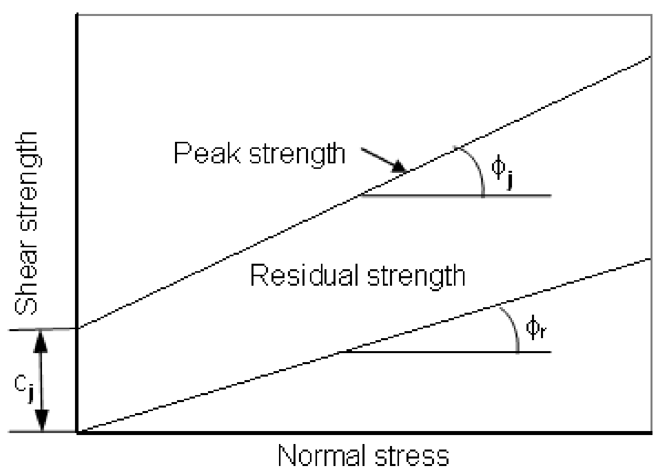

Fig. 3b. Failure envelopes of shear strength for rough rock joint

2.2 Patton's Model

Patton (1966) conducted experiments by simulating the asperities in the form of saw-tooth specimens (Fig. 4) and suggested a criterion. The model considers two failure modes i.e. *either sliding (at low normal stress) along the discontinuities or shearing (at high normal stress) of the asperities material*. The following bilinear model was suggested:

$$\tau_f = \sigma_n \tan(\phi_\mu + i) \quad \text{for low } \sigma n \tag{3}$$

$$\tau_f = c_j + \sigma_n \tan(\phi_r) \quad \text{for high } \sigma_n \tag{4}$$

where i defines the roughness angle, and ϕ_μ and ϕ_r are the basic angle (friction on horizontal plane) and residual friction angle respectively.

From practical standpoint it is difficult to assess the normal stress level at which transition from sliding to shearing takes place. In reality, there is no such distinct and clear-cut normal stress level, which defines the boundary between the two failure modes.

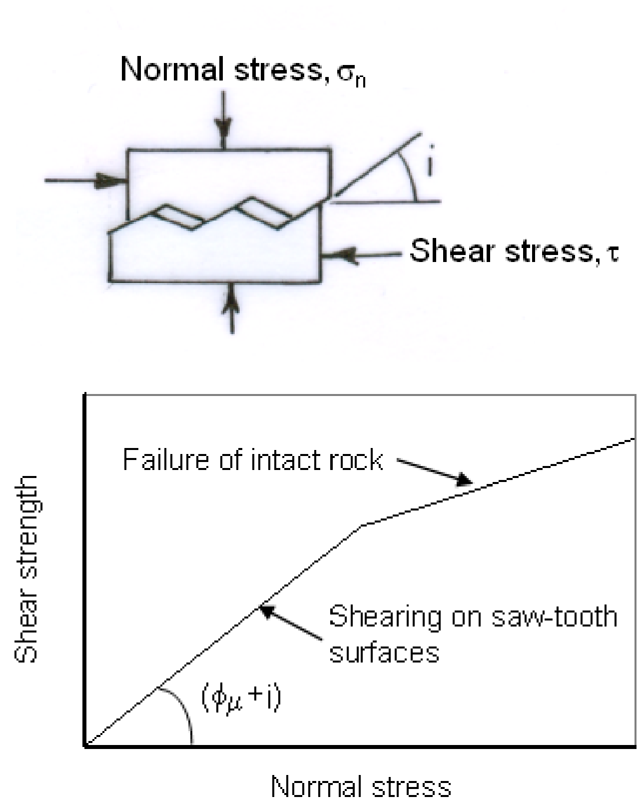

Fig. 4. Patton's simulation of asperities and bi-linear shear strength model for rock joints

2.3 Barton (1973) Model

It is the most widely used strength criterion for assessing the shear strength along discontinuity surfaces. The criterion is an extension of the Patton's model and considers simultaneous occurrence of sliding and shearing. The roughness angle i, was assumed to be constant in Patton's model. It was observed by Barton (1973) that the roughness i varies with normal stress level as given below:

$$i = \mathrm{JRC}\log_{10}\frac{\mathrm{JCS}}{\sigma_n} \tag{5}$$

Barton's shear strength model (Fig. 5) is thus expressed as:

$$\tau_f = \sigma_n \tan\left(\phi_r + \mathrm{JRC}\log_{10}\frac{\mathrm{JCS}}{\sigma_n}\right) \tag{6}$$

where JRC is the joint roughness coefficient, which is a measure of the initial roughness (in degrees) of the discontinuity surface. JRC is assigned a value in the range of 0–20, by matching the field joint surface profile with the standard surface profiles on a laboratory scale of 10 cm (Barton and Choubey, 1977) as shown in Fig. 6. JCS is the joint wall compressive strength of the discontinuity surface, and σ_n is the effective normal stress acting across the discontinuity surface.

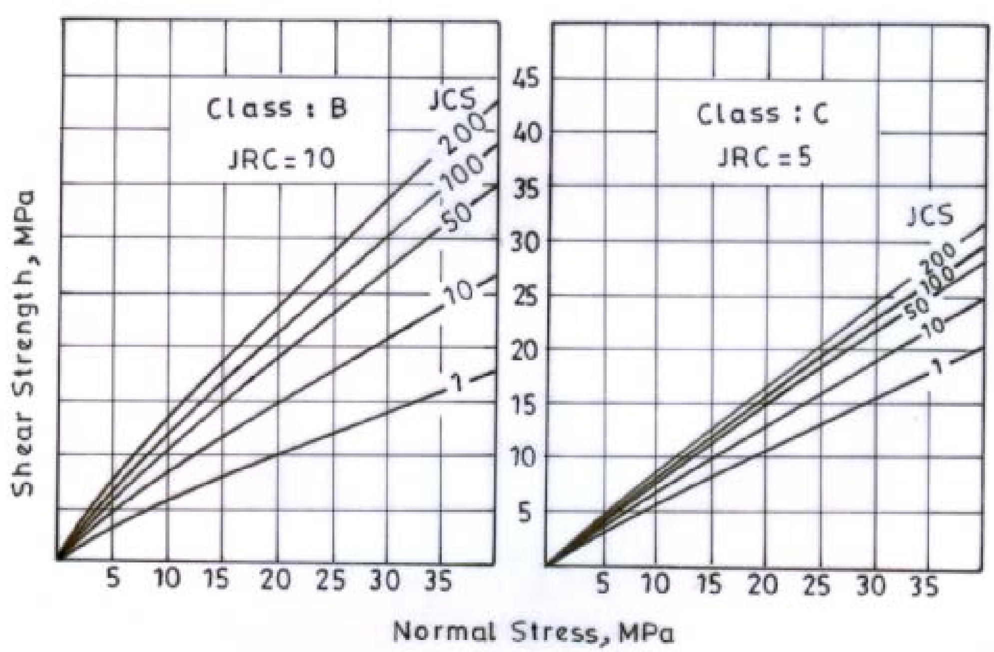

Fig. 5. Barton's shear strength model for rock joints

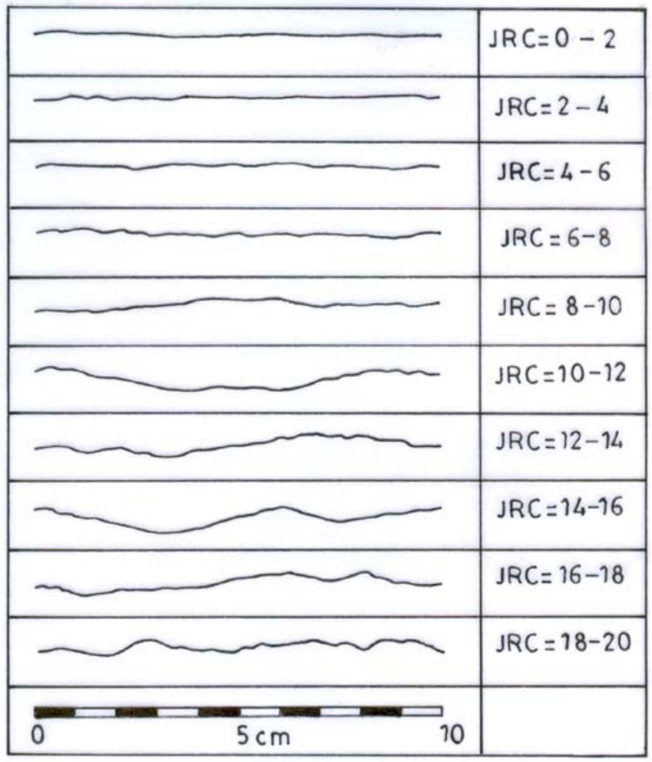

Fig. 6. Roughness profiles to estimate JRC (Barton and Choubey, 1977)

3.0 SHEAR STRENGTH OF ROCK MASS

In case of failure of rock mass, intact rock blocks forming the mass may slide, translate, rotate, shear or split. Consequently, the shear behaviour of the mass is substantially different from that of a single discontinuity. At low normal stress level, shearing results in dilation of the mass associated with rotation of blocks. The friction angle of mass at low normal stress is high, whereas the cohesion is low. As the normal stress increases, the dilation is suppressed and shearing of the intact rock material commences. This results in a relatively higher cohesion and lower friction angle. Due to continuous change in mechanism of failure, the shear strength envelope of rock mass is highly curvilinear, especially in low normal stress range.

It is not feasible to prepare and test specimens of rock mass in the field. Strength criteria are used to simulate triaxial strength tests on the rock mass. Using the results of simulated triaxial strength tests, a relationship between normal stress across the failure surface and the corresponding shear strength is derived for analysis. These relationships are generally non-linear. Most of the non-linear strength criteria for rock masses are generally expressed in terms of the major and minor principal stresses. Some of the strength criteria are discussed in the following sections:

3.1 Linear Strength Criterion

Coulomb's linear strength criterion is the most widely used criterion for jointed rock and rock masses as well. The criterion is also referred to as Mohr-Coulomb criterion. According to this criterion, rock mass is treated as an isotropic material and the shear strength along the failure surface is expressed as follows:

$$\tau_f = c_m + \sigma_n \tan \phi_m \tag{7}$$

where c_m and ϕ_m are Mohr-Coulomb shear strength parameters for jointed rock or rock mass. For rock mass in field, the values of c_m and ϕ_m may be obtained from field shear tests on rock mass. Indirectly classification approaches also provide a rough estimate of the shear strength and some of these methods are given below.

3.1.1 Rock Mass Rating

Bieniawski (1973, 1989 and 1993) has suggested a classification system popularly known as Rock Mass Rating (RMR) system to characterise the quality of the rock mass. The parameters used are UCS of intact rock material, Rock Quality Designation (RQD), spacing of discontinuities, condition of discontinuities, groundwater condition and orientation of discontinuities. The ratings for individual parameters are summed up to get the RMR of the mass. The values of shear strength parameters c_m, ϕ_m for five levels of rock mass ratings are presented in Table 1 (Bieniawski, 1989).

Table 1: Mohr-Coulomb parameters from RMR (After Bieniawski, 1989)

Class number	I	II	III	IV	V
Cohesion of rock mass (kPa)	>400	300 – 400	200 – 300	100 – 200	<100
Friction angle of rock mass (deg)	>45	35 – 45	25 - 35	15 – 25	<15

Mehrotra (1992), based on experience from Indian project sites, observed that the shear strength is under-predicted by expressions suggested by Bieniawski (1989). Figure 7 may be used for assessing the shear strength parameters of rock masses especially for slopes.

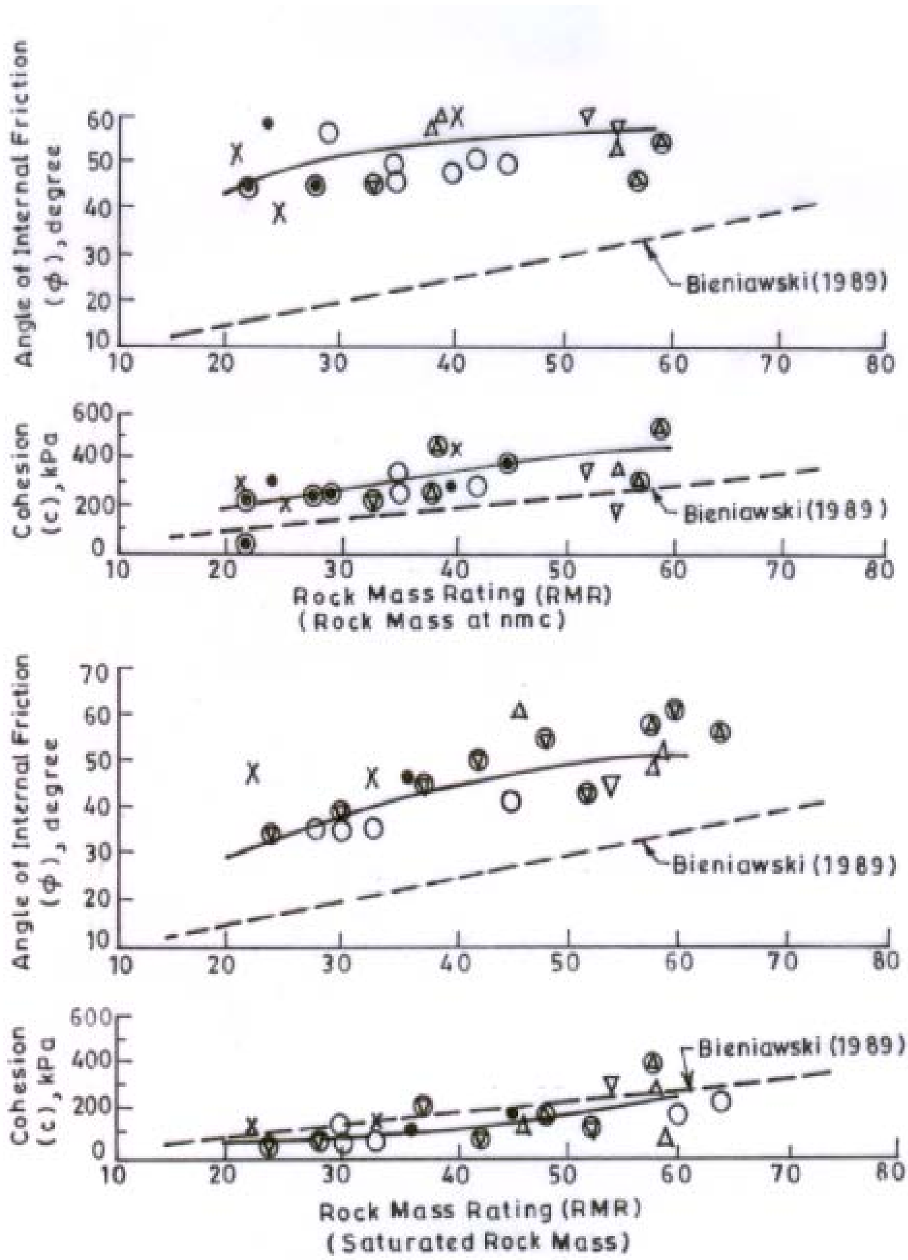

Fig. 7. Estimation of friction angle of rock mass from RMR (Mehrotra, 1992)

3.1.2 Q index

Rock mass quality index, Q (Barton et al., 1974) can also be used to obtain shear strength parameters (Barton, 2002) as:

$$c_m = \left(\frac{RQD}{J_n}\right)\left(\frac{1}{SRF}\right)\left(\frac{\sigma_{ci}}{100}\right) \quad \text{MPa} \tag{8}$$

$$\phi_m = \tan^{-1}\left(\frac{J_r}{J_a} J_w\right) \qquad (9)$$

where c_m is the cohesion of the undisturbed rock mass; ϕ_m, the friction angle of the mass; RQD, the rock quality designation (Deere, 1963); J_n, the joint set number; J_r, the joint roughness number; J_a, the joint alteration number; J_w, the joint water reduction factor, and σ_{ci} is the uniaxial compressive strength of intact rock material.

The shear strength parameters obtained from Q are suggested by Barton (2002) for analyzing underground openings. If used for slopes, an overestimation in the strength may be expected. *For slopes, it is felt that the relationship between shear strength parameters and RMR as suggested by Mehrotra (1992) will be more appropriate for the Himalayan rock masses.* The relationships were developed based on extensive in-situ direct shear tests on saturated rock masses in Himalayas.

3.2 Non-Linear Strength Criteria

One major limitation of the Mohr-Coulomb strength criterion is that it considers the rock mass shear strength as a linear function of normal stress σ_n. It is well established that the shear strength response is highly non-linear and the parameters c, ϕ change with the range of confining pressure used in estimating these parameters. Consequently, several non-linear strength criteria have been proposed for jointed rocks and rock masses. Only those criteria, whose parameters are easy to obtain in the field, are presented in the subsequent sections.

3.2.1 Based on RMR and Q

Based on extensive in-situ direct shear testing of rock masses in the Himalayas, Mehrotra (1992) has suggested the non-linear variation of shear strength as:

$$\frac{\tau_f}{\sigma_{ci}} = A\left(\frac{\sigma_n}{\sigma_{ci}} + B\right)^C \qquad (10)$$

where A, B and C are empirical constants and depend on RMR or Q. Their values for different moisture contents, RMR and Q index are presented in Table 2.

Table 2: Shear strength parameters for jointed rock masses (after Mehrotra, 1992)
(S- degree of saturation; NMC- natural moisture content; S_{av} - average value of degree of saturation)

Rock type quality	Limestone	Slate, Xenolith, Phyllite	Sandstone, Quartzite	Trap, Metabasic
Good rock mass RMR= 61-80 Q>10	NMC A = 0.38, B = 0.005, C = 0.669 Saturated (S=1) A = 0.35, B = 0.004, C = 0.669	NMC A =0.42, B=0.004, C = 0.683 Saturated (S=1) A=0.38, B=0.003, C = 0.683	NMC A= 0.44, B = 0.003, C=0.695 Saturated (S=1) A=0.43, B=0.002, C=0.695	NMC (S_{av} = 0.30) A = 0.50, B=0.003, C= 0.698 Saturated (S=1) A=0.49,B = 0.002, C = 0.698
Fair rock mass RMR =41-60 Q = 2-10	NMC A=2.60, B = 1.25 , C = 0.662 Saturated (S=1) A = 1.95, B = 1.20, C = 0.662	NMC A = 2.75, B = 1.15 , C=0.675 Saturated (S=1) A = 2.15, B = 1.10, C= 0.675	NMC (S_{av} = 0.15) A = 2.85, B =1.10 , C=0.685 Saturated (S=1) A = 2.25, B = 1.05, C =0.688	NMC (S_{av} = 0.35) A = 3.05, B =1.00 ,C = 0.691 Saturated (S=1) A = 2.45, B=0.95, C = 0.691
Poor rock mass RMR = 21-40 Q = 0.5 – 2	NMC (S_{av} = 0.25) A = 2.50, B = 0.80, C = 0.646 Saturated (S=1) A=1.50, B = 0.75, C = 0.646	NMC (S_{av} = 0.40) A = 2.65, B = 0.75, C =0.655 Saturated (S=1) A = 1.75, B = 0.70, C= 0.655	NMC (S_{av} = 0.25) A = 2.85, B= 0.70, C = 0.672 Saturated (S=1) A = 2.00, B = 0.65, C =0.672	NMC (S_{av} = 0.15) A = 3.00, B = 0.65, C= 0.676 Saturated (S=1) A = 2.25, B = 0.50, C= 0.676
Very poor rock mass RMR<21 Q <0.5	NMC A = 2.25; B = 0.65, C=0.534 Saturated (S=1) A = 0.80, B = 0.0, C = 0.534	NMC A = 2.45; B = 0.60, C=0.539 Saturated (S=1) A = 0.95, B = 0.0, C = 0.539	NMC A = 2.65; B = 0.55, C=0.546 Saturated (S=1) A = 1.05, B = 0.0, C = 0.546	NMC A = 2.90; B = 0.50, C=0.548 Saturated (S=1) A = 1.25, B = 0.0, C = 0.548

3.2.2 *Hoek-Brown Strength Criterion*

The strength criterion was initially proposed for intact rocks by Hoek and Brown (1980). The criterion is expressed as:

$$\sigma_1 = \sigma_3 + \sqrt{m_i \sigma_{ci} \sigma_3 + \sigma_{ci}^2} \quad (11)$$

where σ_1 is the effective major principal stress at failure; σ_3 is the effective minor principal stress at failure; m_i is a criterion parameter; and σ_{ci} is the UCS of the intact rock, which is also treated as a criterion parameter.

A few triaxial tests may be conducted on the intact rock specimens, and the criterion may be fitted into the triaxial test data to obtain σ_{ci} and m_i. Approximate values of parameters m_i can also be obtained from Table 3 (Hoek, 2000), if triaxial test data is not available.

The strength criterion was extended to heavily jointed isotropic rock masses also (Hoek and Brown, 1980). The latest form of the criterion (Hoek et al., 2002) is expressed as:

$$\sigma_1 = \sigma_3 + \sigma_{ci} \left(m_j \frac{\sigma_3}{\sigma_{ci}} + s_j \right)^a \quad (12)$$

Where m_j is an empirical constant, which depends upon the rock type; and s_j is an empirical constant, which varies between 0 (for crushed rock) to 1 (for intact rock) depending upon the degree of fracturing.

$$a = \frac{1}{2} + \frac{1}{6} \left(e^{-GSI/15} - e^{-20/3} \right) \quad (13)$$

where GSI is the Geological Strength Index which depends on the structure of mass and surface characteristics of the discontinuities (Fig. 8).

To obtain parameter m_j and s_j, the use of a classification index, Geological Strength Index (GSI), has been suggested (Hoek and Brown, 1997; Hoek et al., 2002). The expressions for m_j and s_j are given as:

$$m_j = m_i \exp\left(\frac{GSI - 100}{28 - 14D} \right) \quad (14)$$

$$s_j = \exp\left(\frac{GSI - 100}{9 - 3D} \right) \quad (15)$$

Table 3: Approximate estimation of parameter m_i (After Hoek, 2000)
(Note: Values in parenthesis are estimates)

Rock type	Class	Group	Texture: Coarse	Texture: Medium	Texture: Fine	Texture: Very fine
SEDIMENTARY	Clastic		Conglomerate (22)	Sandstone 19	Siltstone 9	Claystone 4
SEDIMENTARY	Clastic			------ Greywacke ------- (18)		
SEDIMENTARY	Non-clastic	Organic		---------- Chalk -------- 7 ---------- Coal --------- (8 – 21)		
SEDIMENTARY	Non-clastic	Carbonate	Breccia (20)	Sparitic Limestone (10)	Micritic Limestone 8	
SEDIMENTARY	Non-clastic	Chemical		Gypstone 16	Anhydrite 13	
METAMORPHIC	Non Foliated		Marble 9	Hornfels (19)	Quartzite (24)	
METAMORPHIC	Slightly Foliated		Migmatite (30)	Amphiboli te (25-31)	Mylonites (6)	
METAMORPHIC	Folaited*		Gneiss 33	Schists 4-8	Phyllites (10)	Slate 9
IGNEOUS	Light		Granite 33		Rhyolyte (16)	Obsidian (19)
IGNEOUS	Light		Granodiorite (30)		Dacite (17)	
IGNEOUS	Dark		Diorite (28)		Andesite 19	
IGNEOUS	Dark		Gabbro 27	Dolerite (19)	Basalt (17)	
IGNEOUS	Dark		Norite 22			
IGNEOUS	Extrusive Pyroclastic type		Agglomerate (20)	Breccia (18)	Tuff (15)	

* These values are for intact rock specimens tested normal to bedding or foliation. The value will be significantly different if failure occurs along a weakness plane.

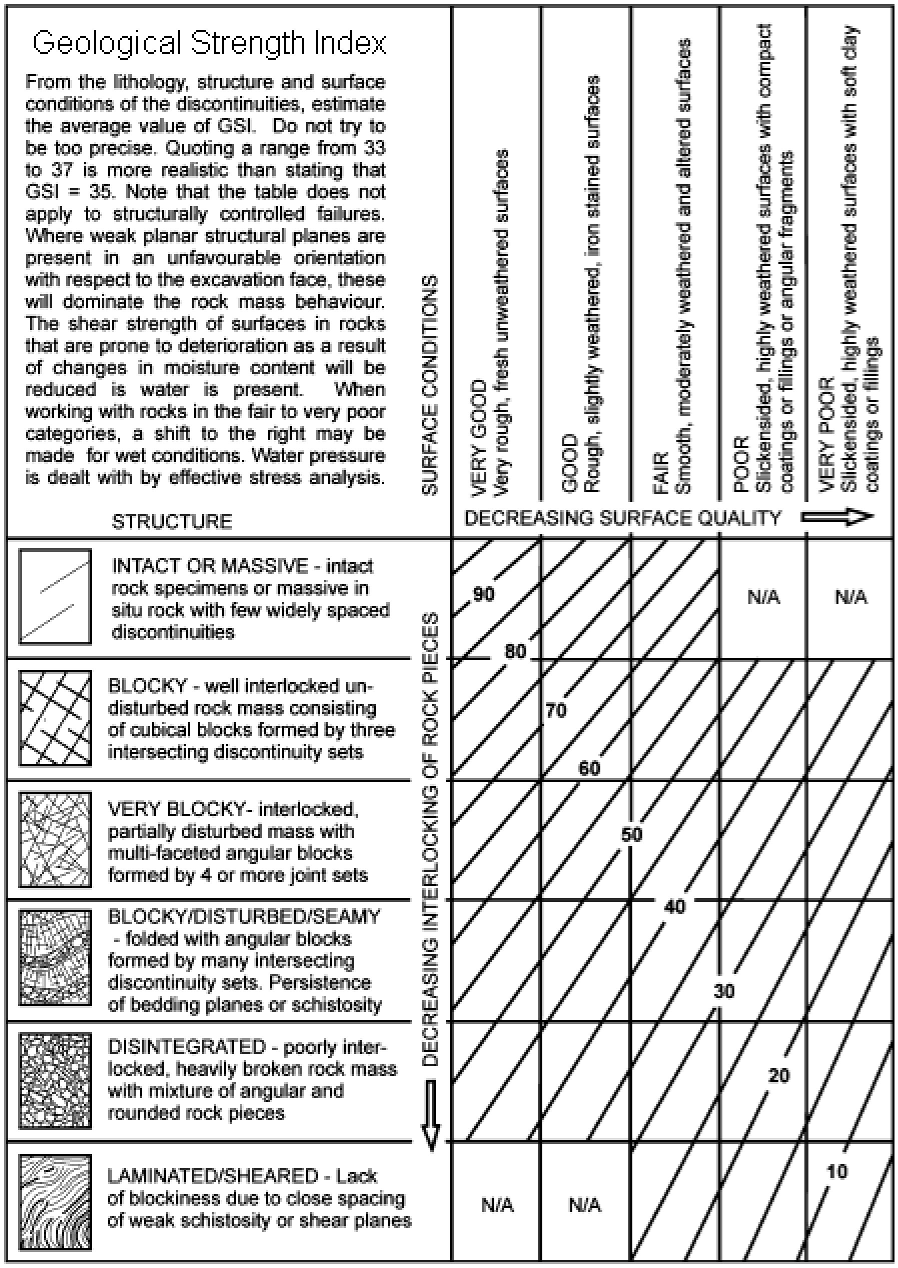

Fig. 8. Estimation of Geological Strength Index (redrawn from Marinos et al., 2005)

where m_i is the Hoek-Brown parameter for intact rock to be obtained from triaxial test data; *D* is a factor which depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation. It varies from zero for undisturbed in situ rock masses to one for very disturbed rock masses. For blasted rock slopes, D is taken in the range 0.7 to 1.0.

The limitation of the GSI approach is that the GSI is estimated only from geological features and disturbance to the mass, and no measurements e.g. joint mapping are done in the field.

3.2.3 Ramamurthy Criterion

Ramamurthy and co-workers (Ramamurthy, 1993; Ramamurthy, 1994; Ramamurthy and Arora, 1994; Ramamurthy, 2014) have suggested the following non-linear strength criterion for intact isotropic rocks:

$$\left(\frac{\sigma_1 - \sigma_3}{\sigma_3 + \sigma_t}\right) = B_i \left(\frac{\sigma_{ci}}{\sigma_3 + \sigma_t}\right)^{\alpha_i} \tag{16}$$

where σ_3 and σ_1 are the minor and major principal stresses at failure; σ_t is the tensile strength of intact rock; σ_{ci} is the UCS of the intact rock; and α_i, B_i are the criterion parameters.

Parameters α_i and B_i should be obtained by fitting the criterion into the laboratory triaxial test data for intact rock. In the absence of triaxial test data, the following approximate correlations may be used:

$$\alpha_i = 2/3; \text{ and } B_i = 1.1\left(\frac{\sigma_{ci}}{\sigma_t}\right)^{1/3} \text{ to } 1.3\left(\frac{\sigma_{ci}}{\sigma_t}\right)^{1/3} \tag{17}$$

For jointed rocks and rock masses, the strength criterion proposed for intact rocks has been extended to jointed rocks as:

$$\left(\frac{\sigma_1 - \sigma_3}{\sigma_3}\right) = B_j \left(\frac{\sigma_{cj}}{\sigma_3}\right)^{\alpha_j} \tag{18}$$

where α_j and B_j are the criterion parameters for jointed rock; and σ_{cj} is the UCS of the jointed rock.

Based on extensive laboratory testing, the following correlations were suggested to obtain criterion parameters α_j and B_j:

$$\frac{\alpha_j}{\alpha_i} = \left(\frac{\sigma_{cj}}{\sigma_{ci}}\right)^{0.5} \tag{19}$$

$$\frac{B_i}{B_j} = 0.13 \exp\left[2.037\left(\frac{\sigma_{cj}}{\sigma_{ci}}\right)^{0.5}\right] \tag{20}$$

where parameters α_i and B_i are obtained from laboratory triaxial tests performed on intact rock specimens. The UCS of the rock mass, $\sigma_{cj,}$ which is popularly known as rock mass strength, is an important input parameter to this strength criterion and has been discussed later.

3.2.4 Modified Mohr Coulomb Criterion

Singh and Singh (2012) have suggested a strength criterion as an extension of Mohr-Coulomb strength criterion. Mohr-Coulomb linear criterion may be expressed in terms of σ_3 and σ_1 as follows.

$$(\sigma_1 - \sigma_3) = \frac{2c_m \cos\phi_m}{1\text{-}\sin\phi_m} + \frac{2\ \sin\phi_m}{1\text{-}\sin\phi_m}\sigma_3 \tag{21}$$

Where, c_m and ϕ_m are Mohr-Coulomb shear strength parameters of the rock mass; the term (σ_1-σ_3) is the deviatoric stress at failure; σ_3 and σ_1 are the minor and major effective principal stresses at failure.

The linear failure criterion was extended to incorporate non-linear strength behaviour and the Modified Mohr-Coulomb (MMC) criterion was expressed as:

$$(\sigma_1 - \sigma_3) = \sigma_{cj} + \frac{2\sin\phi_{m0}}{1 - \sin\phi_{m0}}\sigma_3 - \frac{1}{\sigma_{ci}}\frac{\sin\phi_{m0}}{(1 - \sin\phi_{m0})}\sigma_3^2 \quad \text{for } 0 \le \sigma_3 \le \sigma_{ci} \tag{22}$$

Where σ_{cj} is the UCS of the rock mass; ϕ_{m0} is the friction angle of the rock mass corresponding to very low confining pressure range ($\sigma_3 \rightarrow 0$) and can be obtained as:

$$\sin\phi_{m0} = \frac{(1 - SRF) + \dfrac{\sin\phi_{i0}}{1 - \sin\phi_{i0}}}{(2 - SRF) + \dfrac{\sin\phi_{i0}}{1 - \sin\phi_{i0}}} \tag{23}$$

where $SRF = \text{Strength Reduction Factor} = \sigma_{cj} / \sigma_{ci}$; ϕ_{i0} is friction angle obtained by conducting triaxial strength tests on intact rock specimens at low confining pressures ($\sigma_3 \rightarrow 0$).

If triaxial test data on intact rock is not available, the following non-linear form of the criterion may be used (Singh and Singh, 2004; Singh and Rao, 2005a):

$$\sigma_1 = A\,(\sigma_3)^2 + (1-2A\sigma_{ci})\sigma_3 + \sigma_{cj}; \qquad \sigma_3 \leq \sigma_{ci} \tag{24}$$

Where A is criterion parameter and may be estimated from the experimental value of σ_{ci}, using the following expressions:

For average σ_1 $\qquad A = -1.23\,(\sigma_{ci})^{-0.77}$ (25)

For lower bound σ_1 $\qquad A = -0.43\,(\sigma_{ci})^{-0.72}$ (26)

For design purposes, the lower bound values of σ_1 are recommended to be used. It is, however, suggested that a range of σ_1 values, varying from lower bound to the average, be worked out to observe variation in strength behaviour.

3.3 Rock Mass Strength (σ_{cj})

The UCS of rock mass, σ_{cj} is an important input parameter in the strength criteria. The accuracy of shear strength prediction depends on how precisely σ_{cj} has been estimated. Ramamurthy criterion and parabolic criterion (Singh and Singh, 2004; Singh and Rao, 2005a; Singh and Singh, 2012) consider the rock mass to be anisotropic in strength response and value of σ_{cj} should be obtained in a particular direction. The following methods can be used to determine the UCS of the rock mass:

i) Joint Factor concept, J_f

ii) Rock quality designations, RQD

iii) Rock mass quality, Q

iv) Rock mass rating, RMR

v) Modulus ratio concept (Strength reduction factor)

3.3.1 Joint Factor Concept

Ramamurthy and co-workers (Arora, 1987; Ramamurthy, 1993; Ramamurthy and Arora, 1994; Singh, 1997; Singh et al., 2002) have suggested a weakness coefficient, Joint Factor to characterise effect of fracturing in rocks. The most important properties of joints which affect the rock mass strength are frequency, orientation and shear strength along the joints. The weakness coefficient, Joint Factor was defined by considering the combined effect of frequency, orientation and shear strength of joints as:

$$J_f = \frac{J_n}{n\ r} \tag{27}$$

where, J_n = joint frequency, *i.e.*, number of joints / metre, which take care of RQD and joint sets and joint spacing; n is inclination parameter, depends on the inclination of sliding plane with respect to the major principal stress direction (Table 4); the joint or set which is closer to $(45 - \phi_j/2)^o$ with the major principal stress will be the most critical one to experience sliding at first; r is a parameter for joint strength; it takes care of the influence of closed or filled up joint, thickness of gouge, roughness, extent of weathering of joint walls and cementation along the joint. The joint strength parameter, r is obtained from direct shear tests conducted along the joint surface at low normal stress levels and is given by:

$$r = \frac{\tau_j}{\sigma_{nj}} = \tan\phi_j \tag{28}$$

where τ_j is the shear strength along the joint; σ_{nj} is the normal stress across the joint surface; and ϕ_j is the equivalent value of friction angle incorporating the effect of asperities (Ramamurthy, 2001). The tests should be conducted at very low normal stress levels so that the initial roughness is reflected through this parameter. For cemented joints, the value of ϕ_j includes the effect of cohesion intercept also. In case the direct shear tests are not possible and the joint is tight, a rough estimate of ϕ_j may be obtained from Table 5 (Ramamurthy, 2001). If the joints are filled with gouge material and have reached the residual shear strength, the value of r may be assigned from Table 6 (Ramamurthy, 2001).

The σ_{cj} for jointed rocks has been linked to J_f based on vast experimental data of granite, sandstones and various grades of plaster of Paris, with σ_{ci} varying from 10 MPa to 123 MPa; the number of joints varied from 13 to 92 joints / meter, and friction on the joints varied from 20 to 45 degrees. Joints were developed by cutting, breaking in the desired direction, stepped and berm shaped and with and without gouge materials. The test data in uniaxial compression of jointed specimen of size 38 mm diameter and 76 mm height, 150mm x 150mm x 150mm and 620mm x 620mm x 1200mm were analysed. The strength and moduli of jointed specimens were expressed in non-dimensional form with σ_{ci} and linked to J_f.

Table 4: Joint inclination parameter n, (Ramamurthy, 1993)

Orientation of joint θ°	Inclination parameter n	Orientation of joint θ°	Inclination parameter n
0	1.00	50	0.071
10	0.814	60	0.046
20	0.634	70	0.105
30	0.465	80	0.460
40	0.306	90	0.810
θ = Angle between the normal to the joint plane and major principal stress direction			

Table 5: Values of joint strength parameter, r for different values of σ_{ci} (After Ramamurthy(1993, 2001))

Uniaxial compressive strength of intact rock, σ_{ci} (MPa)	Joint strength parameter, r	Remarks
2.5	0.30	Fine grained micaceous to coarse grained
5.0	0.45	
15.0	0.60	
25.0	0.70	
45.0	0.80	
65.0	0.90	
100.0	1.0	

Table 6: Joint strength parameter, r for filled-up joints at residual stage (After Ramamurthy, (1993, 2001))

Gouge material	Friction angle (ϕ_j)	r = tan ϕ_j
Gravelly sand	45°	1.00
Course sand	40°	0.84
Fine sand	35°	0.70
Silty sand	32°	0.62
Clayey sand	30°	0.58
Clay silt Clay - 25% Clay - 50% Clay – 75%	 25° 15° 10°	 0.47 0.27 0.18

The UCS of the rock mass (rock mass strength) may be obtained as:

$$\sigma_{cj} = \sigma_{ci} \exp(a J_f) \qquad (29)$$

where a is an empirical coefficient equal to -0.008.

Singh (1997) and Singh et al. (2002) suggested that the empirical coefficient would depend on potential failure mode in the mass. The values suggested are presented in Table 7 for different failure modes. The failure mode may be decided as per guideline given below (Singh,1997; Singh and Rao, 2005a). If it is not possible to assess the failure mode, an average value of the empirical constant, 'a' may be taken equal to -0.017.

Table 7: Empirical constant 'a' for estimating σ_{cj}

Failure Mode	Coefficient a
Splitting/Shearing	- 0.0123
Sliding	- 0.0180
Rotation	- 0.0250

Let θ be the angle between the normal to the joint plane and the major principal stress direction:

(i) For θ = 0° to 10°, the failure is likely to occur due to *splitting* of the intact material of blocks.

(ii) For θ = 10° to ≈ 0.8 ϕ_j, the mode of failure shifts from splitting (at θ = 10°) to sliding (at θ ≈ 0.8 ϕ_j).

(iii) For θ = 0.8ϕ_j to 65°, the mode of failure is expected to be sliding only.

(iv) For θ = 65° to 75°, the mode of failure shifts from sliding (at θ = 65°) to rotation of blocks (at θ = 75°).

(v) For θ = 75° to 85°, the mass fails due to rotation of blocks only. Geometry of the blocks is an important parameter in governing the strength behaviour of the mass. In the study conducted, it was assumed that the mass consists of blocks of square section. In case of slender columns, the mass can fail due to buckling if the joints are open. Theory of long columns can be used in this case and this mode was excluded in the study.

(vi) For θ = 85° to 90°, the failure mode shifts from *rotation* at θ = 85° to *shearing* at θ = 90°.

3.3.2 Rock Quality Designation, RQD

If no data is available on orientation of discontinuities or orientation of discontinuities is not important, Rock Quality Designation (Deere, 1963) may be used to assess the UCS of the rock mass. Zhang (2009) has proposed the following correlation for scaling down the intact rock strength to obtain the rock masses strength.

$$\frac{\sigma_{cj}}{\sigma_{ci}} = 10^{(0.013RQD - 1.34)} \tag{30}$$

3.3.3 Rock Mass Quality, Q

An estimate of rock mass strength can also be made by using the rock mass quality index, Q (Barton et al., 1974). Singh et al. (1997) have proposed correlations of rock mass strength, σ_{cj} with Q by analysing block shear tests in the field.

$$\sigma_{cj} = 0.38\gamma Q^{1/3} \text{MPa for slopes} \tag{31}$$

$$\sigma_{cj} = 7\gamma Q^{1/3} \text{ MPa for tunnels} \tag{32}$$

Barton (2002) modified the above equation for tunnels and suggested the expression:

$$\sigma_{cj} = 5\gamma\left(\frac{Q\sigma_{ci}}{100}\right)^{1/3} \text{MPa for tunnels} \tag{33}$$

where γ is the unit weight of rock mass in gm/cm^3; and Q is the Barton’s rock mass quality index.

Ramamurthy (2014) has stated that the Q system was developed for stability of tunnels. It is expected that it’s use for foundations and open excavations may result in overproduction of strength of the rock mass.

3.3.4 Rock Mass Rating, RMR

The shear strength parameters c_m and ϕ_m may be obtained from RMR (Bieniawski, 1973, 1989, 1993) and the rock mass strength σ_{cj} may be obtained as:

$$\sigma_{cj} = \frac{2c_m \cos\phi_m}{1 - \sin\phi_m} \tag{34}$$

It has been observed (Ramamurthy, 2014) that the shear strength parameters recommended by Bieniawski (1973, 1989, 1993) appear to be on lower side resulting in very low values of σ_{cj}. The other commonly used correlations are as follows:

i) Kalamaras and Bieniawski (1993)

$$\frac{\sigma_{cj}}{\sigma_{ci}} = \exp\left(\frac{RMR - 100}{24}\right) \quad (35)$$

ii) Sheorey (1997)

$$\frac{\sigma_{cj}}{\sigma_{ci}} = \exp\left(\frac{RMR - 100}{20}\right) \quad (36)$$

3.3.5 Strength Reduction Factor

Theoretically, the best estimates of rock mass strength, σ_{cj} can only be made in the field through large size field-testing in which the mass may be loaded upto failure to determine rock mass strength. It is, however, extremely difficult, time consuming and expensive to load a large volume of jointed mass in the field upto ultimate failure. Singh (1997), Ramamurthy (2004), Singh and Rao (2005b) have discussed that a better alternative is to get the deformability properties of rock mass by stressing a limited area of the mass upto a certain stress level, and then relate the ultimate strength of the mass to the laboratory UCS of the rock material through a strength reduction factor, SRF. It has been shown by Singh and Rao (2005b) that the Modulus Reduction Factor, MRF and Strength Reduction Factor, SRF are correlated with each other by the following expression approximately:

$$SRF = (MRF)^{0.63} \quad (37)$$

$$\Rightarrow \frac{\sigma_{cj}}{\sigma_{ci}} = \left(\frac{E_j}{E_i}\right)^{0.63} \quad (38)$$

where SRF is the ratio of rock mass strength to the intact rock strength; MRF is . the ratio of rock mass modulus to the intact rock modulus; σ_{cj} is the rock mass strength; σ_{ci} is the intact rock strength; E_j is the elastic modulus of rock mass; and E_i is the intact rock modulus available from laboratory tests and taken equal to the tangent modulus at stress level equal to 50% of the intact rock strength.

It is strongly recommended that field deformability tests should invariably be conducted on project sites. The elastic modulus of rock mass, E_j may be obtained in the field by conducting uniaxial jacking tests (IS:7317, 1974) in drift excavated for the purpose. The test consists of stressing two parallel flat rock faces (usually the roof and invert) of a drift by means of a hydraulic jack (Mehrotra, 1992). The stress is generally applied in two or more cycles as shown in Fig. 9. The second cycle of the stress deformation curve is recommended for computing the field modulus as:

$$E_j = \frac{m(1-\vartheta)^2\, P}{\sqrt{A}\,\delta_e} \tag{39}$$

where E_j is the elastic modulus of the rock mass in kg/cm^2; υ is the Poisson's ratio of the rock mass (= 0.3); P is the load in kg; δ_e is the elastic settlement in cm; A is the area of plate in cm^2; and m is an empirical constant (=0.96 for circular plate of 25mm thickness).

The size of the drift should be sufficiently large as compared to the plate size so that there is little effect of confinement. The confinement may result in over prediction of the modulus values.

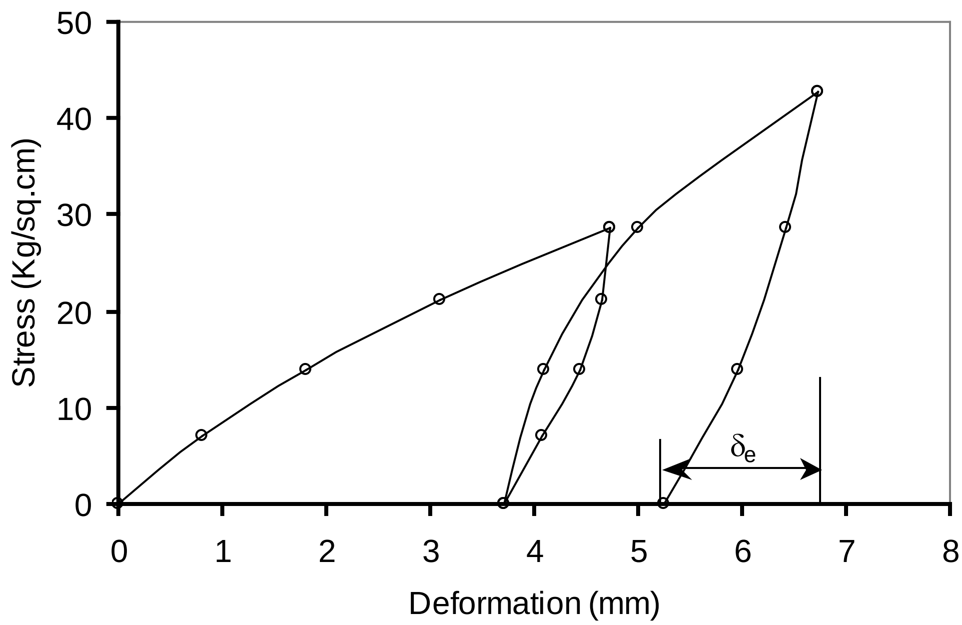

Fig. 9 Field modulus of elasticity.

A number of methods for assessing the rock mass strength, σ_{cj} have been discussed above. It is desirable that more than one method be used for assessing the rock mass strength and generating the failure envelopes. A range of values will thus be obtained and design values may be taken according to experience and confidence of the designer.

4.0 ROCK MASS MODULUS

The strength alone cannot represent the overall quality of a rock mass. Strength and modulus will combinedly give a better understanding of the mass. The best estimate of rock mass modulus can only be made from the results of the field tests. In the absence of such data, deformability characteristics may be estimated approximately from Joint factor concept, rock mass classifications and laboratory test data using following approaches:

4.1 Joint Factor Concept

The concept of Joint Factor (Arora, 1987; Ramamurthy, 1993; Ramamurthy and Arora, 1994) has already been explained in previous sections. The Joint factor may be computed from the field. The following expressions may be used for determination of rock mass modulus:

$$E_j / E_i = \exp[-0.0115\, J_f] \qquad (40)$$

Singh (1997) and Singh et al. (2002) based on tests conducted on blocky mass specimens have suggested the following expressions for different probable failure modes:

$E_j / E_i = \exp[-0.020\, J_f]$ For splitting and shearing failure modes (41)

$E_j / E_i = \exp[-0.035\, J_f]$ For sliding failure mode (42)

$E_j / E_i = \exp[-0.040\, J_f]$ For rotational failure mode (43)

4.2 Modulus Ratio Concept

Deere and Miller (1966) presented a classification of intact rocks based on modulus value (E_i) at 50% of the failure stress and the unconfined compressive strength (σ_{ci}). Vast experimental data of 613 rock specimens from different locations covering 176 igneous, 193 sedimentary, 167 metamorphic and 77 limestone and dolomite specimens, were presented by them classifying intact rocks on the basis of σ_{ci} and modulus ratio, M_{ri}

($= E_i / \sigma_{ci}$). It was observed that for basalts and limestones, one could expect M_{ri} values upto 1600, whereas for shales this value could be close to 60. Even weathered Keuper showed M_{ri} close to 50 (Hobbs, 1974).

Even though the original classification due to Deere & Miller was suggested only for intact rocks, it was modified to classify rock masses as well, Ramamurthy (2004) (Table 8a). The main advantage of such a classification is that it not only takes into account two important engineering properties of rock mass but also gives an assessment of the failure strain (ε_f) which the rock mass is likely to exhibit in uniaxial compression, where in the stress-strain response is nearly linear. The modulus ratio is defined as:

$$M_{rj}=E_{tj}/\sigma_{cj} = 1/\varepsilon_{fj} \tag{44}$$

Further, the ratio of the failure strain of the jointed rock to that of intact rock is given by

$$\varepsilon_{f\,i} / \varepsilon_{f\,j} = M_{r\,j} / M_{r\,i} = \exp(-3.50 \times 10^{-3} J_f) \tag{45}$$

On the basis of experimental data, the following simple expression was suggested for the failure stain of rock mass (Ramamurthy, 2001),

$$\varepsilon_{f\,j} = 50(M_{r\,j})^{-0.75} \text{ per cent.} \tag{46}$$

Modulus ratio classification of intact and jointed rocks is presented in Table 8b. A modulus ratio of 500 would mean a minimum failure strain of 0.2 %, whereas a ratio of 50 corresponds to a minimum failure strain of 2 %. Very soft rocks and dense/compacted soils would show often failure strains of the order of 2%. Therefore, the modulus ratio of 50 was chosen as the lower limiting value for rocks. That is, the Soil-Rock boundary occurs not only when σ_{cj} = 1MPa but also when M_{rj} = 50 and J_f = 300 per meter.

The influence of confining pressure on E_j was estimated as (Ramamurthy,2001)

$$E_{j0} / E_{j3} = 1 - \exp[-0.10\, \sigma_{cj} / \sigma'_3] \tag{47}$$

where, the subscript 0 & 3 refer to $\sigma'_3 = 0$ and $\sigma'_3 > 0$; σ'_3 is the effective confining stress.

Table 8a: Strength classification of intact and jointed rocks (Ramamurthy, 2004)

Class	Description	$\sigma_{ci,j}$ (MPa)
A	Very high strength	> 250
B	High strength	100-250
C	Moderate strength	50-100
D	Medium strength	25-50
E	Low strength	5-25
F	Very low strength	<5

Table 8b: Modulus ratio classification of intact and jointed rocks

Class	Description	Modulus ratio of rock $M_{ri,j}$
A	Very high modulus ratio	> 500
B	High modulus ratio	200-500
C	Medium modulus ratio	100-200
D	Low modulus ratio	50-100
E	Very low modulus ratio	< 50

4.3 From RMR, Q, RQD and GSI

In the absence of field deformability tests, the following correlations may also be used to assess the deformability characteristics of rock mass in the field:

i) Hoek and Diederichs (2006)

$$E_{mass} = E_i\left(0.02 + \frac{1 - D/2}{1 + \exp((60 + 15D - GSI)/11)}\right) \tag{48}$$

$$E_{mass} = 1x10^5\left(\frac{1 - D/2}{1 + \exp((75 + 25D - GSI)/11)}\right) \tag{49}$$

where E_i is the intact rock modulus and D is the damage factor, which varies from zero for undisturbed in situ rock masses to one for very disturbed rock masses.

ii) Serafim and Pereirra (1983)

$$E_{mass} = 10^{(RMR-10)/40}\ \text{GPa} \tag{50}$$

iii) Mehrotra (1992):

$$E_{mass} = 10^{(RMR-25)/40}\ \text{GPa} \tag{51}$$

iv) Zhang (2009)

$$\frac{E_{mass}}{E_i} = 10^{(0.0186\,RQD-1.91)} \tag{52}$$

(To be used in absence of discontinuity orientation data)

vi) Barton (2002)

$$E_j = 10\ Q_c^{1/3},\ \text{GPa}, \tag{53}$$

where , $Q_c = Q\left(\frac{\sigma_{ci}}{100}\right)$; σ_{ci} = UCS of intact rock in MPa. (54)

5.0 WEATHERED ROCKS

Weathering is an inevitable process of nature, gradually alters rock from its original hard state (fresh) to residual (soil) material and as a consequence, changes its engineering behaviour. Most of the rocks encountered are weathered to some extent and it is universally recognized that this process will have affected many of the engineering properties. In common field practice, rock masses are assessed through a classification system. Few rock mass classifications, such as RMR, RSR, Q, and GSI have been popular for last couple of decades. They all involve little consideration of the influence of weathering, especially chemical weathering, in overall performance of rock mass.

For quantification of weathering, an index "Strength Rating" R_s is defined as

$$R_s = \frac{\sigma_{cw}}{\sigma_{cf}} \text{x}100 \tag{55}$$

where σ_{cw} and σ_{cf} are the unconfined compressive strength values of the weathered and corresponding fresh rock respectively. The degree of weathered can be expressed using the R_s index successfully. The following criterion (Rao, 1984, Rao et al.1985) may be used for predicting the triaxial strength of weathered rocks:

$$\frac{\sigma_1 - \sigma_3}{\sigma_3} = B_w\left(\frac{\sigma_c}{\sigma_3}\right)^{\alpha_w} \tag{56}$$

where, B_w and α_w are weathered material constants. Based on the available data and test results, the following equations were suggested for evaluating the material constants:

$$\frac{B_w}{B_i} = \exp\left[\frac{R_w - 100}{30}\right] \quad (57)$$

$$\frac{\alpha_w}{\alpha_i} = \exp\left[\frac{R_w - 100}{140}\right] \quad (58)$$

Where R_w is rating through weathering classification.

In-situ deformability can be estimated using the following relationship:

$$E_t\,(\text{In} - \text{situ}) = \exp\left[\frac{R_w - 27}{16}\right] \text{GPa} \quad (59)$$

Further details can be obtained from Gupta and Rao (1998, 2000, 2001).

6.0 CONCLUDING REMARKS

The stability of a structure in rock mass is governed by the characteristics of discontinuities and rock material present in the mass. Adequate understanding of strength behaviour of the rock mass subject to given confinement is essential for analysis and design of the structure. The potential failure surface may occur along a dominating persistent discontinuity or through blocks of the rock mass. Accordingly, discontinuity shear strength or rock mass strength will be mobilised. Various approaches available for obtaining the shear strength of an individual discontinuity or of a mass as a whole have been discussed. Special emphasis has been given to non-linear strength behaviour and only those approaches have been presented for which the parameters are easily obtainable in the field. A small discussion has also been given on obtaining the deformability characteristics of the rock mass. Effect of weathering on strength response has also been discussed. It is expected that shear strength values obtained from different approaches may vary over a range. It is suggested that this range of values, rather than a single value of shear strength should be used to solve a real life problem in the field. Parametric analysis should be done to examine the behaviour of the structure for the range of values to gain more confidence in the design.

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